"Because the details of mental representations can differ dramatically from field to field, it's hard to offer an overarching definition that is not too vague, but in essence these representations are preexisting patterns of information — facts, images, rules, relationships, and so on — that are held in long-term memory and that can be used to respond quickly and effectively in certain types of situations. The thing all mental representations have in common is that they make it possible to process large amounts of information quickly, despite the limitations of short-term memory."

-- Peak: Secrets from the New Science of Expertise by Ericsson and Pool

CSCI 136: Data Structures and Advanced Programming

Lecture 8

Asymptotic analysis

Instructor: Kelly Shaw

Williams

Topics

- Measuring time (and space)

Your to-dos

1. Lab 2, **due Tuesday 9/27 by 10pm** ([random] partner lab!)
2. Read **before Wed**: Bailey, Ch 7.1–7.2.

Announcements

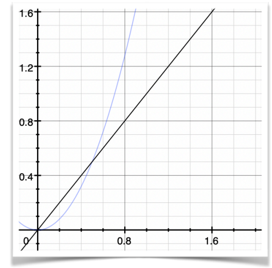
- CS Colloquium this **Friday, Sept 30 @ 2:35pm in Wege Auditorium (TCL 123)**

Sonia Roberts (Northeastern University)

Sonia is a postdoctoral research associate working on **soft sensors** based on origami and **knitted** structures for soft robots at Northeastern University as part of the Institute for Experiential Robotics.

Sonia's research focuses on the morphological design and control of robots, asking questions like how detailed a model of the environment a robot needs, why a robot might need legs or wheels for different tasks, and what the trade-off is between robustness and plasticity when implementing aspects of a robot's control using morphology versus actuated degrees of freedom.

Asymptotic analysis



We measure **time** and **space** similarly.

(I'll focus on **time** today)

How do we know if an algorithm is faster than another?

Why can't we just measure "wall time"?

Why can't we just measure "wall time"?

- Other things are happening **at the same time**
- Total running time **often varies by input size**
- **Different computers** usually produce **different results**!

Let's just count "steps", then

- If we count steps, then…
  - what is a "**step**"?
  - what about steps **inside loops**?

A little context

- How **accurate** do we need to be?
  - If one algorithm takes **64 steps** and another **128 steps**, do we need to know the precise number?

What we do

Instead of precisely counting steps, we usually develop an **approximation** of a program's **time** or **space complexity**.

This approximation **ignores small details** and focuses on the big picture:

How do time and space requirements grow as a function of the size of the input?

Operations we assume to have unit cost

Accessing an element of an array.

```
arr[5]
```

Assigning a value to a variable.

```
int x = 10;
```

Reading a class field.

```
foo.some_data;
```

Elementary mathematical operations.

```
x + 1
y * z
```

Returning something.

```
return x;
```

Example

```
// pre: array length n > 0
public static int findPosOfMax(int[] arr) {
    int maxPos = 0
    for (int i = 1; i < arr.length; i++)
        if (arr[maxPos] < arr[i]) maxPos = i;
    return maxPos;
}
```

- Can we count steps exactly? Do we even want to?
  - `if` complicates counting
- Idea: **overcount**: assume `if` block always runs
  - in the worst case, it does
- Overcounting gives **upper bound** on run time
- Can also **undercount** for the **lower bound**

Overcounting Example

```
// pre: array length n > 0
public static int findPosOfMax(int[] arr) {
    int maxPos = 0 ........................................ line 1 cost: c1
    for (int i = 1; i < arr.length; i++) ................... line 2 cost: nc2
        if (arr[maxPos] < arr[i]) .......................... line 3 cost: nc3
            maxPos = i; .................................... line 4 cost: nc4
    return maxPos; ......................................... line 5 cost: c5
}
```

Total cost: $c_1 + nc_2 + nc_3 + nc_4 + c_5$

$= c_1 + n(c_2 + c_3 + c_4) + c_5$

$= n(c_2 + c_3 + c_4) + c_1 + c_5$

$\approx O(n)$

(as you shall see)

Focus is on order of magnitude

We can do this analysis for the **best**, **average**, and **worst** cases. We often focus on the best and worst cases.

Average case analysis is interesting and extremely useful, but it's beyond the scope of this course.

Big-O notation

Let **f** and **g** be real-valued functions that are defined on the same set of real numbers. Then **f** is of order **g**, written **f(n) is O(g(n))**, if and only if there exists a positive real number **c** and a real number n_0 such that for all **n** in in the common domain of **f** and **g**,

$$|f(n)| \le c \times |g(n)|, \text{ whenever } n > n_0.$$

We read this as: "**f(n) is O(g(n))**"
as "**f** of **n** is big-oh of **g** of **n**."

English, please!

$$|f(n)| \le c \times |g(n)|, \text{ whenever } n > n_0.$$

Intuition:
"at some point, f(n) is **always bounded from above** by g(n)."

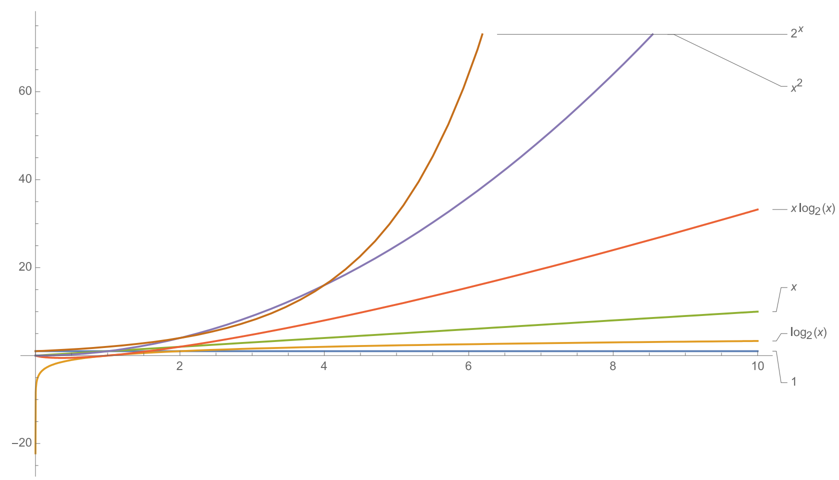
What we want: some **g(n)** that is both:
- Always bigger than **f(n)** (after some value n_0)
- Close to **f(n)**

If so, f is **O(g(n))**.

Function growth

Consider the following functions, for $x \ge 1$
- $f(x) = 1$
- $g(x) = \log_2(x)$ // Reminder: if x=2^n, $\log_2(x) = n$
- $h(x) = x$
- $m(x) = x \log_2(x)$
- $n(x) = x^2$
- $p(x) = x^3$
- $r(x) = 2^x$

Function growth



Function growth & Big-O

- Rule of thumb: **ignore multiplicative constants**
- Examples:
  - Treat n and n/2 as same order of magnitude
  - $n^2/1000$, $2n^2$, and $1000n^2$ are "pretty much" just n^2
  - $a_0n^k + a_1n^{k-1} + a_2n^{k-2} + \cdots a_k$ is roughly n^k
- The key is to find the **most significant** or **dominant term**
- Ex: $\lim_{x\to\infty} (3x^4 - 10x^3 - 1) = x^4$ (Why?)
  - So $3x^4 - 10x^3 - 1$ grows "like" x^4

Why base of log doesn't matter

- In CS, we generally use **$\log_2$**
- But for asymptotic analysis, **the base does not matter**.
- Proof:

$\log_2(x) = \log_{10}(x)/\log_{10}(2)$

$\log_{10}(2) \cdot \log_2(x) = \log_{10}(x)$

$c \cdot \log_2(x) = \log_{10}(x)$

$\log_2$ and $\log_{10}$ are **asymptotically the same!**

Think about the following for next class

Example

```
x + 1
```

What does this operation do?
(i.e., what is our desired "post condition"?)

Recap & Next Class

Today:

- Asymptotic analysis

Next class:

- Pre/post conditions
- Recursion