CSCI 136:
Data Structures
and
Advanced Programming

Lecture 23

Trees, part 2

Instructor: Kelly Shaw

Williams

Topics

Tree terminology

Your to-dos

1. Read **before Wed**: Bailey, Ch 14.4.
2. Lab 7 (partner lab), **due Tuesday 11/8 by 10pm**.

Announcements

CSCI 136 final exam
Saturday, December 17 at 1:30pm
Room TBD

Tree ADT

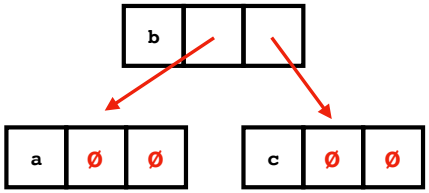
A **tree** is a recursive data structure that stores information hierarchically. A tree is either:

- **empty** (i.e., Ø), or
- a **node** containing a **value** and references to one or more **trees**.

The empty tree:

Ø

A non-empty **binary** tree:

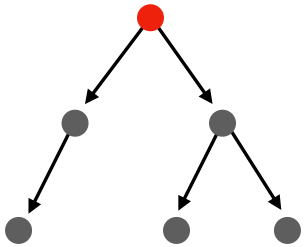


Binary Tree

Let's implement this together.

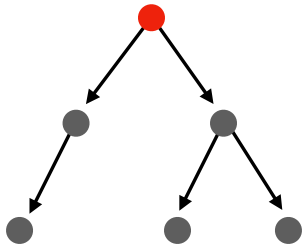
Terminology

The topmost node is called the **root**.



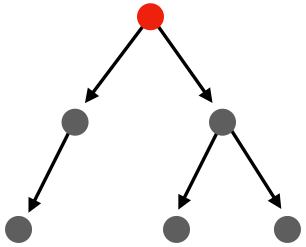
Properties of trees

Connected: every node in a tree is **reachable** by following a single unique **path** starting from the **root** node.



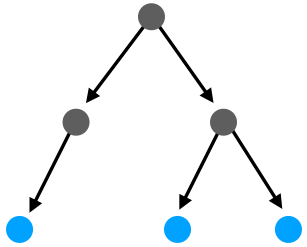
Properties of trees

 # edges: a tree having **n vertices** always has **n-1 edges**.



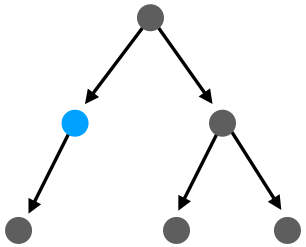
Terminology

The nodes at the bottom of a tree are called **leaves**.



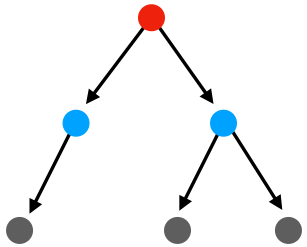
Terminology

Any node that is not a leaf is an **interior node**.



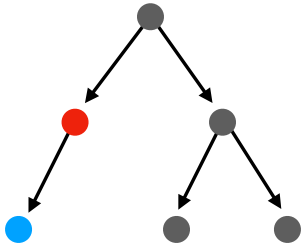
Terminology

A **node** may have **children**.



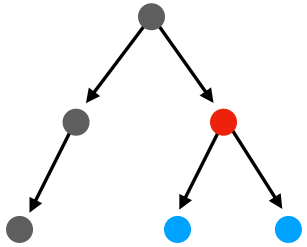
Terminology

A **node** may have **children**.



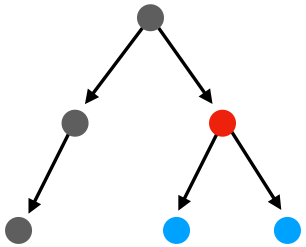
Terminology

A **node** may have **children**.



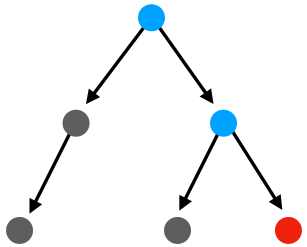
Terminology

A **node** that has **children** is called the **parent** of those children.



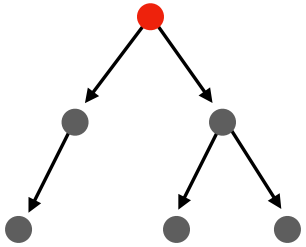
Terminology

For a **given node**, all of the nodes above it are called **ancestors**.



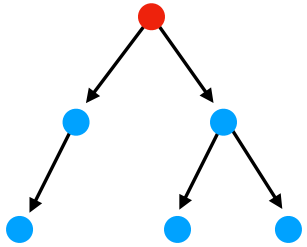
Properties of trees

Single ancestor: every node in a tree has at most one ancestor.



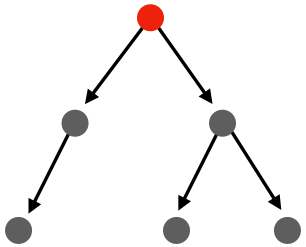
Terminology

For a **given node**, all of the nodes below it are called **descendants**.



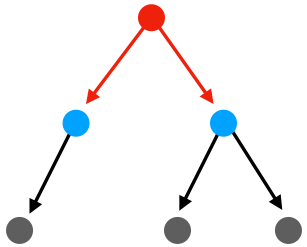
Properties of trees

Subtrees: the descendents of every tree (except the empty tree) are also trees.



Terminology

The **degree** of a tree is the maximum number of **children** had by any node.

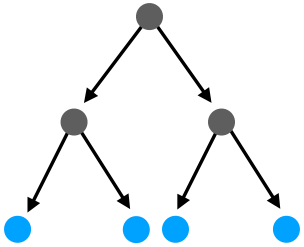


Degree of this tree: 2

Degree 2 trees are common: we call them **binary trees**.

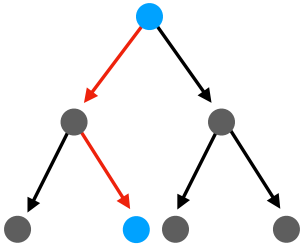
Terminology

A tree that is missing no leaves is **full**.



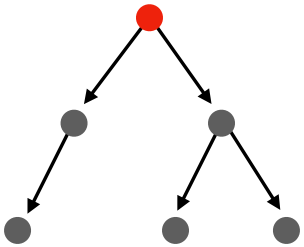
Terminology

A **path** is a sequence of edges between **two nodes**.



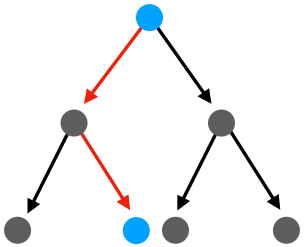
Properties of trees

Cycle-free: no path will ever revisit the same node.



Terminology

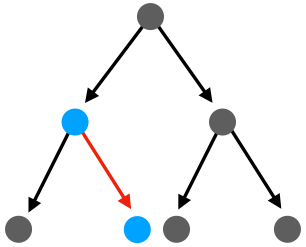
The **length** of a path is the **number of edges** in the path.



Length = **2**

Terminology

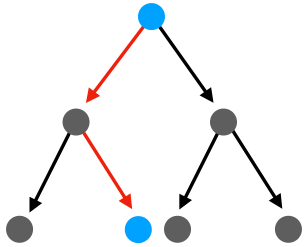
The **height** of **node n** is the length of the longest path between **n** and **any leaf**.



Height of **n** = **1**

Terminology

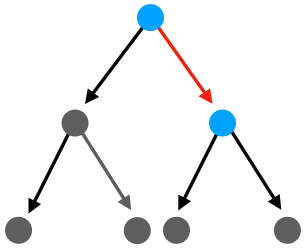
The **height** of **a tree** is the length of the longest path between **the root** and **any leaf**.



Height of **tree** = **2**

Terminology

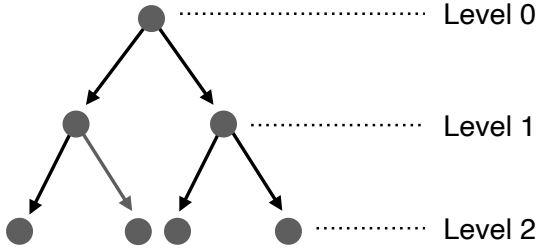
The **depth** of **node n** is the length of the longest path between **the root** and **n**.



Depth of **n** = **1**

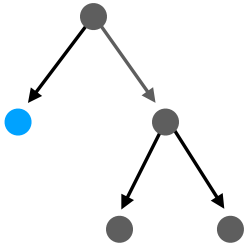
Terminology

The **level** of **any node** is its depth.



Terminology

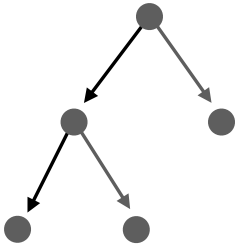
The depth of **n** + the height of **n** ≤ the height of the tree.



(depth of **n**: **1**) + (height of **n**: **0**) ≤ (height of **tree**: **2**)

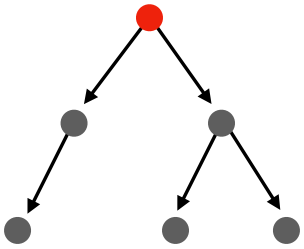
Terminology

A **complete** tree of **height h** is a full tree with zero or more rightmost leaves of **level h** removed.



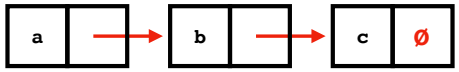
Properties of trees

Directed or **undirected**: trees can be either directed, meaning that traversals can only happen in one direction, or undirected, meaning that traversals can happen in any direction.



The tree shown here is directed.
We can represent an undirected tree using back edges.

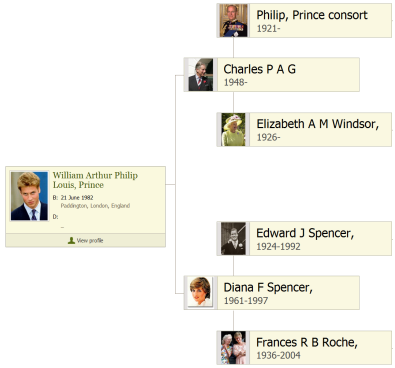
Is a **list** a **tree**?



Yes, a list is a tree whose nodes have **degree** 1.

We call such trees **degenerate**.

Activity



Encode this binary tree using BinaryTree<T>

Recap & Next Class

Today:

Tree terminology

Next class:

Tree traversals