CSCI 136:
Data Structures
and
Advanced Programming

Lecture 23

Trees, part 3

Instructor: Kelly Shaw

Williams

Topics

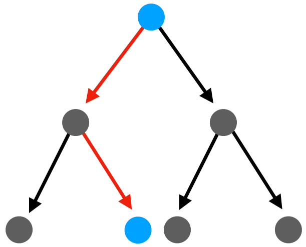
Tree terminology

Your to-dos

1. Read **before Fri**: review Bailey, Ch 14.
2. Lab 8 **(solo lab)**, **due Tuesday 11/15 by 10pm**.
   Note: you will implement a tree data structure called a ***trie*** *for lab 8; the structure is described in the lab handout. Please bring a short design document to your lab meeting.*

Activity: Binary Tree Height

The **height** of **a tree** is the length of the longest path between **the root** and **any leaf**.



Height of **tree** = **2**

Binary Tree Height

Let's think about some corner cases.

What is the height of a tree with just one node?

The **height** of **a tree** is the length of the longest path between **the root** and **any leaf**.

Height of **tree** = **0**

Binary Tree Height

Let's think about some corner cases.

What about the empty tree?

The **height** of **a tree** is the length of the longest path between **the root** and **any leaf**.

Height of **tree** = **-1**

Binary Tree Height

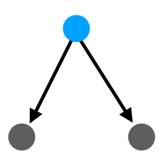
Here's a more formal definition.

The **height** of a tree is defined as:

- **-1** if the tree is empty, or
- **height(left)** or **height(right)**, whichever is bigger, **+ 1**

empty tree: -1

just a root: 0



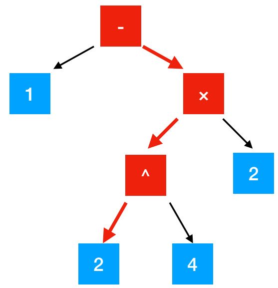
any other tree: longest path

Activity: Binary Tree Height

How might we implement `getHeight()`?

Height

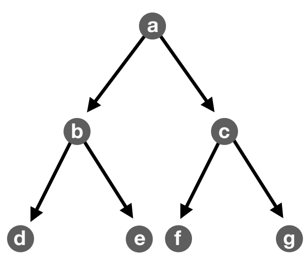
$$1 - 2^4 \times 2$$



Binary tree traversals

Binary tree traversals

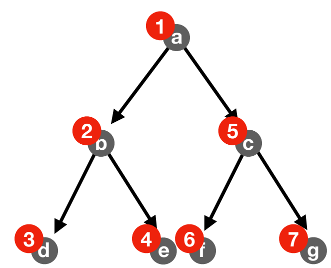
Suppose you are asked to write an `Iterator<T>` for a binary tree. What order do you choose?



Remember that tree nodes store data (`T`). A **traversal** corresponds with the order that data is returned.

Binary tree traversals

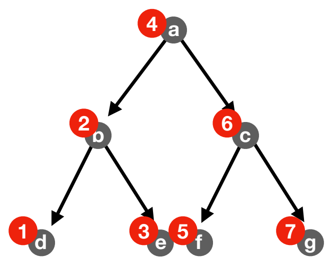
Pre-order traversal: Return data from each node **before its children**, and then return child data from **left to right**.



Returns the sequence: **a, b, d, e, c, f, g**

Binary tree traversals

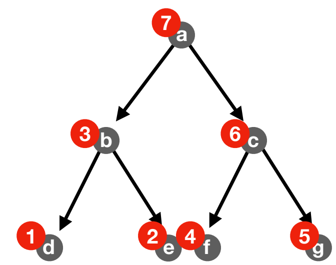
In-order traversal: Return data from each node **after its left child** and **before its right child**.



Returns the sequence: **d, b, e, a, f, c, g**

Binary tree traversals

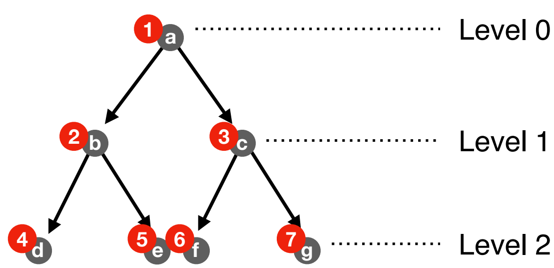
Post-order traversal: Return data from each node **after its children**; return child data from **left to right**.



Returns the sequence: **d, e, b, f, g, c, a**

Binary tree traversals

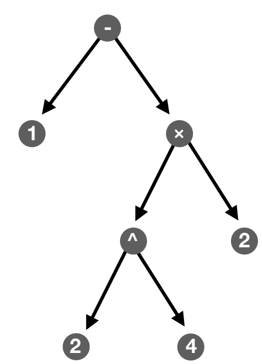
Level-order traversal (aka **breadth-first order**): Return data from each node in **level *i*** before data in **level *i+1***.



Returns the sequence: **a, b, c, d, e, f, g**

Activity: What traversal should I use?

Suppose I encode the arithmetic expression $1 - 2^4 \times 2$ using the following tree.



Ordered Trees

Binary search tree

A **binary search tree** is a binary tree that maintains the **binary search property** as elements are added or removed. In other words, the **key** in each node:

- must be > any **key** stored in the left subtree, and
- must be ≤ any **key** stored in the right subtree.

As with other ordered structures, order is maintained **on insertion.**

Binary search tree (alternative)

A **binary search tree** is a binary tree that maintains the **binary search property** as elements are added or removed. In other words, the **key** in each node:

- must be ≥ any **key** stored in the left subtree, and
- must be < any **key** stored in the right subtree.

As with other ordered structures, order is maintained **on insertion.**

Key, Value nodes

Note that I said **key** instead of **element**.

Storing a **key** and a **value** in each node allows the greatest flexibility when arranging a tree. I.e., the key type K need not be the value type V.

Restriction: keys must be **comparable** in some way (e.g., `Comparable<K>` or `Comparator<K>`).

Example

Insert the following elements: `71,20,27,17,91,14,87`

Assume K and V are the same.

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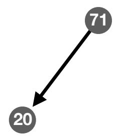

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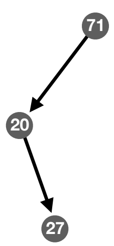

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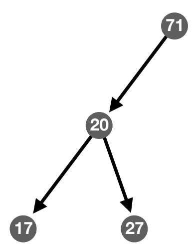

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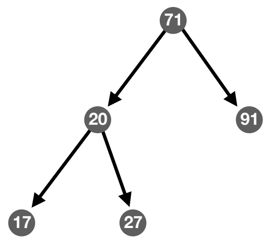

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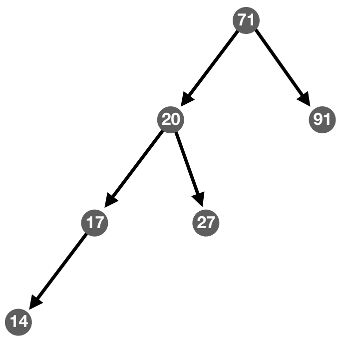

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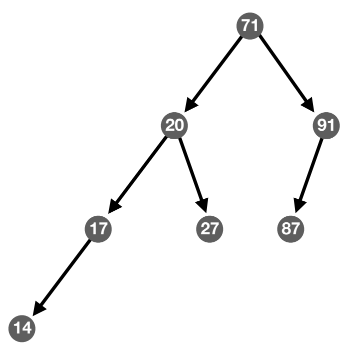

Activity

Insert the following elements:

Assume K and V are the same.

Binary Search Tree

Let's start implementing this together.

Recap & Next Class

Today:

Tree terminology

Tree traversals

Next class:

Binary search trees