Lec 16: Sorting

Sam McCauley

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Admin

- Lab today; remember that you need to do the lab in-person during t
- Quiz Friday: Sorting and big-*O* notation
  - I will ask you to prove the big-*O* bound for an algorithm
  - Be sure to know the definition, and how to apply it
- Anything else?

Sorting

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  - Code it up in Java
  - Analyze the running time

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Maximum so far:

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Maximum so far: 10 at pos 0

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Maximum so far: 21 at pos 1

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- (This is essentially a recursive algorithm)

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- Let’s assume we have a `swap(int[], int, int)` method that swaps two indices of an array

```
1  public void swap(int[] arr, int index1, int index2) {
2      int temp = arr[index1];
3      arr[index1] = arr[index2];
4      arr[index2] = temp;
5  }
```

Selection Sort Code

```
1  public static void selectionSort(int data[]) {
2      int numUnsorted = data.length;
3      int index; // general index
4      int max; // index of largest value
5      while (numUnsorted > 0) {
6          // determine maximum value in array
7          max = 0;
8          for (index = 1; index < numUnsorted; index++) {
9              if (data[max] < data[index]) {
10                 max = index;
11             }
12         }
13         swap(data, max, numUnsorted-1);
14         numUnsorted--;
15     }
16 }
```

Selection Sort Code: Method Version

```
1  public static int findMax(int data[], int maxIndex) {
2      int max = 0;
3      for (int index = 1; index < maxIndex; index++) {
4          if (data[max] < data[index]) {
5              max = index;
6          }
7      }
8      return max;
9  }
10
11 public static void selectionSort(int data[]) {
12     int numUnsorted = data.length;
13     while (numUnsorted > 0) {
14         int max = findMax(data, numUnsorted);
15         swap(data, max, numUnsorted-1);
16         numUnsorted--;
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- Idea: after the loop iterates *i* times,
  - The last *i* slots of the array contain the *i* largest elements of the array in sorted order

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- Idea: after the loop iterates i times,
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- When $i = n$ we are done

Using Generics for Selection Sort

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- Can we use the power of interfaces to write code that will work for a wide variety of objects?
- What method(s) do we need the objects to have to use selection sort?

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All we do with `data` is see if `data[max] < data[index]`.

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- Let's look at how the selection sort code changes to use a comparable object
- `Integer` already implements `Comparable<Integer>` so we can already sort `Integers`; same with `Strings`

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- Really: want to make one method generic. Can we do this in Java?
- Yes! Looks something like this:
  - `public static <E> void SelectionSort(ArrayList<E> list)`
- Problem: can't sort objects of *any* class E. Needs to be comparable with other objects of class E

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  - (Yes, it's `extends` and not `implements`. There are some good back-end reasons for this.)
- What do we want for our `selectionSort` method?
  - Want `<E extends Comparable<E>>`
  - That is to say: we want a type `E` that implements `Comparable<E>`. That is to say: need that objects of type `E` have a `compareTo` method that takes objects of type `E` as argument

Generic Selection Sort Code

```
1  public static <E extends Comparable<E> > void selectionSort(
       ArrayList<E> data) {
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3      int index; // general index
4      int max; // index of largest value
5      while (numUnsorted > 0) {
6          // determine maximum value in array
7          max = 0;
8          for (index = 1; index < numUnsorted; index++) {
9              if (data.get(max).compareTo(data.get(index)) < 0)
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- What are the downsides of this?
    - What if we want to sort objects that aren't already comparable and we don't want to modify the class?
    - Can only sort objects one way. (What if we want to sort `Employees` by age rather than name? Would need to rewrite the `Employee` class!)

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- In general: `Comparator<T>` has a `int compare(T, T)` method
  - `compare` returns < 0 if first is smaller; 0 if equal; > 0 if second is smaller
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  - Need to `import java.util.Comparator`

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- Goal 2: sort employees by *name*

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- Let’s say we want a sort method that uses a comparator, rather than a comparable object
- (Our sort should work for *any* comparator)
- Idea: take a comparator object as an argument. Then we can use its compare method to compare the objects!
- Let’s write a selection sort method that uses a comparator. And, let’s write two comparators for employees: one that compares by name; another that compares by age.

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- Comparator:
  - Need to write a new class to compare two objects
  - But: don’t need to modify the original class
  - Can be used to compare objects of a class multiple different ways

Merge Sort

Merge Sort

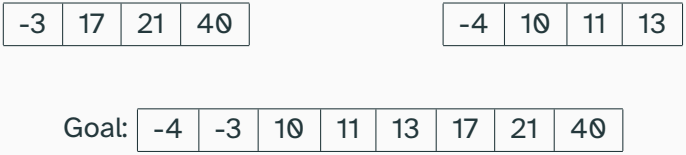
- We can sort faster than $O(n^2)$!
    - Can sort in $O(n \log n)$ time
- Merge sort is a classic way to do that
- Very fast in practice; used as the basis of many state of the art sorting algorithms

Sorting Recursively?

- Can we use recursion to sort?
- Base case?
  - One-element array is always sorted.
- How can we create a smaller subproblem?
  - Could do one element smaller, but that gives us $O(n^2)$ (like in selection sort)
  - Can we divide the size by 2 like in Binary Search?

Merge Sort Hint

- Let's say I have two sorted arrays
- How fast can I sort the concatenation of these arrays?



- Hint: where does the *smallest* element between the two arrays reside?
- Repeatedly take the smaller of the first remaining element in the two arrays. Takes $O(n)$ time

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- If array has size 1, just return
- Split array into two halves
- Sort each half
  - How?
  - Using Merge Sort!
- Then merge the resulting arrays together in $O(n)$ time by walking through them

Merge Sort PseudoCode

```
1  mergeSortRecursive(list):
2      if list has length 1
3          return
4
5      list firstHalf =  first half of list
6      mergeSortRecursive(firstHalf);
7
8      list secondHalf = second half of list
9      mergeSortRecursive(secondHalf);
10
11     merge firstHalf and secondHalf into list
```

Merge PseudoCode

```
merge(list, firstHalf, secondHalf):
    firstIndex = 0
    secondIndex = 0
    for each slot in list:
        if firstHalf[firstIndex] < secondHalf[secondIndex]:
            list[slot] = firstHalf[firstIndex]
            firstIndex++
        else:
            list[slot] = secondHalf[secondIndex]
            secondIndex++
```

This pseudocode has an out-of-bounds error. Let's fix it!

Merge PseudoCode: Option 1

```
merge(list, firstHalf, secondHalf):
    firstIndex = 0
    secondIndex = 0
    for each slot in list:
        if secondIndex >= secondHalf.length or (firstIndex <
            firstHalf.length and firstHalf[firstIndex] <
           secondHalf[secondIndex]):
            list[slot] = firstHalf[firstIndex]
            firstIndex++
        else:
            list[slot] = secondHalf[secondIndex]
            secondIndex++
```

The extra checks, with short-circuit evaluation, mean that we no longer have out-of-bounds issues.

Merge PseudoCode: Option 2

```
merge(list, firstHalf, secondHalf):
    firstIndex = 0
    secondIndex = 0
    for each slot in list:
        if secondIndex >= secondHalf.length:
            list[slot] = firstHalf[firstIndex]
            firstIndex++
        else if firstIndex >= firstHalf.length:
            list[slot] = secondHalf[secondIndex]
            secondIndex++
        else if firstHalf[firstIndex] < secondHalf[
            secondIndex]):
            list[slot] = firstHalf[firstIndex]
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        else:
            list[slot] = secondHalf[secondIndex]
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```

This strategy has a little redundancy, but it's easier to read how the out-of-bounds

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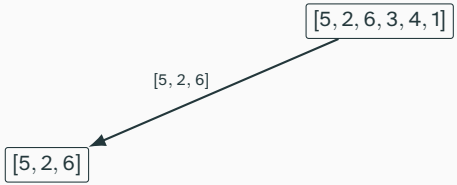

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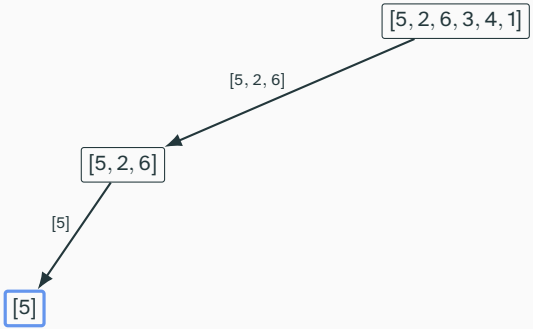

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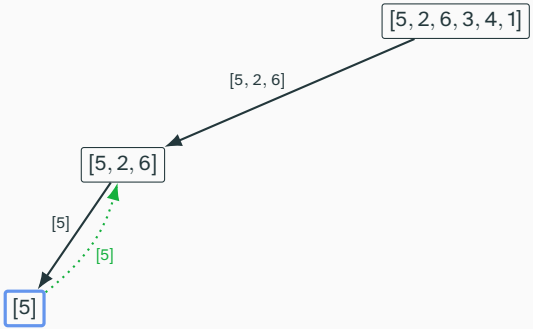

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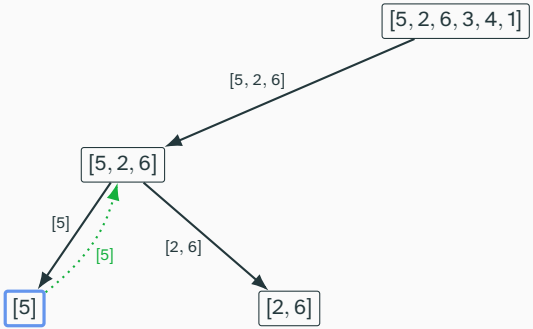

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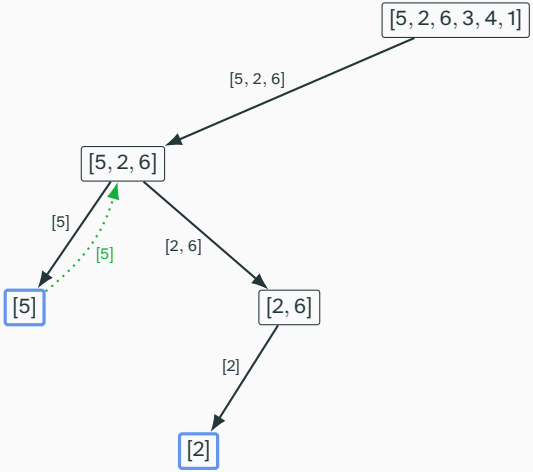

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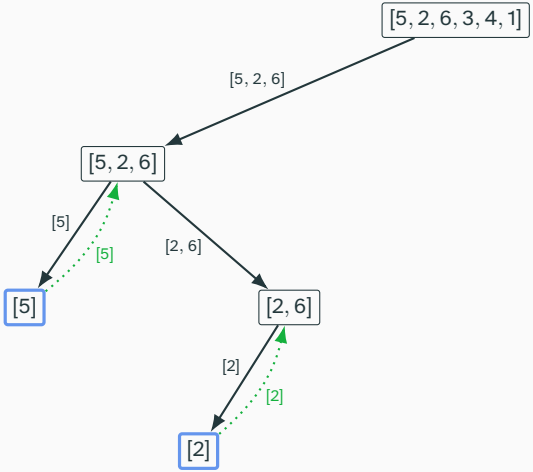
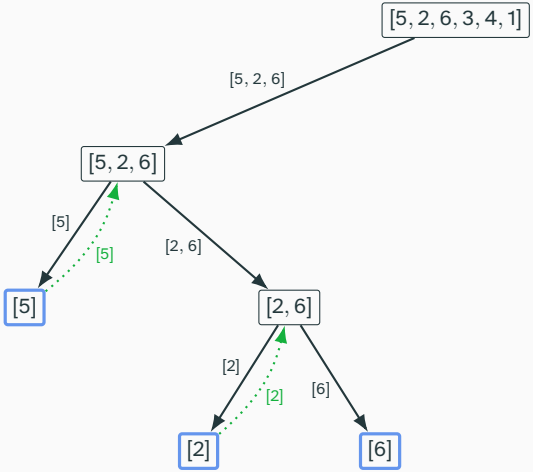


Merge Sort



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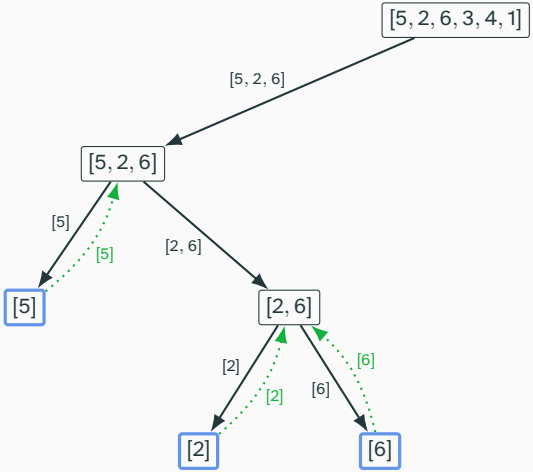

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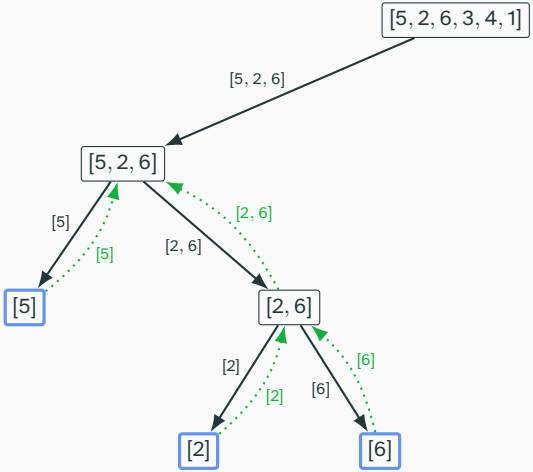
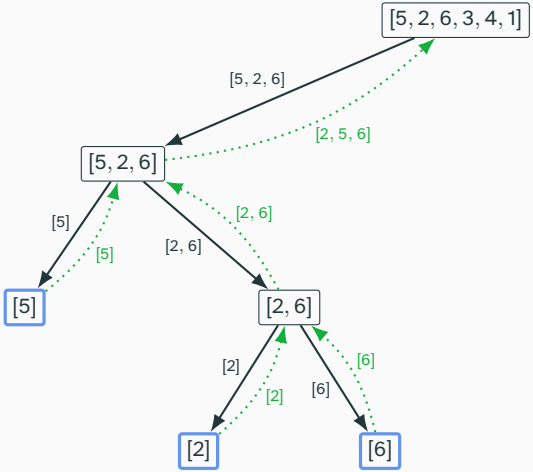
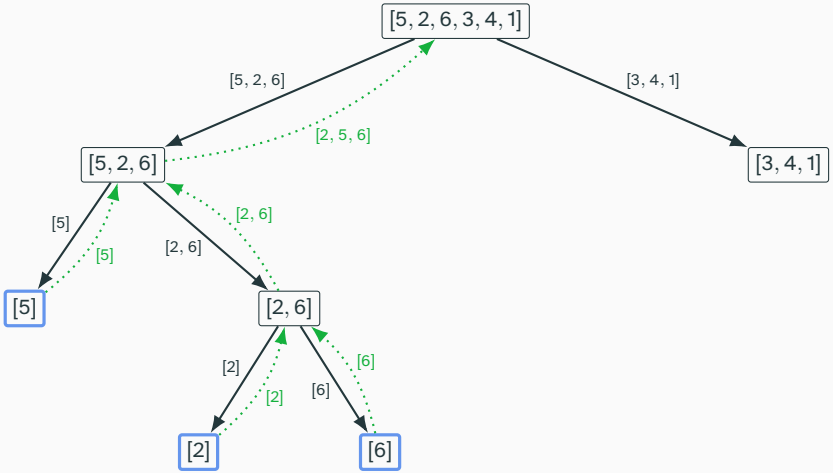


Merge Sort



Merge Sort



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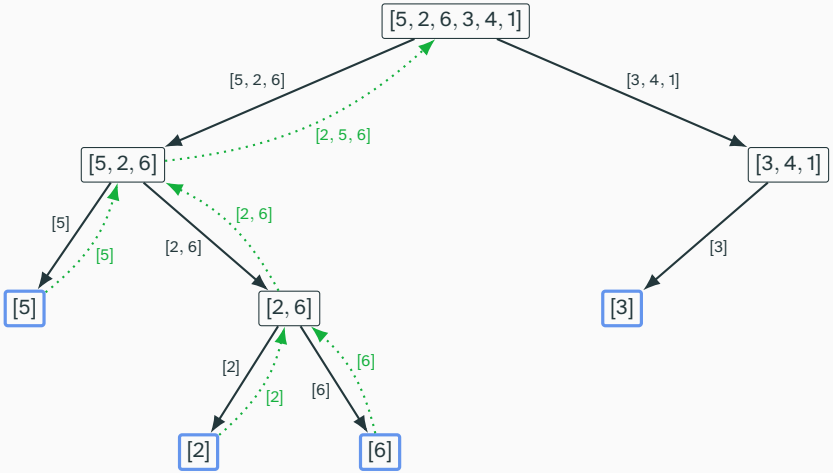

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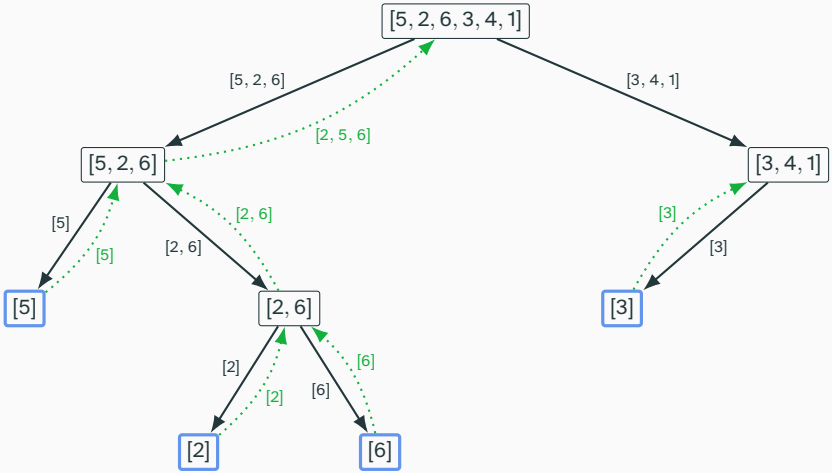

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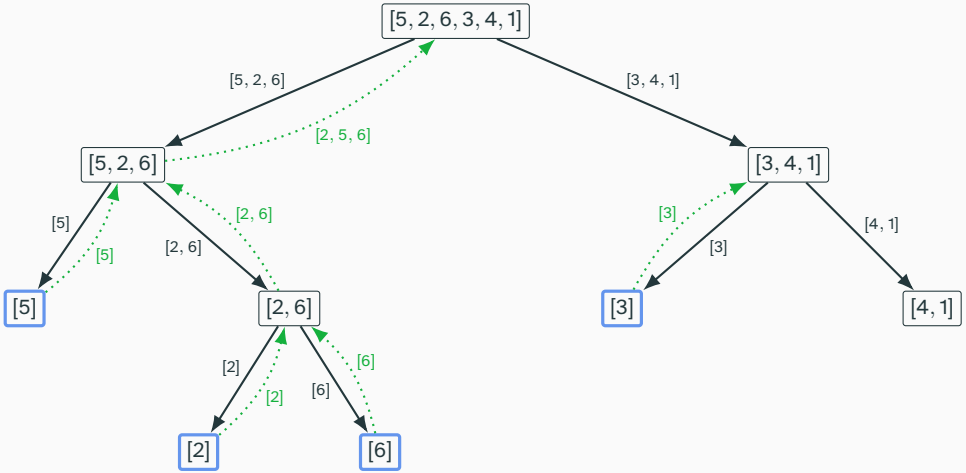

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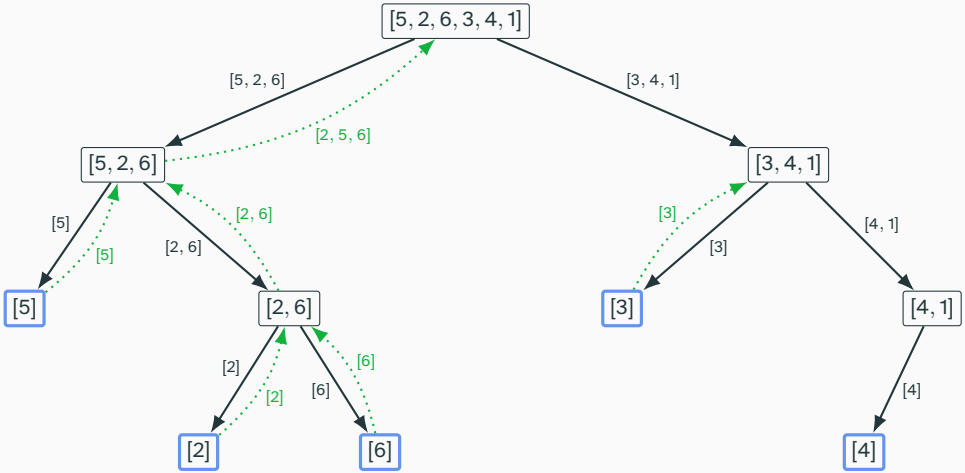

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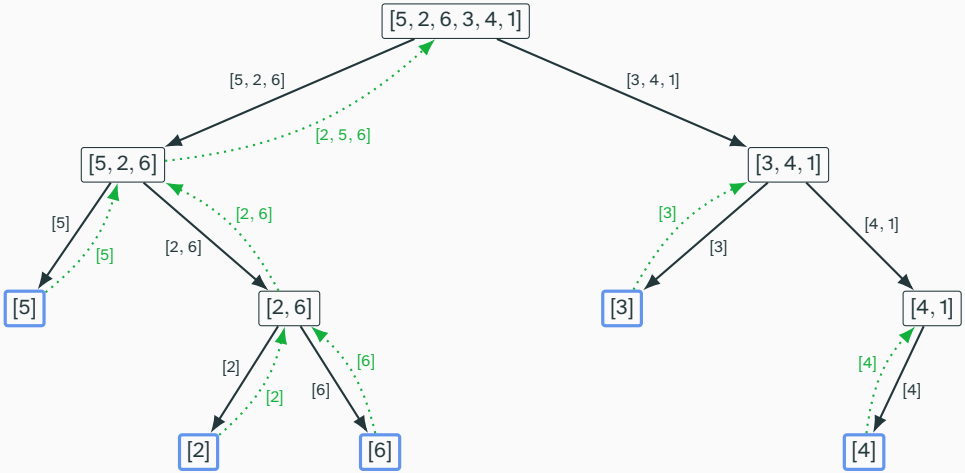

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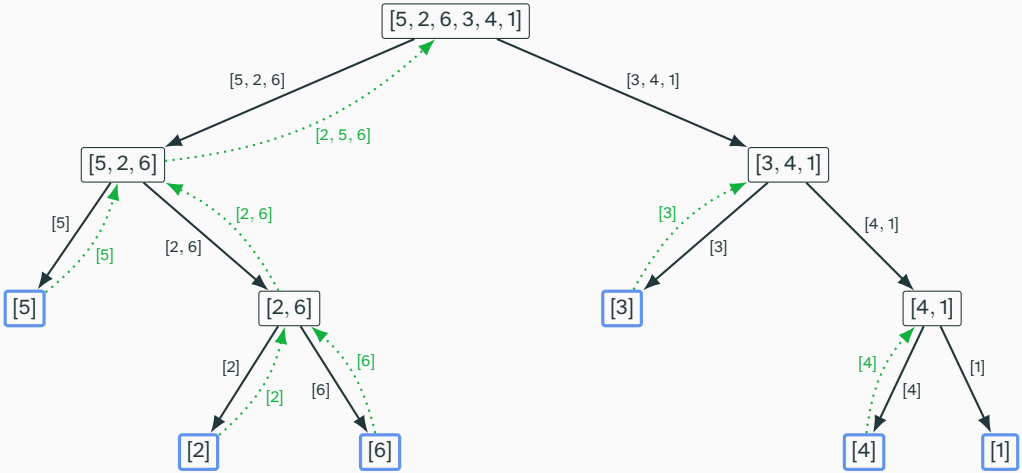

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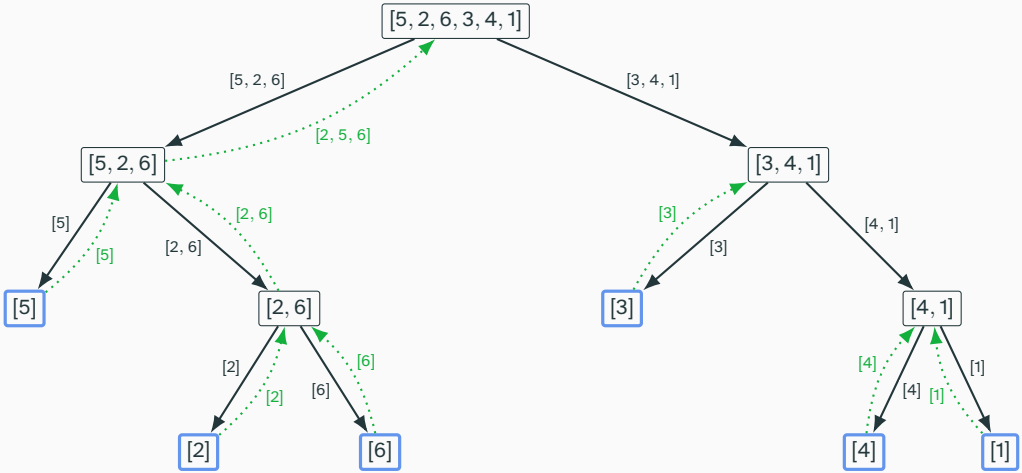
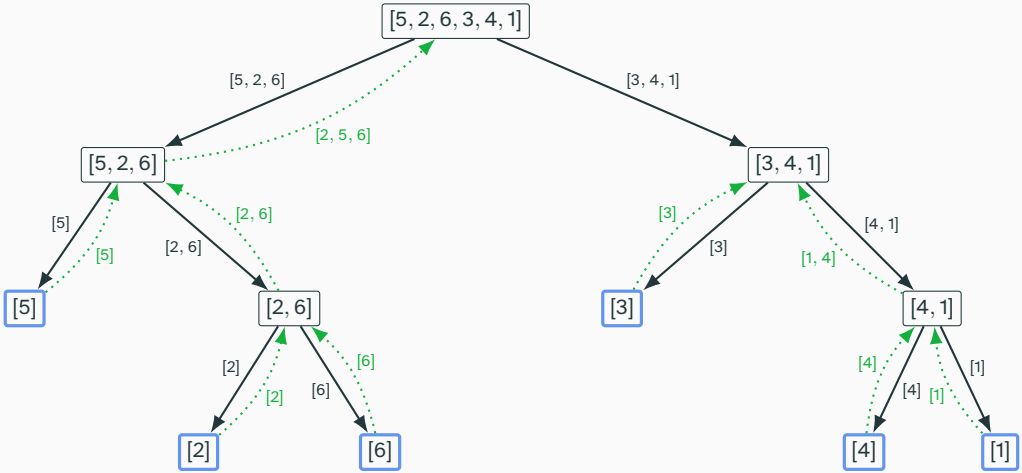


Merge Sort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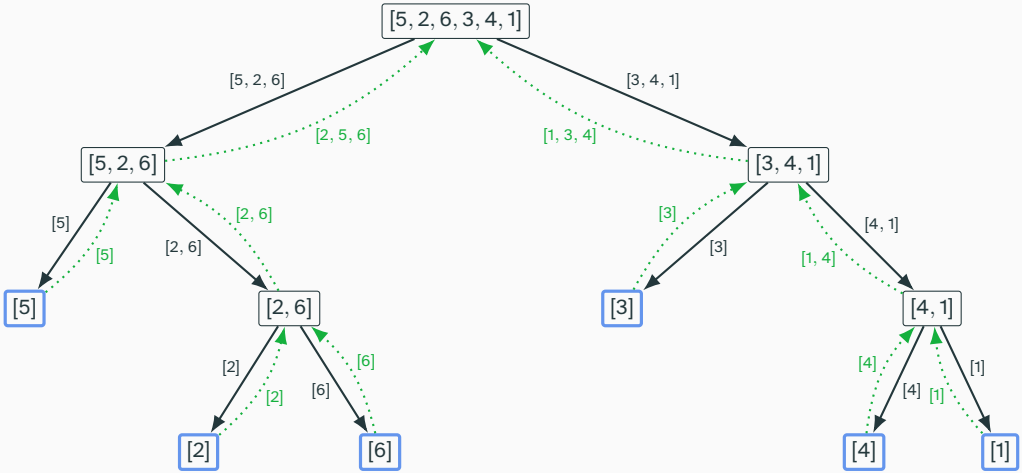
[5, 2, 6]
[3, 4, 1]
[2, 5, 6]
[5, 2, 6]
[3, 4, 1]
[5]
[5]
[2, 6]
[2, 6]
[3]
[3]
[1, 4]
[4, 1]
[5]
[2, 6]
[3]
[4, 1]
[2]
[2]
[6]
[6]
[4]
[4]
[1]
[1]
[2]
[6]
[4]
[1]

Merge Sort


[5, 2, 6, 3, 4, 1]
[5, 2, 6]
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[5, 2, 6]
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[5]
[5]
[2, 6]
[2, 6]
[3]
[3]
[4, 1]
[1, 4]
[5]
[2, 6]
[3]
[4, 1]
[2]
[2]
[6]
[6]
[4]
[4]
[1]
[1]
[2]
[6]
[4]
[1]

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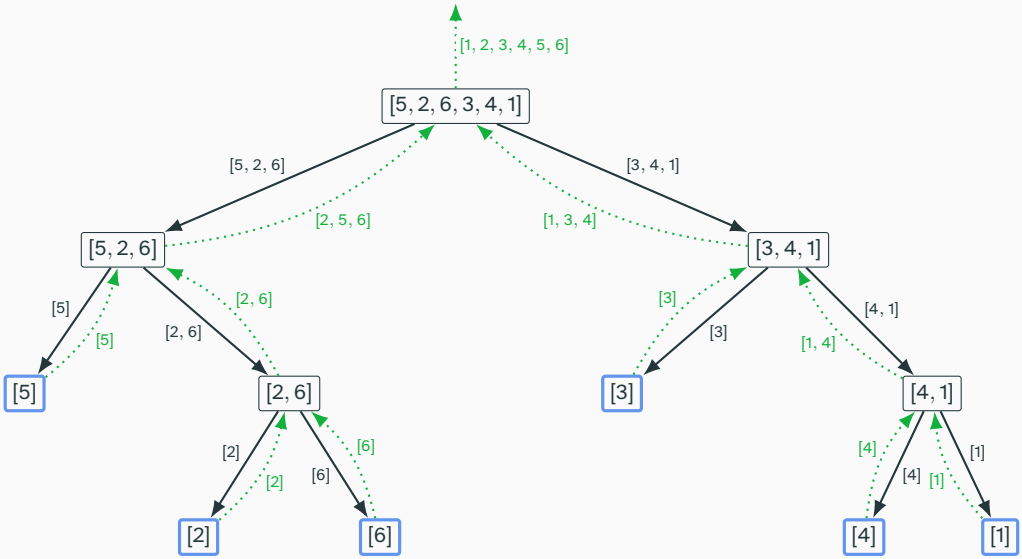
[1, 2, 3, 4, 5, 6]
[5, 2, 6, 3, 4, 1]
[5, 2, 6]
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[5]
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- *All work* is done during copying: either creating `firstHalf` and `secondHalf`, or merging the two halves into `list`

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- “Splitting” part of the algorithm just recurses down to arrays of size 1

Merge Sort Running Time

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1  merge(list, firstHalf, secondHalf):
2      firstHalfIndex = 0
3      secondHalfIndex = 0
4      for each slot in list:
5          if secondIndex >= secondHalf.length or (firstIndex <
               firstHalf.length and firstHalf[firstIndex] <
              secondHalf[secondIndex]):
6              list[slot] = firstHalf[firstIndex]
7              firstIndex++
8          else:
9              list[slot] = secondHalf[secondIndex]
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How much time does a merge take for a `list` of length n?

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- $O(n)$ time outside of the merge.
- $O(n)$ time for the merge.
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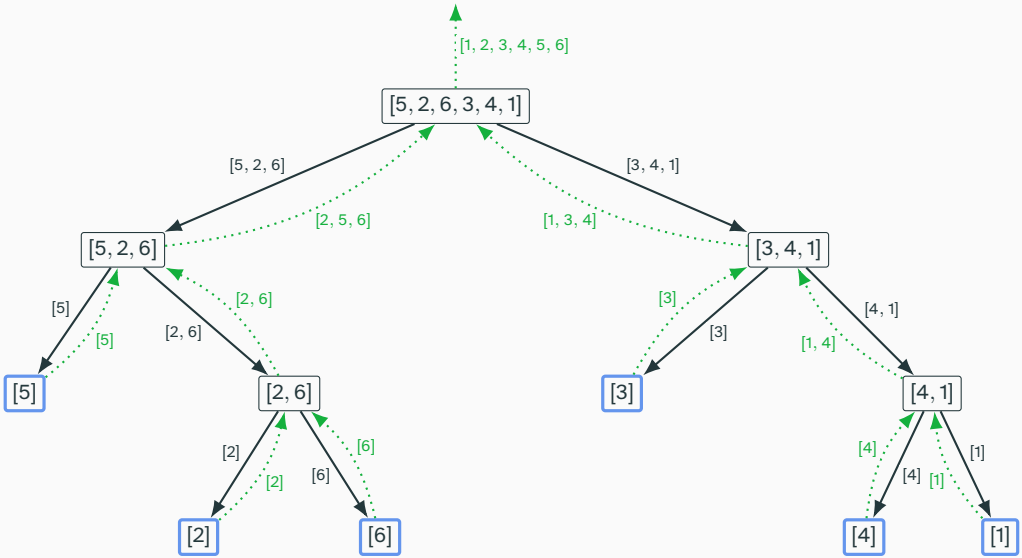
- Total: $O(n \log n)$ time

Merge Sort Analysis



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- This time we'll start with why it works, and derive the algorithm

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-3	10	21	40	17	13	11	-4

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- Let's say it works for k. What does the $k + 1$st loop need to accomplish to maintain the invariant?

  - Needs to insert the $k + 1$st item among the first k items in sorted order.

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- Inner loop: store element we are trying to insert. Shift elements down while it is smaller.

Insertion Sort Code

```java
public static void insertionSort(int data[]) {
    int numSorted = 1; // number of values in place
    int index; // general index
    while (numSorted < data.length) {
        int temp = data[numSorted]; // first unsorted value
        for (index = numSorted; index > 0; index--) {
            if (temp < data[index-1]) {
                data[index] = data[index-1];
            } else {
                break;
            }
        }
        data[index] = temp; // reinsert value
        numSorted++;
    }
}
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7
8
9              } else {
10                 break;
11             }
12         }
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```

Can we get rid of the `break` command in this code?

Insertion Sort Code # 2

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3      while (numSorted < data.length) {
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5          int index = numSorted;
6          while(index > 0 && temp < data[index - 1]) {
7              data[index] = data[index-1];
8              index--;
9          }
10         data[index] = temp; // reinsert value
11         numSorted++;
12     }
13 }
```

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- To be clear: both are still $O(n^2)$ in terms of worst-case performance. Insertion sort just has better constants, and better best-case performance

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  - Tradeoff point n somewhere in the range 32–256
- Bears out in practice: insertion sort is better on small lists
- A well-implemented Merge Sort switches to Insertion Sort as a base case