CSCI 136
Data Structures & Advanced Programming

Traversing Trees using Iterators

Designing Tree Iterators

- Goal: design iterators to dispense items in the same order that the different tree traversal algorithms visit nodes.

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- Methods provided by `BinaryTree` class:

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- **Goal**: design iterators to dispense items in the same order that the different tree traversal algorithms visit nodes.

- Methods provided by `BinaryTree` class:
  - `preorderIterator()`
  - `inorderIterator()`
  - `postorderIterator()`
  - `levelorderIterator()`

Implementing the Iterators

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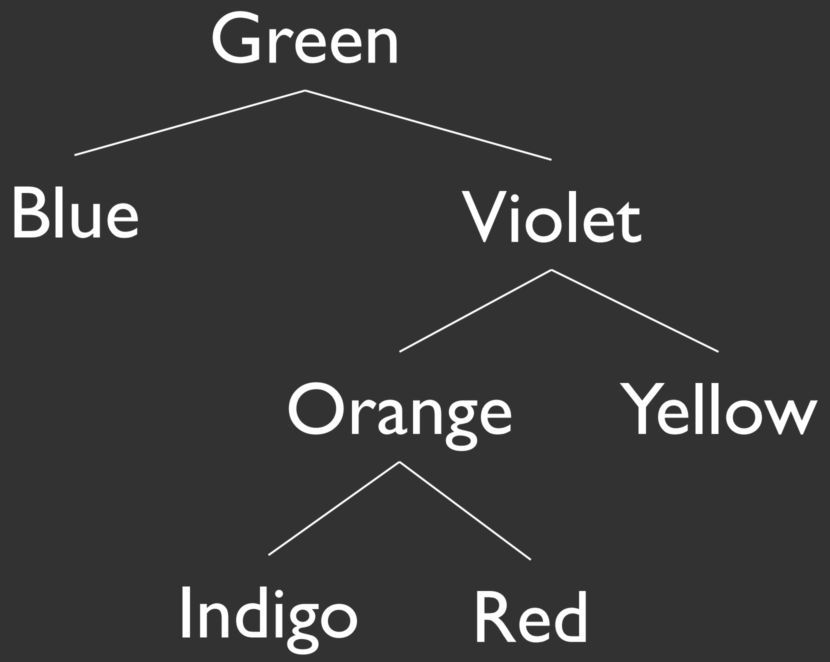
- Iterators should dispense values in same order as the corresponding traversal method

Implementing the Iterators

- Iterators should dispense values in same order as the corresponding traversal method
- Challenge: We must phrase algorithm steps in terms of `next()` and `hasNext()`
  - Recursive methods don't convert as easily, so, let's start with the most "straightforward" traversal order: level-order!

Level-Order Traversal



G B V O Y I R

Level-Order Iterator

Level-Order Iterator

- Should return elements in same order as processed by level-order traversal method
  - Visit all nodes at depth *i* before visiting any node at depth *i+1*

Level-Order Iterator

- Should return elements in same order as processed by level-order traversal method
  - Visit all nodes at depth *i* before visiting any node at depth *i+1*
- Must phrase in terms of `next()` and `hasNext()`
  - Basic Idea: We "capture" our traversal in a queue
    - The queue holds "to be visited" nodes

Level-Order Iterator

```
public BTLevelorderIterator(BinaryTree<E> root) {



}

public void reset() {




}
```

Level-Order Iterator

```
public BTLevelorderIterator(BinaryTree<E> root) {
        todo = new QueueList<BinaryTree<E>>();
        this.root = root; // needed for reset
        reset();
}

public void reset() {




}
```

Level-Order Iterator

```
public BTLevelorderIterator(BinaryTree<E> root) {
      todo = new QueueList<BinaryTree<E>>();
      this.root = root; // needed for reset
      reset();
}

public void reset() {
      todo.clear();

      // empty queue, add root
      if (!root.isEmpty()) todo.enqueue(root);
}
```

Level-Order Iterator

```
public boolean hasNext() {

}

public E next() {







}
```

Level-Order Iterator

```
public boolean hasNext() {
       return !todo.isEmpty();
}

public E next() {








}
```

Level-Order Iterator

```
public boolean hasNext() {
       return !todo.isEmpty();
}

public E next() {
       BinaryTree<E> current = todo.dequeue();
       E result = current.value();





}
```

Level-Order Iterator

```
public boolean hasNext() {
       return !todo.isEmpty();
}

public E next() {
       BinaryTree<E> current = todo.dequeue();
       E result = current.value();
       if (!current.left().isEmpty())
           todo.enqueue(current.left());




}
```

Level-Order Iterator

```
public boolean hasNext() {
       return !todo.isEmpty();
}

public E next() {
       BinaryTree<E> current = todo.dequeue();
       E result = current.value();
       if (!current.left().isEmpty())
           todo.enqueue(current.left());
       if (!current.right().isEmpty())
           todo.enqueue(current.right());

}
```

Level-Order Iterator

```
public boolean hasNext() {
       return !todo.isEmpty();
}

public E next() {
       BinaryTree<E> current = todo.dequeue();
       E result = current.value();
       if (!current.left().isEmpty())
           todo.enqueue(current.left());
       if (!current.right().isEmpty())
           todo.enqueue(current.right());
       return result;
}
```

Pre-Order Iterator

Pre-Order Iterator

- Should return elements in same order as processed by pre-order traversal method:
  - Visit node, then left subtree, then right subtree

Pre-Order Iterator

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- Must phrase in terms of `next()` and `hasNext()`

Pre-Order Iterator

- Should return elements in same order as processed by pre-order traversal method:
  - Visit node, then left subtree, then right subtree
- Must phrase in terms of `next()` and `hasNext()`

- Basic idea: We "simulate recursion" with stack
  - The stack holds "partially processed" nodes

Pre-Order Iterator

Pre-Order Iterator

- Order: node -> left subtree -> right subtree

Pre-Order Iterator

- Order: node -> left subtree -> right subtree
    1. Constructor: Push root onto TODO stack

Pre-Order Iterator

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    2. On call to `next()`:

Pre-Order Iterator

- Order: node -> left subtree -> right subtree
  1. Constructor: Push root onto TODO stack
  2. On call to `next()`:
     - Pop node from TODO stack

Pre-Order Iterator

- Order: node -> left subtree -> right subtree
  1. Constructor: Push root onto TODO stack
  2. On call to `next()`:
     - Pop node from TODO stack
     - Push right and then left nodes of popped node onto TODO stack

Pre-Order Iterator

- Order: node -> left subtree -> right subtree
  1. Constructor: Push root onto TODO stack
  2. On call to `next()`:
     - Pop node from TODO stack
     - Push right and then left nodes of popped node onto TODO stack
     - Return popped node's value

Pre-Order Iterator

- Order: node -> left subtree -> right subtree
  1. Constructor: Push root onto TODO stack
  2. On call to `next()`:
     - Pop node from TODO stack
     - Push right and then left nodes of popped node onto TODO stack
     - Return popped node's value
  3. On call to `hasNext()`:

Pre-Order Iterator

- Order: node -> left subtree -> right subtree
  1. Constructor: Push root onto TODO stack
  2. On call to `next()`:
     - Pop node from TODO stack
     - Push right and then left nodes of popped node onto TODO stack
     - Return popped node's value
  3. On call to `hasNext()`:
     - return `!stack.isEmpty()`

Pre-Order Iterator

Pre-Order Iterator

Visit node, then each node in left subtree, then each node in right subtree.

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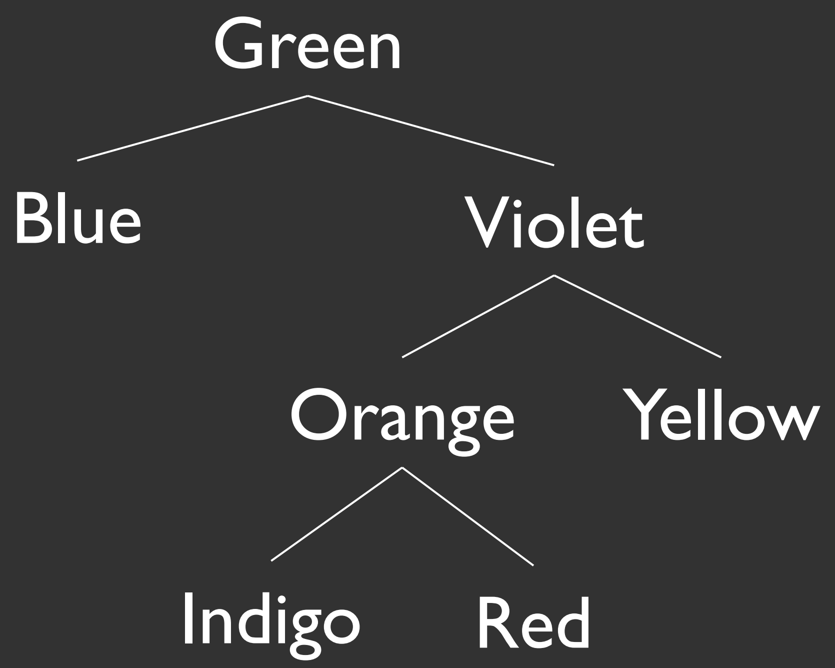
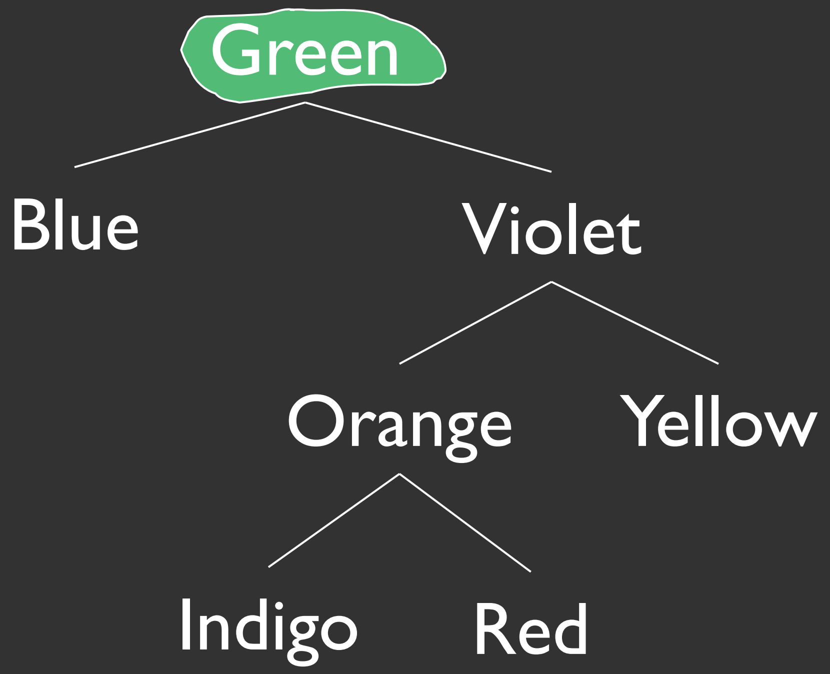


Pre-Order Iterator

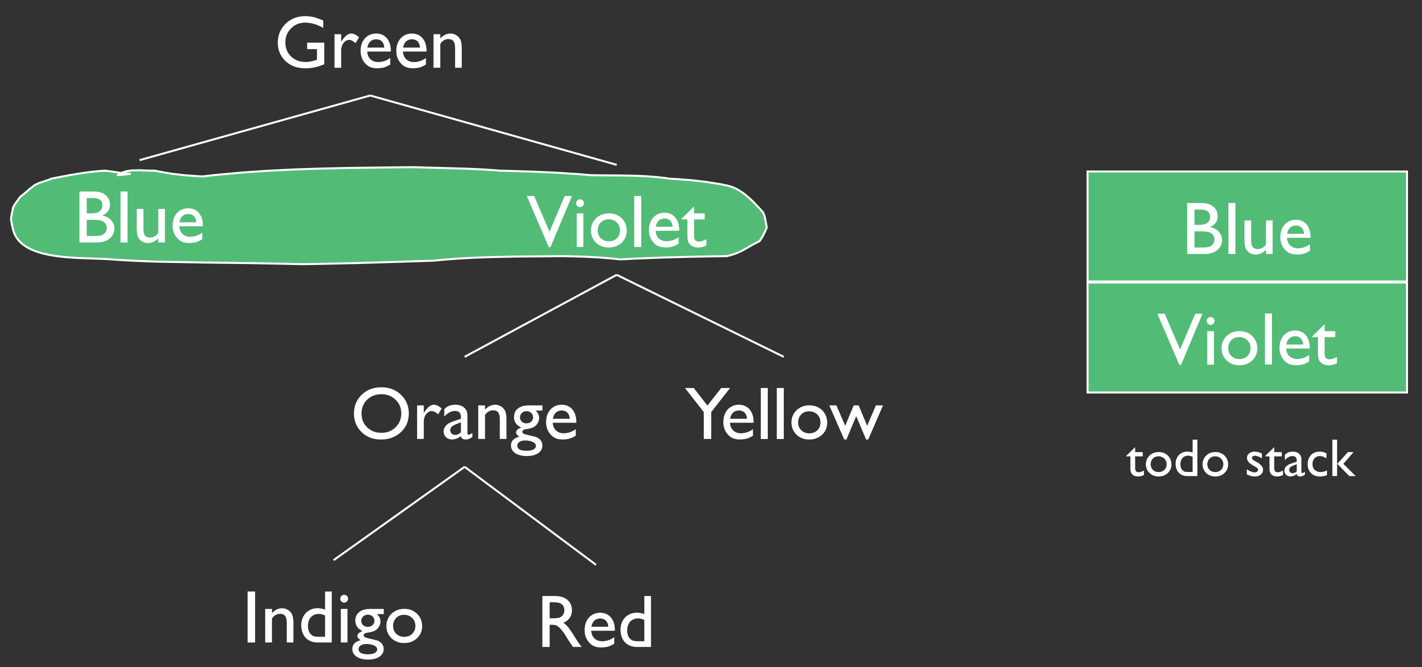
Visit node, then each node in left subtree, then each node in right subtree.



todo stack

Pre-Order Iterator

Visit node, then each node in left subtree, then each node in right subtree.



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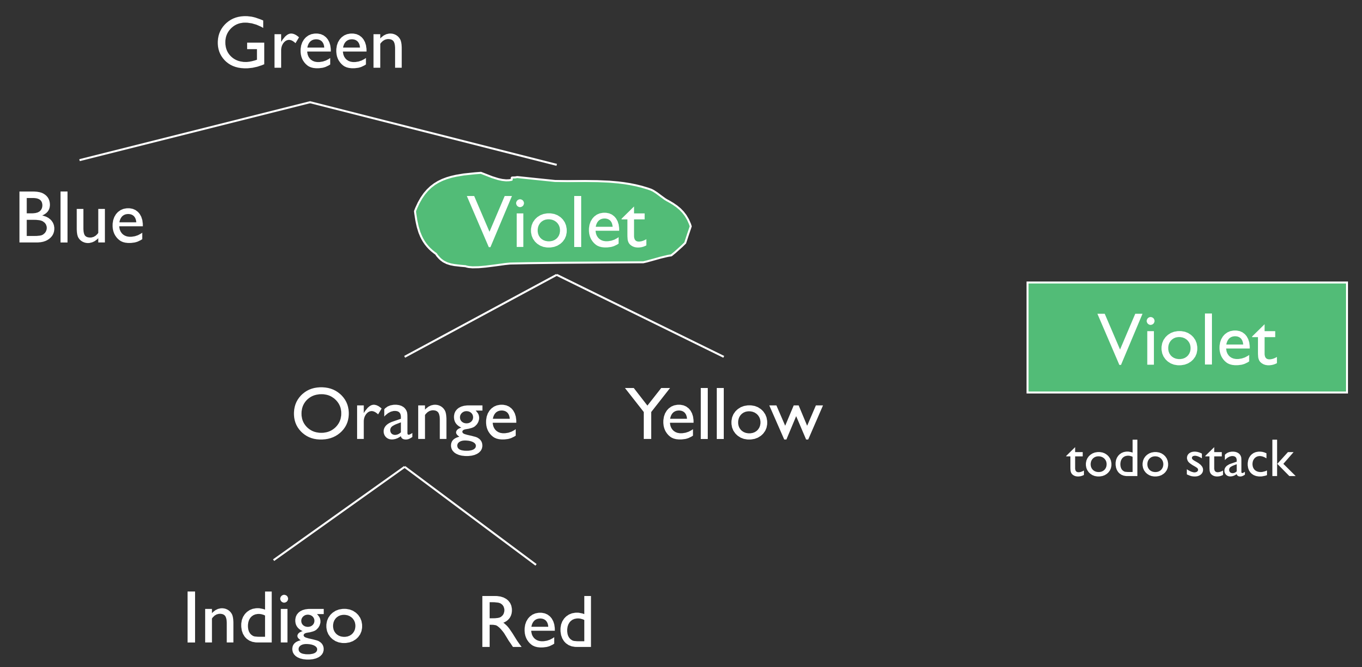
Pre-Order Iterator

Visit node, then each node in left subtree, then each node in right subtree.



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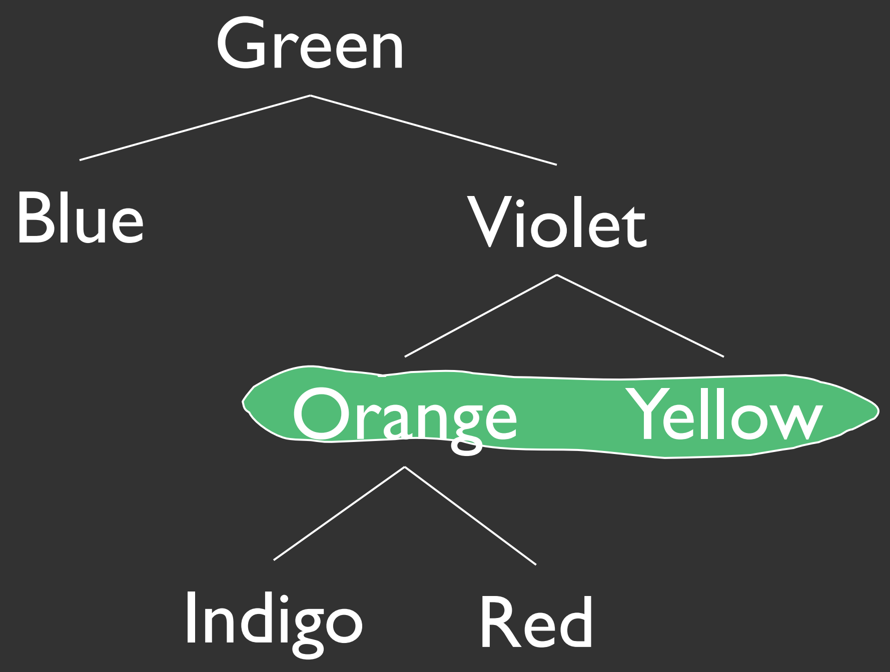
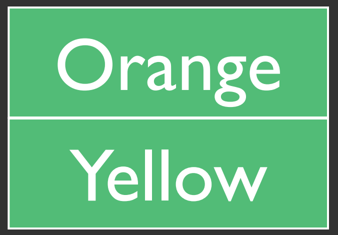
Visit node, then each node in left subtree, then each node in right subtree.



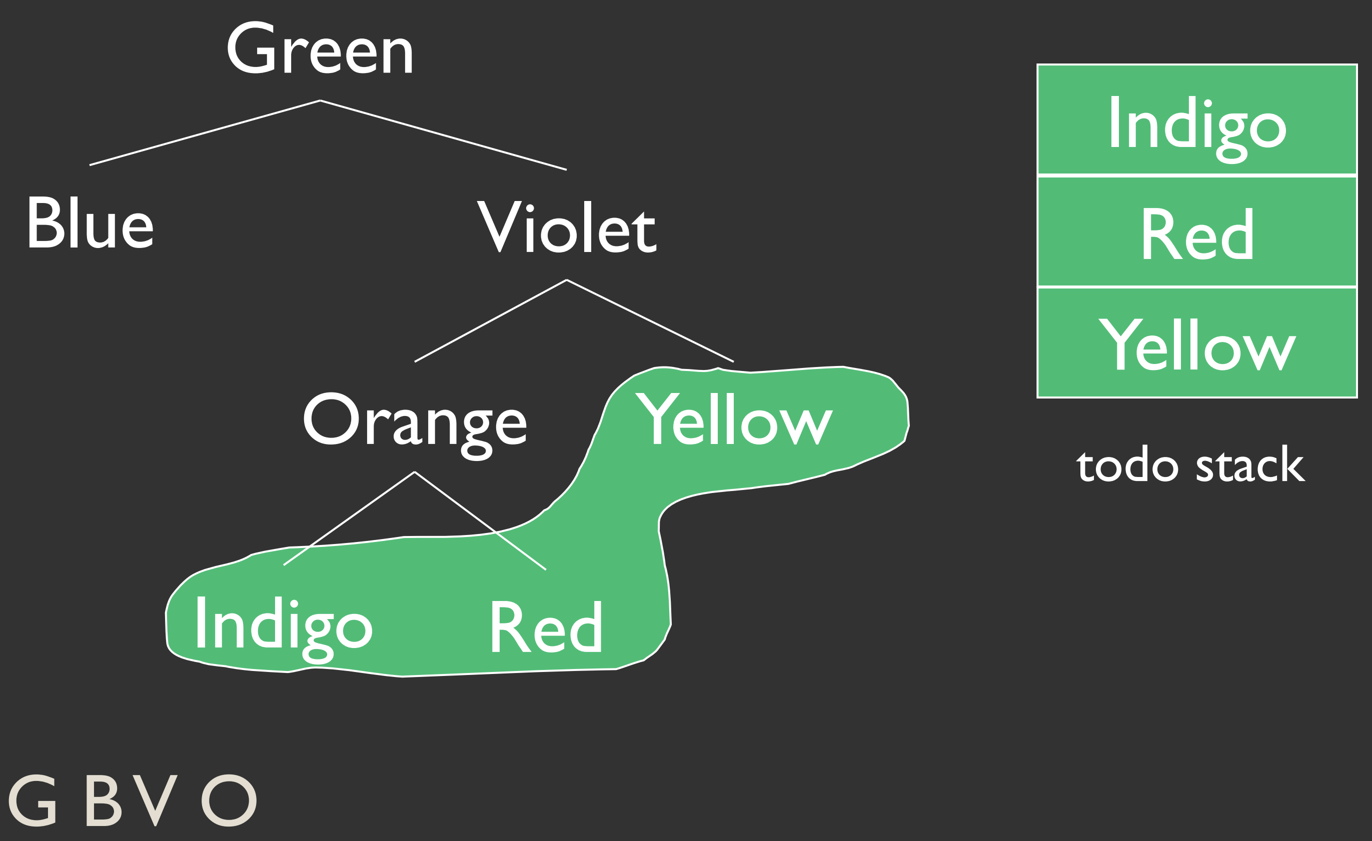


todo stack

G B V

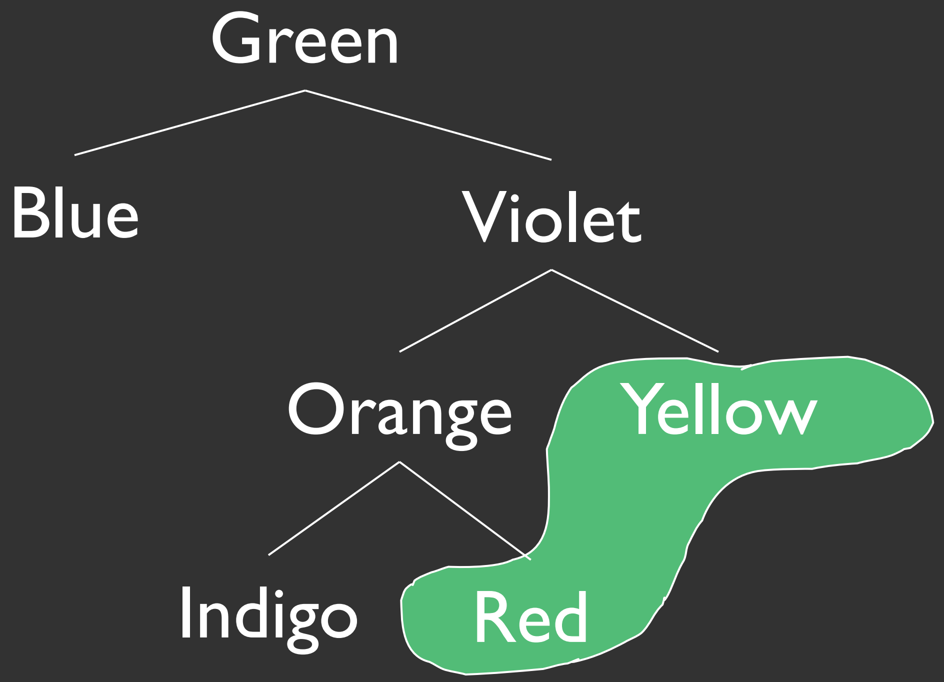
Pre-Order Iterator

Visit node, then each node in left subtree, then each node in right subtree.



Pre-Order Iterator

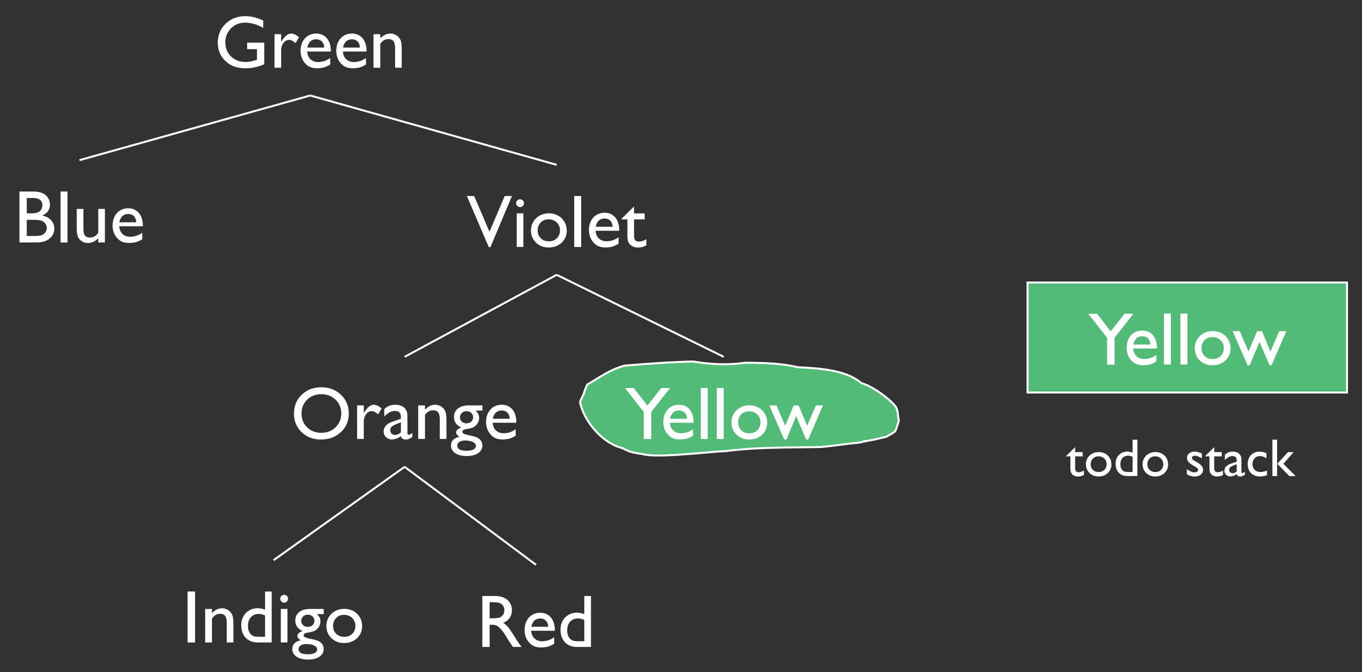
Visit node, then each node in left subtree, then each node in right subtree.



G B V O I

Pre-Order Iterator

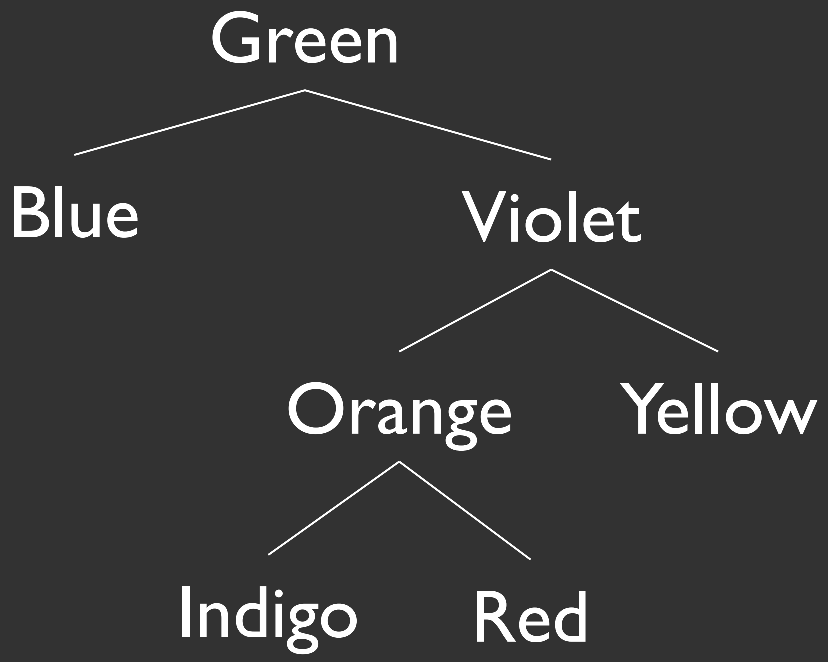
Visit node, then each node in left subtree, then each node in right subtree.



G B V O I R

Pre-Order Iterator

Visit node, then each node in left subtree, then each node in right subtree.



todo stack

G B V O I R Y

Pre-Order Iterator

```
public BTPreorderIterator(BinaryTree<E> root) {



}

public void reset() {





}
```

Pre-Order Iterator

```
public BTPreorderIterator(BinaryTree<E> root) {
        todo = new StackList<BinaryTree<E>>();
        this.root = root;
        reset();
}

public void reset() {






}
```

Pre-Order Iterator

```
 public BTPreorderIterator(BinaryTree<E> root) {
        todo = new StackList<BinaryTree<E>>();
        this.root = root;
        reset();
 }

 public void reset() {
        todo.clear();

        // stack is now empty; push root on TODO
stack
        if ((!root.isEmpty())
             todo.push(root);
 }
```

Pre-Order Iterator

```
public boolean hasNext() {

}

public E next() {





}
```

Pre-Order Iterator

```
public boolean hasNext() {
       return !todo.isEmpty();
}

public E next() {






}
```

Pre-Order Iterator

```
public boolean hasNext() {
        return !todo.isEmpty();
}

public E next() {
     BinaryTree<E> old = todo.pop();
     E result = old.value();







}
```

Pre-Order Iterator

```
public boolean hasNext() {
        return !todo.isEmpty();
}

public E next() {
     BinaryTree<E> old = todo.pop();
     E result = old.value();
     if (!old.right().isEmpty())
        todo.push(old.right());



}
```

Pre-Order Iterator

```
public boolean hasNext() {
       return !todo.isEmpty();
}

public E next() {
     BinaryTree<E> old = todo.pop();
     E result = old.value();
     if (!old.right().isEmpty())
              todo.push(old.right());
     if (!old.left().isEmpty())
              todo.push(old.left());


}
```

Pre-Order Iterator

```
public boolean hasNext() {
        return !todo.isEmpty();
}

public E next() {
     BinaryTree<E> old = todo.pop();
     E result = old.value();
     if (!old.right().isEmpty())
             todo.push(old.right());
     if (!old.left().isEmpty())
             todo.push(old.left());
     return result;
}
```

Tree Traversal Practice Problems

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- Prove that `levelOrder()` is correct: that is, that it touches the nodes of the tree in the correct order (*Hint*: induction by level)

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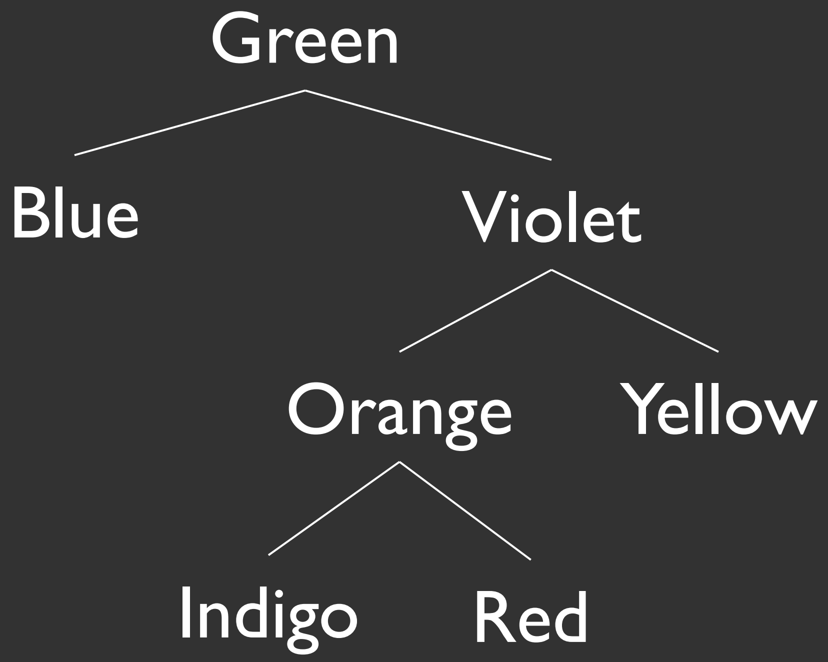
- Prove that `levelOrder()` is correct: that is, that it touches the nodes of the tree in the correct order (*Hint*: induction by level)
- Prove that `levelOrder()` takes O(n) time, where n is the size of the tree

Tree Traversal Practice Problems

- Prove that `levelOrder()` is correct: that is, that it touches the nodes of the tree in the correct order (*Hint*: induction by level)
- Prove that `levelOrder()` takes O(n) time, where n is the size of the tree
- Prove that the PreOrder (or LevelOrder) Iterator visits the nodes in the same order as the PreOrder (or LevelOrder) traversal method

In-Order Iterator

Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree.



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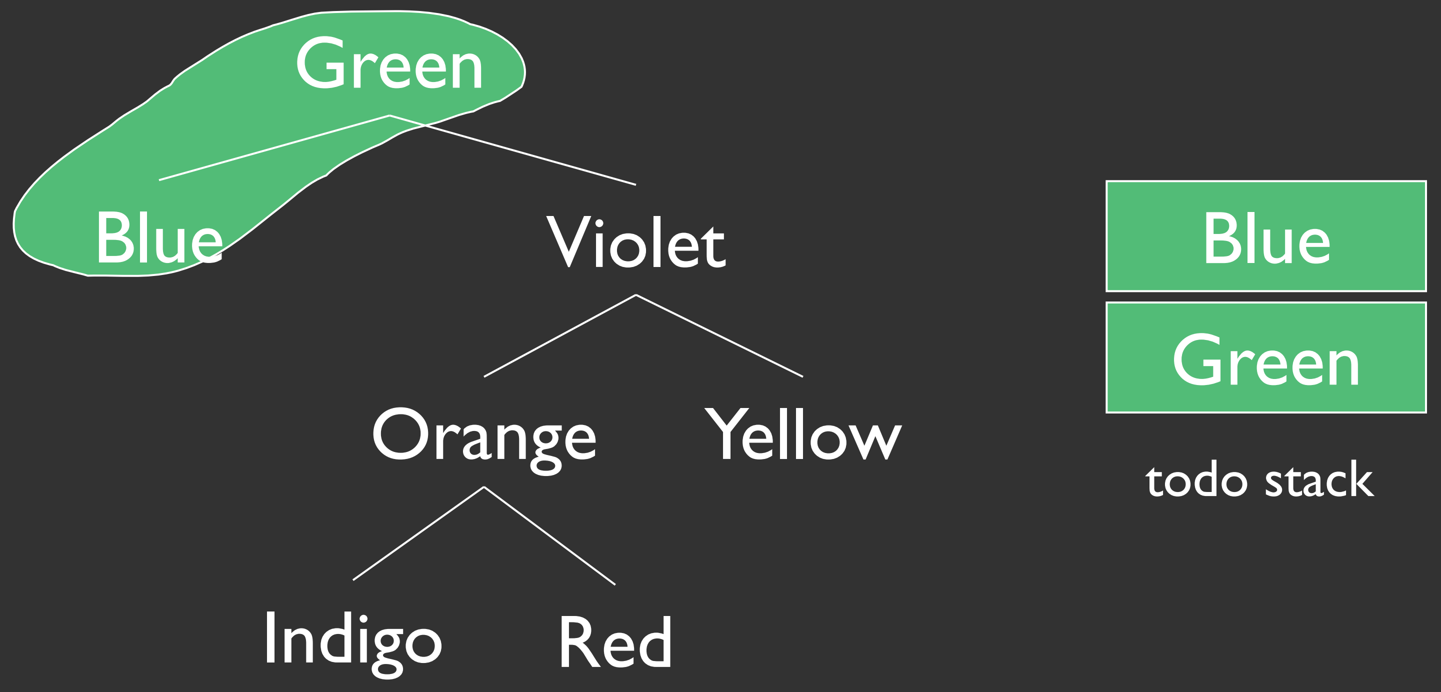

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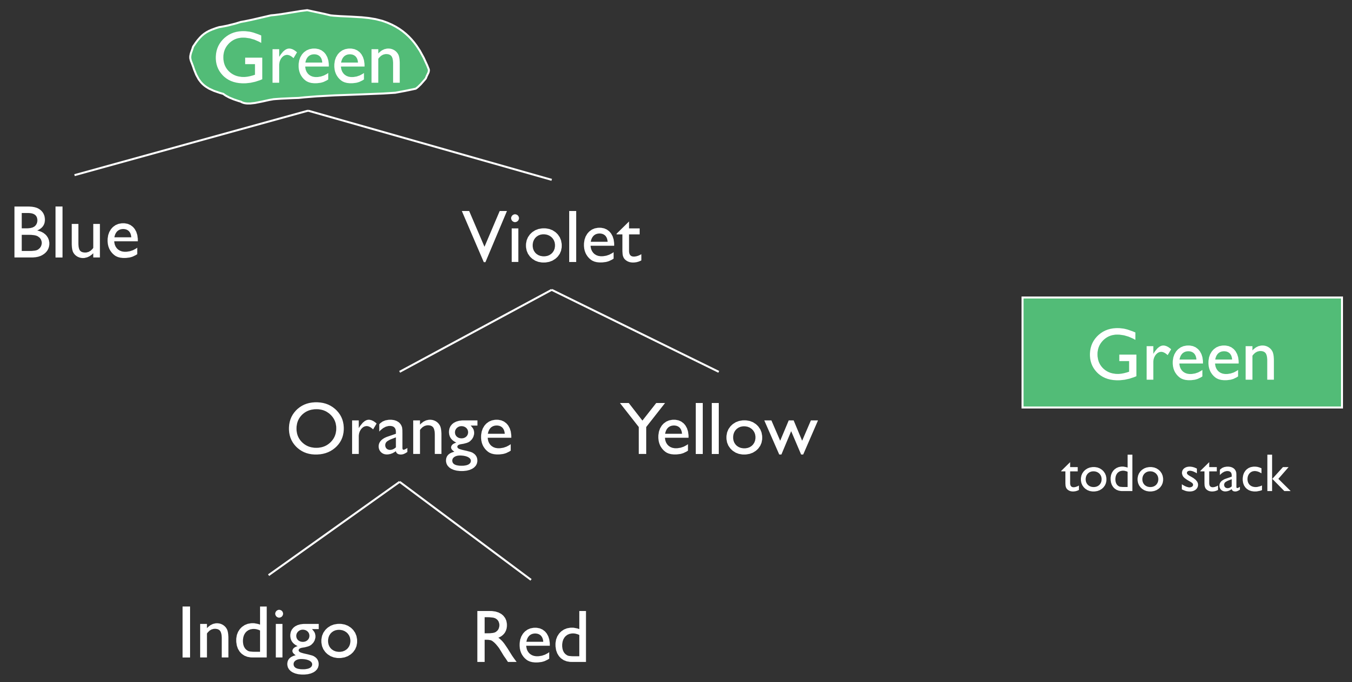
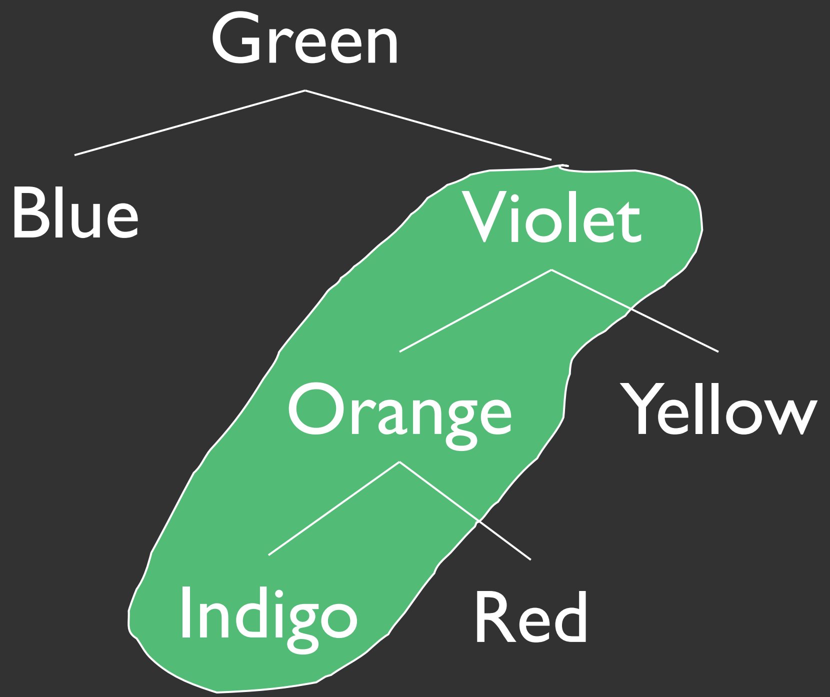


B

In-Order Iterator

Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree.



B G

In-Order Iterator

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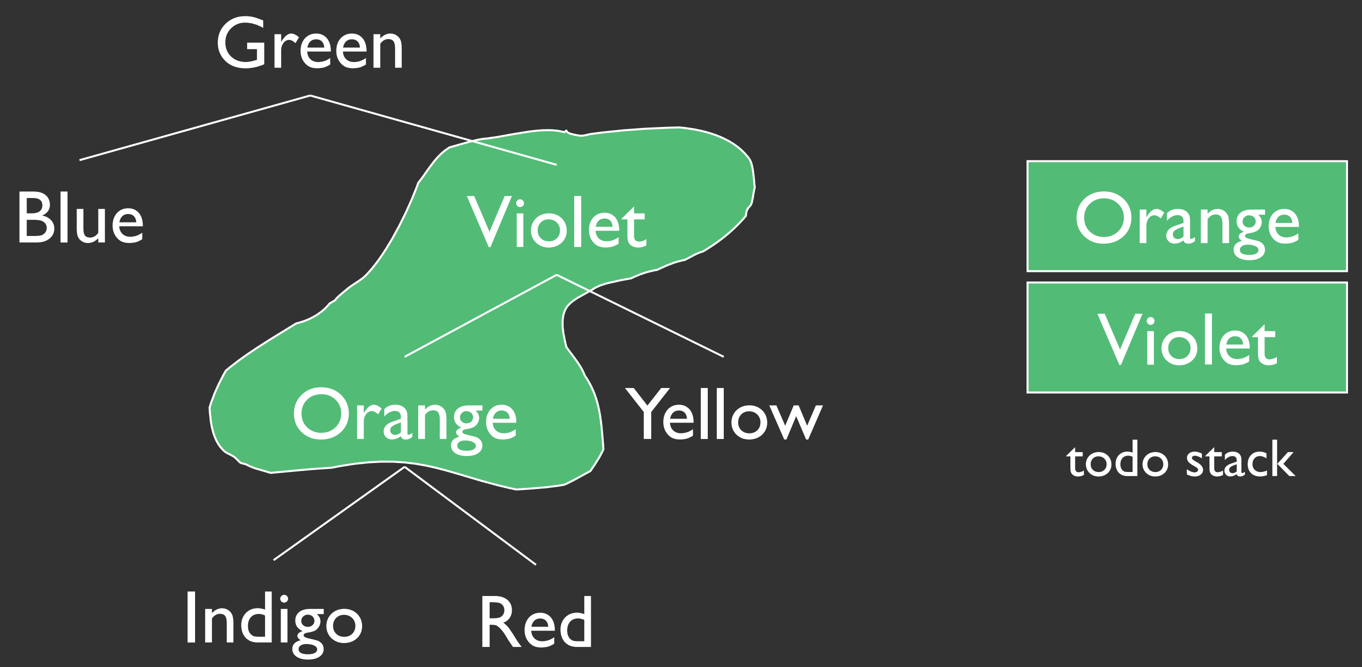

B G I

In-Order Iterator

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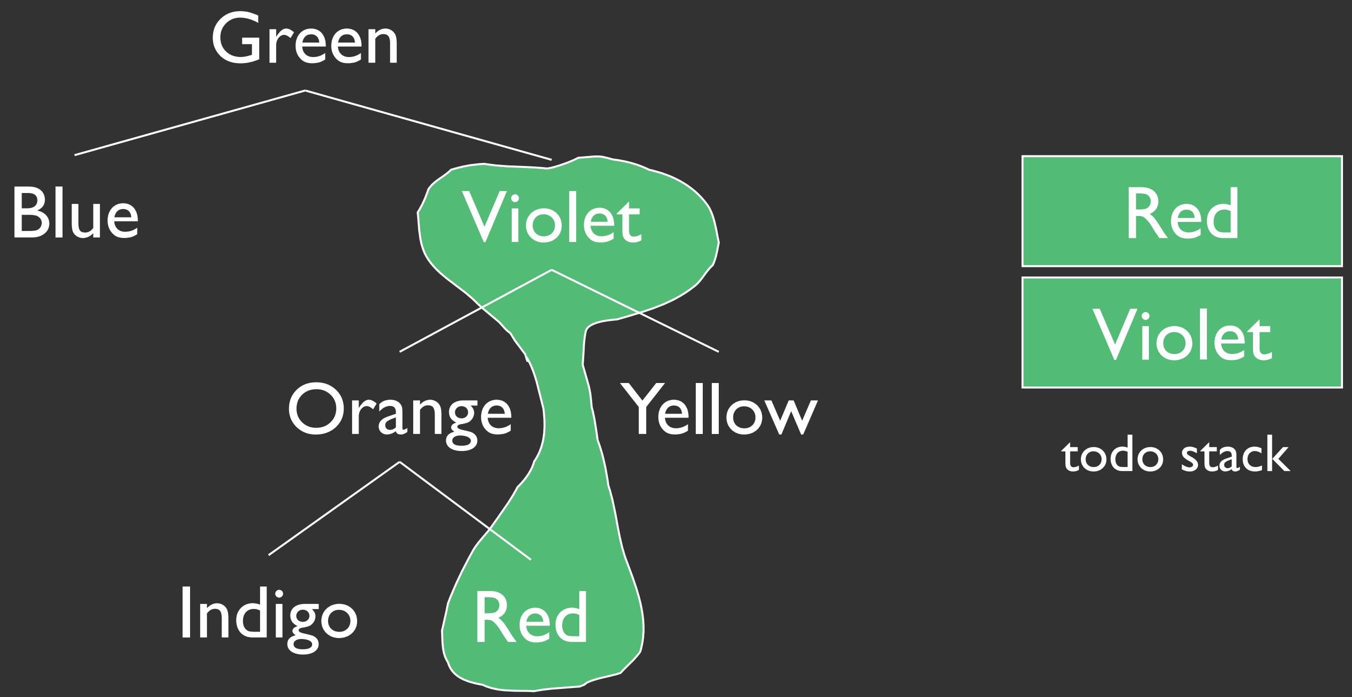

B G I O

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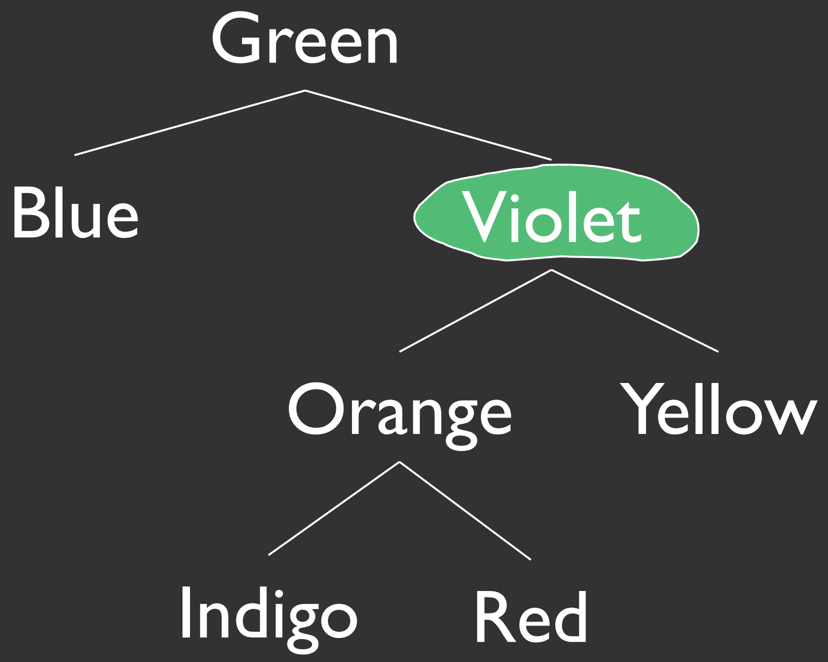
Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree.

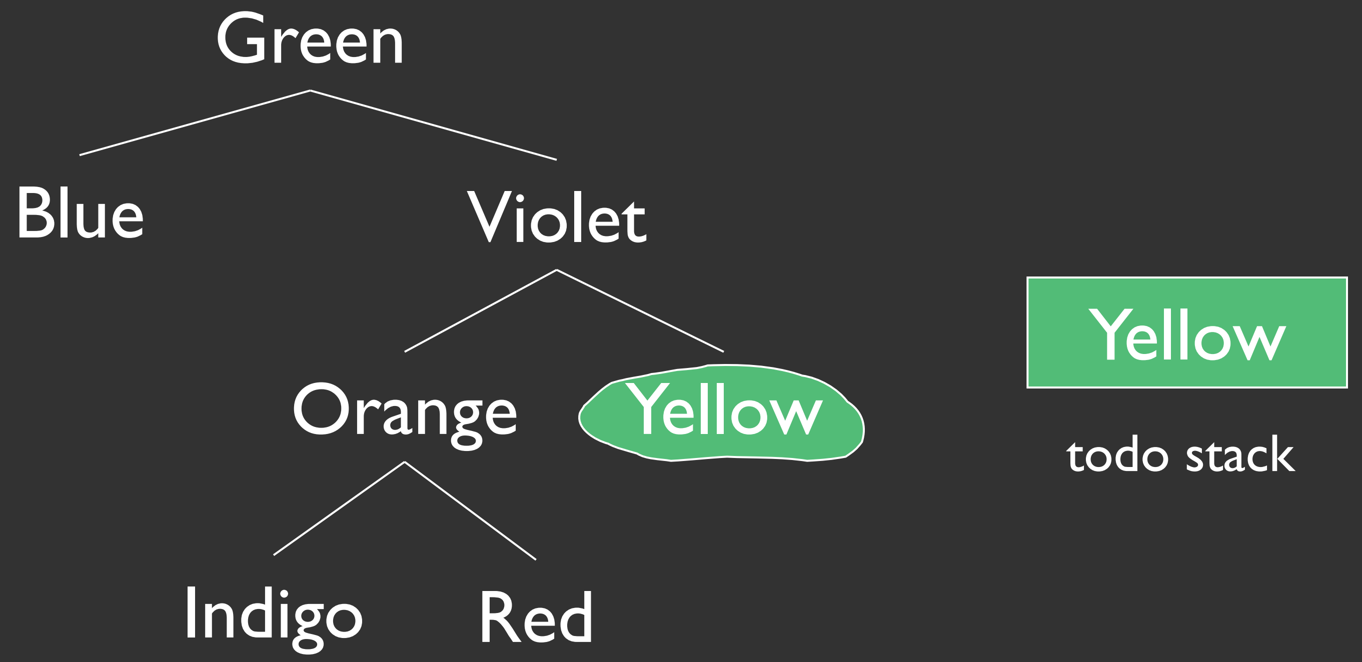


todo stack

B G I O R

In-Order Iterator

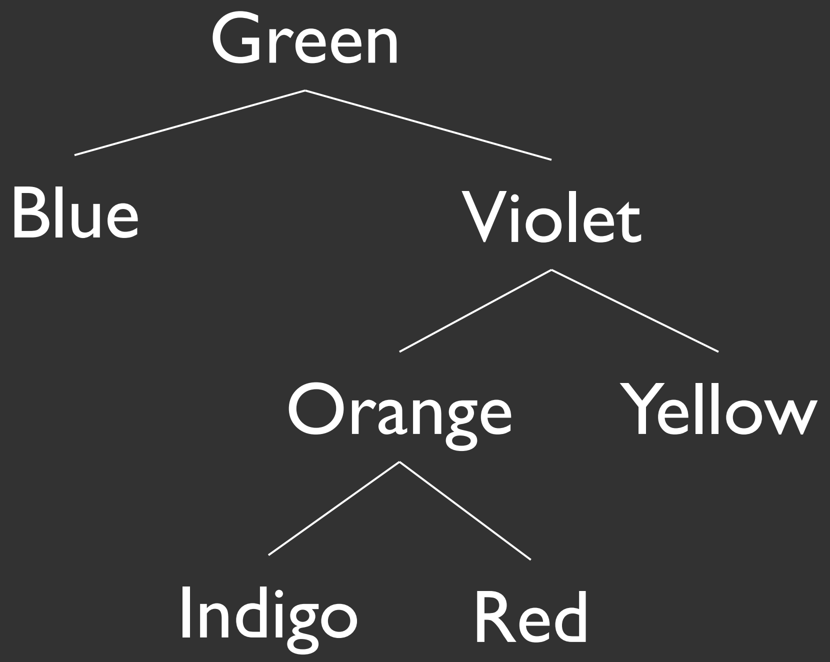
Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree.



B G I O R V

In-Order Iterator

Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree.



todo stack

B G I O R V Y

In-Order Iterator

In-Order Iterator

- Should return elements in same order as processed by in-order traversal method:
  - Traverse left subtree, then node, then right subtree

In-Order Iterator

- Should return elements in same order as processed by in-order traversal method:
  - Traverse left subtree, then node, then right subtree
- Must phrase in terms of `next()` and `hasNext()`

In-Order Iterator

- Should return elements in same order as processed by in-order traversal method:
  - Traverse left subtree, then node, then right subtree
- Must phrase in terms of `next()` and `hasNext()`
- Basic idea: We again “simulate recursion” with stack

In-Order Iterator

- Outline: left -> node -> right

In-Order Iterator

- Outline: left -> node -> right
  1. Push left children (as far as possible) onto stack

In-Order Iterator

- Outline: left -> node -> right
  1. Push left children (as far as possible) onto stack
  2. On call to `next()`:

In-Order Iterator

- Outline: left -> node -> right
  1. Push left children (as far as possible) onto stack
  2. On call to `next()`:
     - Pop node from stack

In-Order Iterator

- Outline: left -> node -> right
  1. Push left children (as far as possible) onto stack
  2. On call to `next()`:
     - Pop node from stack
     - Push right child and follow left children as far as possible

In-Order Iterator

- Outline: left -> node -> right
  1. Push left children (as far as possible) onto stack
  2. On call to `next()`:
     - Pop node from stack
     - Push right child and follow left children as far as possible
     - Return node's value

In-Order Iterator

- Outline: left -> node -> right
  1. Push left children (as far as possible) onto stack
  2. On call to `next()`:
     - Pop node from stack
     - Push right child and follow left children as far as possible
     - Return node's value
  3. On call to `hasNext()`:

In-Order Iterator

- Outline: left -> node -> right
  1. Push left children (as far as possible) onto stack
  2. On call to `next()`:
     - Pop node from stack
     - Push right child and follow left children as far as possible
     - Return node's value
  3. On call to `hasNext()`:
     - return `!stack.isEmpty()`

In-Order Iterator

```
public BTInorderIterator(BinaryTree<E> root) {



}

public void reset() {






}
```

In-Order Iterator

```
public BTInorderIterator(BinaryTree<E> root) {
    todo = new StackList<BinaryTree<E>>();


}

public void reset() {







}
```

In-Order Iterator

```
public BTInorderIterator(BinaryTree<E> root) {
    todo = new StackList<BinaryTree<E>>();
    this.root = root;

}

public void reset() {







}
```

In-Order Iterator

```
public BTInorderIterator(BinaryTree<E> root) {
    todo = new StackList<BinaryTree<E>>();
    this.root = root;
    reset();
}

public void reset() {









}
```

In-Order Iterator

```
public BTInorderIterator(BinaryTree<E> root) {
    todo = new StackList<BinaryTree<E>>();
    this.root = root;
    reset();
}

public void reset() {
        todo.clear();







}
```

In-Order Iterator

```
public BTInorderIterator(BinaryTree<E> root) {
    todo = new StackList<BinaryTree<E>>();
    this.root = root;
    reset();
}

public void reset() {
        todo.clear();
        // stack is empty.  Push on nodes from root along
        // longest “left-only” path




}
```

In-Order Iterator

```
public BTInorderIterator(BinaryTree<E> root) {
    todo = new StackList<BinaryTree<E>>();
    this.root = root;
    reset();
}

public void reset() {
        todo.clear();
        // stack is empty.  Push on nodes from root along
        // longest “left-only” path
        BinaryTree<E> current = root;
        while (!current.isEmpty()) {
            todo.push(current);
            current = current.left();
        }
}
```

In-Order Iterator

```
public E next() {

}
```

In-Order Iterator

```
public E next() {
        BinaryTree<E> old = todo.pop();
        E result = old.value();






    }
```

In-Order Iterator

```
public E next() {
       BinaryTree<E> old = todo.pop();
       E result = old.value();
       // we know this node has no unvisited left children;
       // if this node has a right child,
       //   we push right child and longest “left-only” path
       // else
       //   top element of stack is next node to be visited



   }
```

In-Order Iterator

```
public E next() {
       BinaryTree<E> old = todo.pop();
       E result = old.value();
       // we know this node has no unvisited left children;
       // if this node has a right child,
       //   we push right child and longest “left-only” path
       // else
       //   top element of stack is next node to be visited
       if (!old.right().isEmpty()) {
           BinaryTree<E> current = old.right();
           do {
               todo.push(current);
               current = current.left();
           } while (!current.isEmpty());
       }

   }
```

In-Order Iterator

```
public E next() {
       BinaryTree<E> old = todo.pop();
       E result = old.value();
       // we know this node has no unvisited left children;
       // if this node has a right child,
       //   we push right child and longest “left-only” path
       // else
       //   top element of stack is next node to be visited
       if (!old.right().isEmpty()) {
           BinaryTree<E> current = old.right();
           do {
               todo.push(current);
               current = current.left();
           } while (!current.isEmpty());
       }
       return result;
   }
```

Post-Order Iterator

- Outline: left -> right -> node

Post-Order Iterator

- Outline: left -> right -> node
  1. Push path to leftmost leaf onto stack

Post-Order Iterator

- Outline: left -> right -> node
  1. Push path to leftmost leaf onto stack
  2. On call to `next()`:

Post-Order Iterator

- Outline: left -> right -> node
  1. Push path to leftmost leaf onto stack
  2. On call to `next()`:
     - Pop node from stack

Post-Order Iterator

- Outline: left -> right -> node
  1. Push path to leftmost leaf onto stack
  2. On call to `next()`:
     - Pop node from stack
     - Determine whether it was the left or right node of its parent
       - If left child, push parent's right child and the entire path to leftmost leaf parent's right subtree
     - Return node's value

Post-Order Iterator

- Outline: left -> right -> node
  1. Push path to leftmost leaf onto stack
  2. On call to `next()`:
     - Pop node from stack
     - Determine whether it was the left or right node of its parent
       - If left child, push parent's right child and the entire path to leftmost leaf parent's right subtree
     - Return node's value
  3. On call to `hasNext()`:

Post-Order Iterator

- Outline: left -> right -> node
  1. Push path to leftmost leaf onto stack
  2. On call to `next()`:
     - Pop node from stack
     - Determine whether it was the left or right node of its parent
       - If left child, push parent's right child and the entire path to leftmost leaf parent's right subtree
     - Return node's value
  3. On call to `hasNext()`:
     - return `!stack.isEmpty()`

Post-Order Iterator

```
public BTPostorderIterator(BinaryTree<E> root) {



}
public void reset() {










}
```

Post-Order Iterator

```
public BTPostorderIterator(BinaryTree<E> root) {
    todo = new StackList<BinaryTree<E>>();
    this.root = root;
    reset();
}
public void reset() {








}
```

Post-Order Iterator

```
public BTPostorderIterator(BinaryTree<E> root) {
    todo = new StackList<BinaryTree<E>>();
    this.root = root;
    reset();
}
public void reset() {
    todo.clear();
    BinaryTree<E> current = root;
    while (!current.isEmpty()) {
            todo.push(current); // current now ‘below’
children
            if (!current.left().isEmpty())
                current = current.left();
            else
                current = current.right();
      } // Top of stack is now left-most unvisited leaf
}
```

Post-Order Iterator

Post-Order Iterator

```
public E next() {
```

Post-Order Iterator

```
public E next() {
        BinaryTree<E> current = todo.pop();
        E result = current.value();
```

Post-Order Iterator

```
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- Which is better?
  - Depends on the situation!
    - Does the tree structure represent a concept, e.g., distance or relationship between items?
    - Is the tree "sparse" or "dense"?

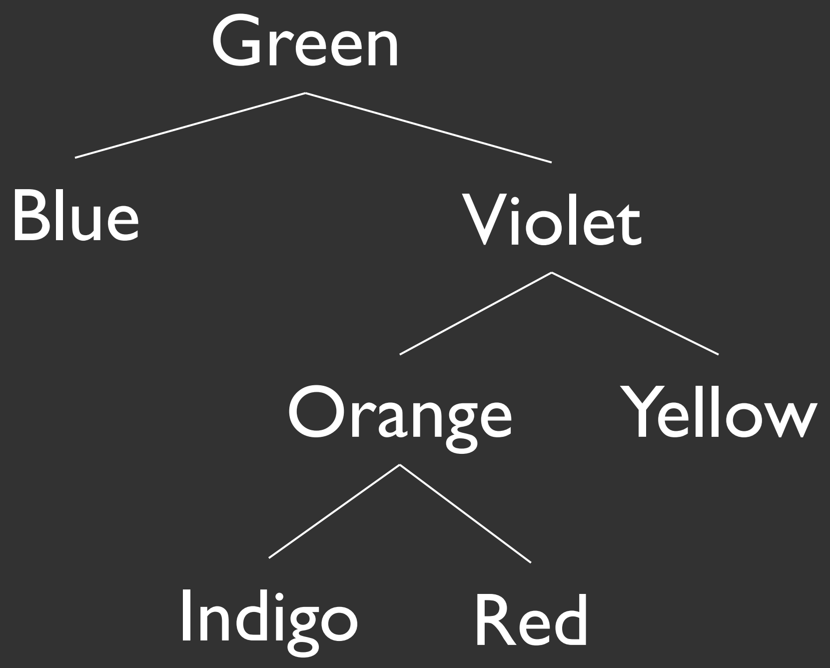
Final Thoughts

- Iterators continue to provide a useful service: common structure to enumerate the contents of a data structures
- We have defined four iterators that let us traverse the nodes of a tree in a variety of principled ways
- The best iterator for the task at hand will depend on our problem and our goals. So think critically!

CSCI 136
Data Structures & Advanced Programming

Alternative Tree Representations

BinaryTree Overheads?



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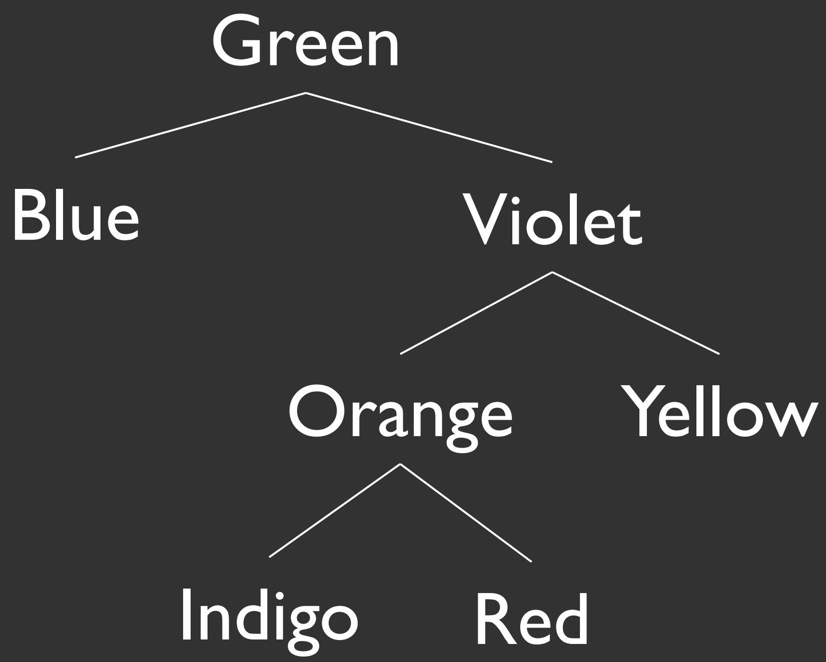

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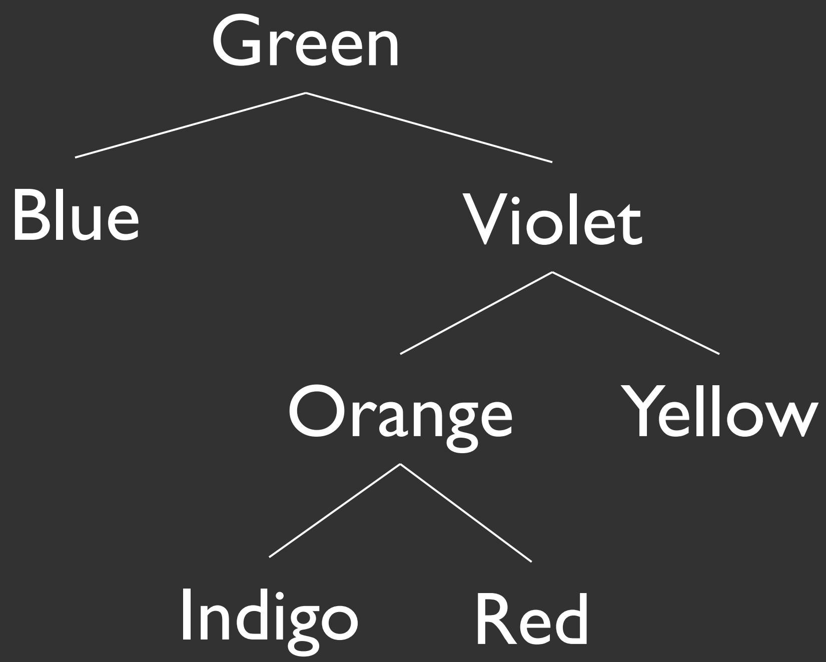

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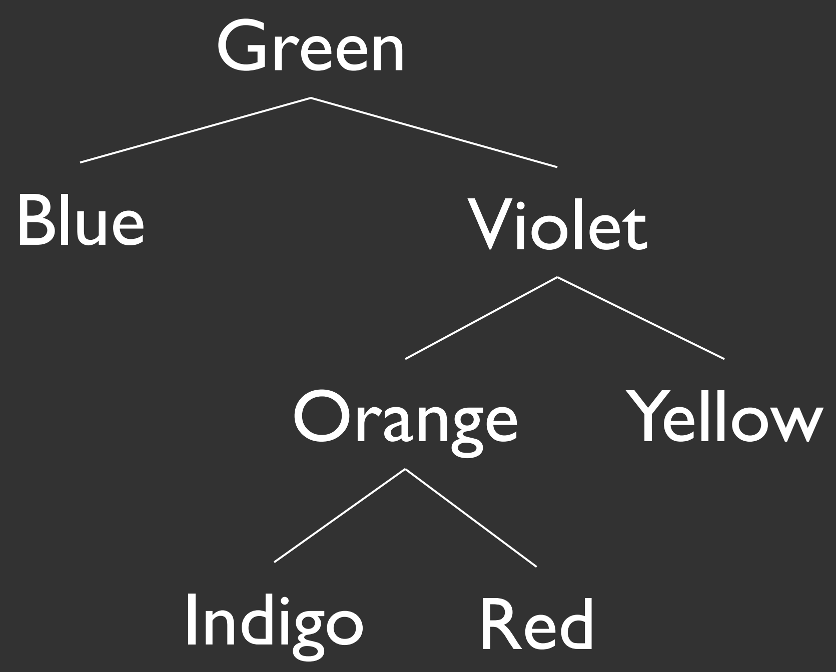

- Total # "references" = 4n
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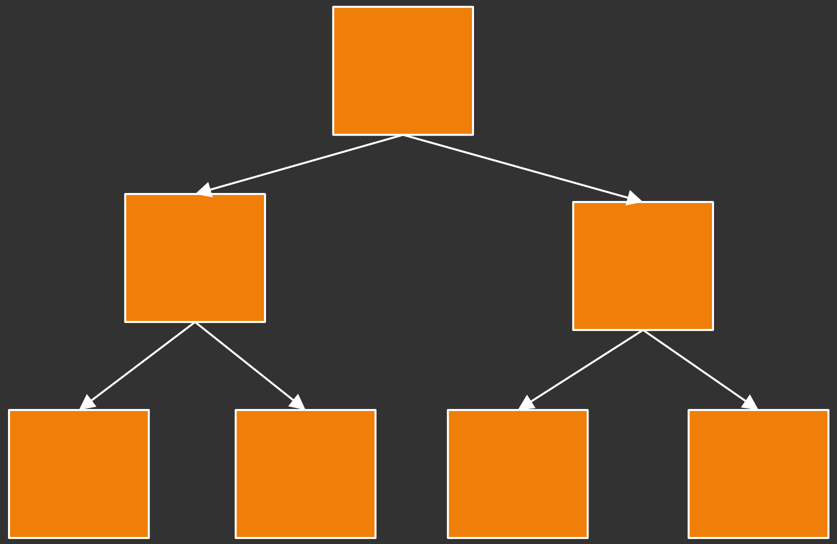
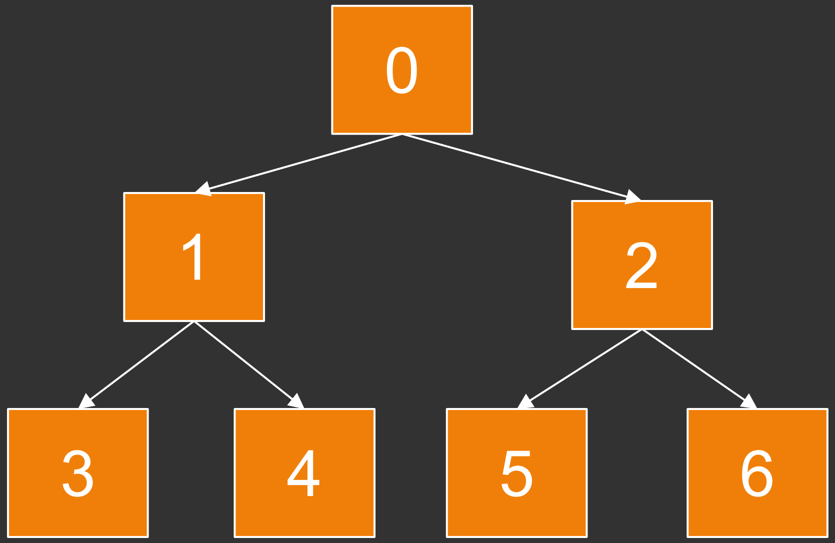
- 2-4x more overhead than vector, SLL, array, …
- But trees capture successor and predecessor relationships that other data structures don't… unless?

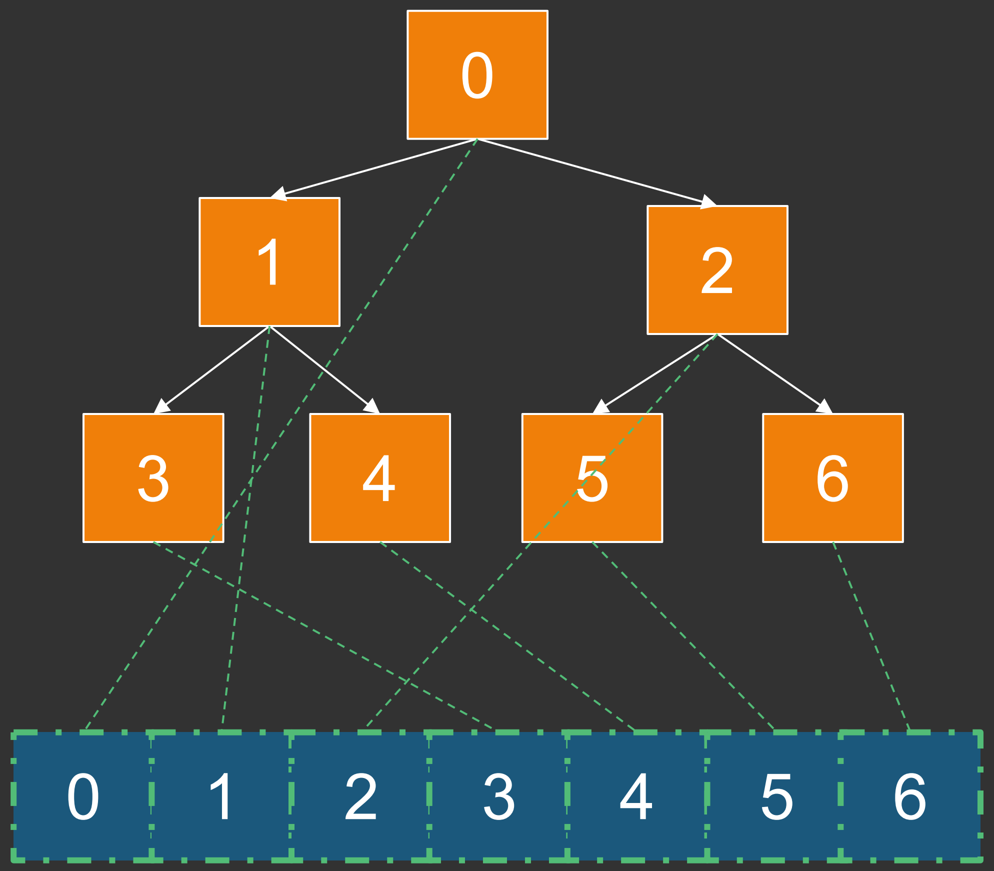
Consider the following (full) tree



Number the Nodes in Level Order



Store them in An Array at that Index!



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  - parent(j): (j-1)/2
  - *Note*: integer truncation takes care of "rounding"

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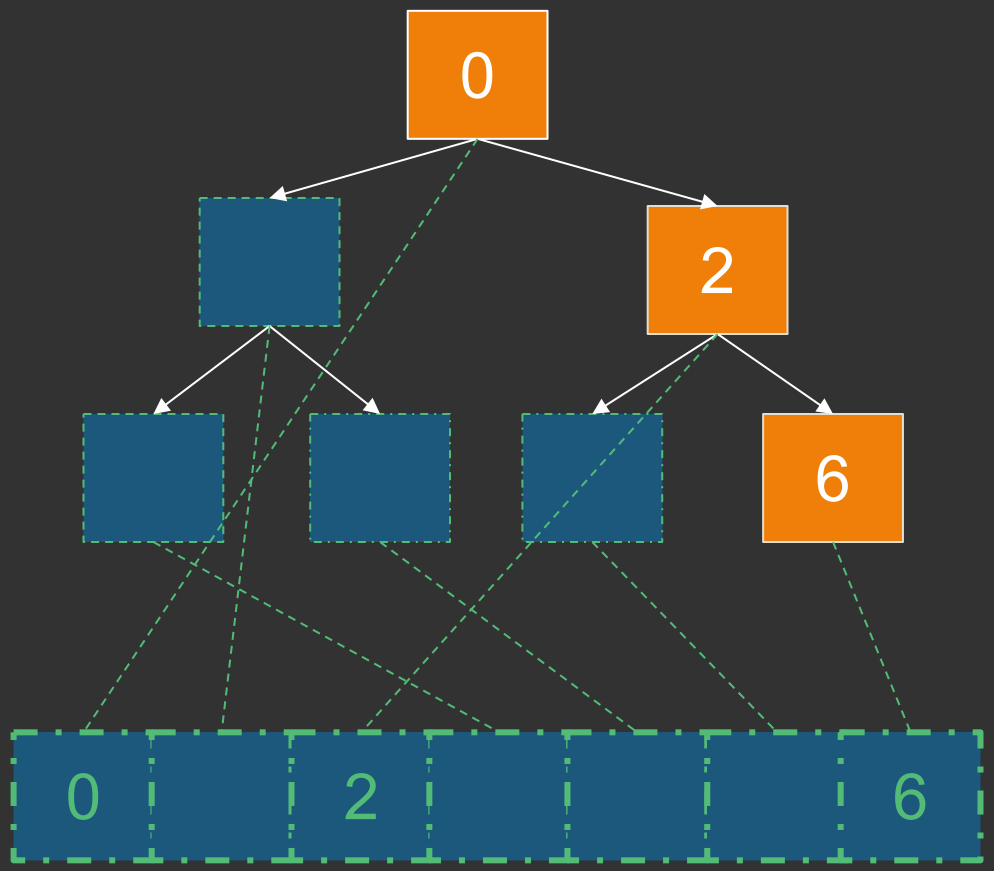
  - Tree of height of n requires $2^{n+1}-1$ array slots even if only O(n) elements

We Leave Gaps for Nodes That Could Exist



Final Thoughts

- For “dense” trees, an array representation is efficient
  - There are many contexts where a dense tree is a reasonable assumption
- If we can design a data structure that always preserves tree completeness, we should strongly consider an array representation
  - (Remember this when we get to heaps!)