CSCI 136:
Data Structures
and
Advanced Programming

Lecture 15

Sorting, part 3

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Williams

Topics

- Sort stability
- Merge sort
- Quick sort

Your to-dos

1. Lab 5 (partner lab), **due Tuesday 10/18 by 10pm**.
2. **Read before Wed:** Bailey, Ch 10.

Sort stability

Unsorted: A

ab	cd	aa	bb
0	1	2	3

Suppose we are sorting on **just the first letter**.

Then **ab** $\nless$ **aa** and **ab** $\ngtr$ **aa**.

Note also the positions of these elements in **A**: 0 **<** 2.

Sorted: A

ab	aa	bb	cd
0	1	2	3

This sort is stable, because the **relative order** of **ab** and **aa** is **the same**.

Sort stability

A sort is **stable** if any two equal (or incomparable) objects **retain their relative order** in a sorted order as in an unsorted order.

Sort stability

More formally,

Let **A** be an **array**, and **i** and **j** indices in that array, s.t. $i \neq j$.

Let $\pi_S(A,i)$ be a function that returns the updated index of **i** **after sorting A** with sorting algorithm **S**.

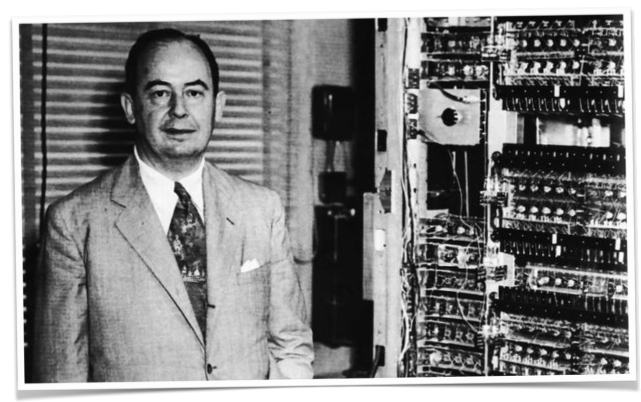
If $i < j$, $A[i] \nless A[j]$, $A[i] \ngtr A[j]$, and $\pi_S(A,i) < \pi_S(A,j)$ then sorting algorithm **S** is **stable**.

<u>Note</u>: people often say $A[i] = A[j]$ instead of $A[i] \nless A[j]$, $A[i] \ngtr A[j]$ even when **A[i]** and **A[j]** may be **incomparable**.

Merge sort

6 5 3 1 8 7 2 4

Merge sort

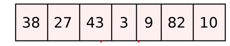


Invented by John von Neumann in 1948.

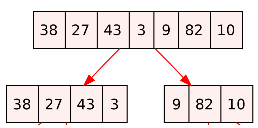
Merge sort

Merge sort is a **sorting algorithm** that uses the **divide and conquer** technique. It works by recursively partitioning data until no further partitioning is possible, then by **merging** elements of the partitions back together in **sorted order**.

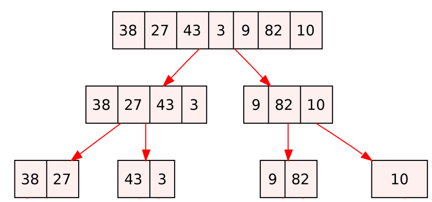
Merge sort



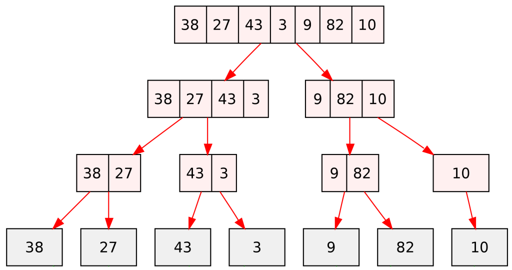
Merge sort



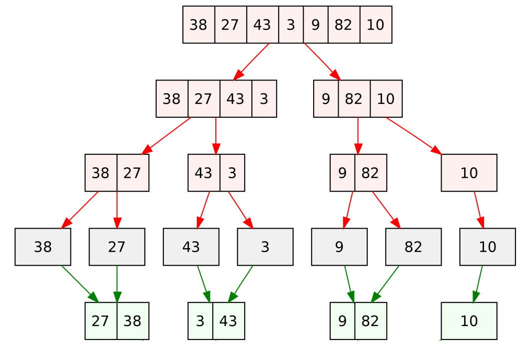
Merge sort



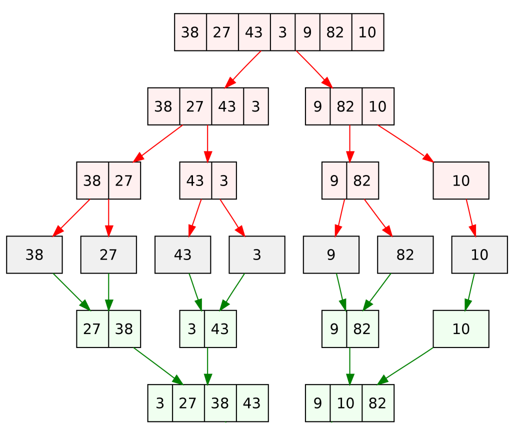
Merge sort



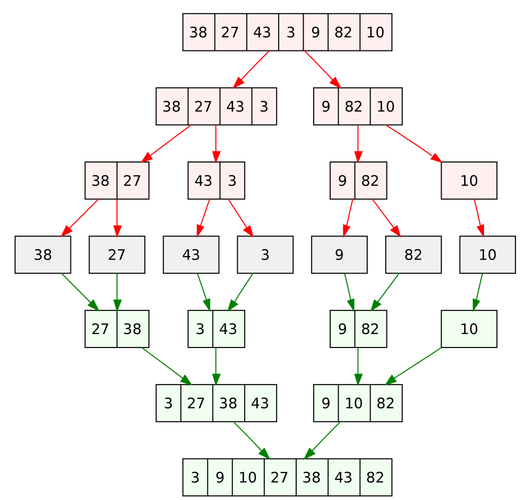
Merge sort



Merge sort



Merge sort



Merge sort

Merge sort takes **O(n × $\log_2$n)** time in the **worst case** (usually written **O(n log n)**).

Merge sort takes **O(n log n)** time in the **best case**.

Merge sort takes **O(n)** auxiliary space because each step makes a **copy of the data being sorted**.

I.e., merge sort is **not** an **in-place sort**. It is **out-of-place**.

Time complexity proof sketch

Divide takes **O(1)** because we are just picking a midpoint.

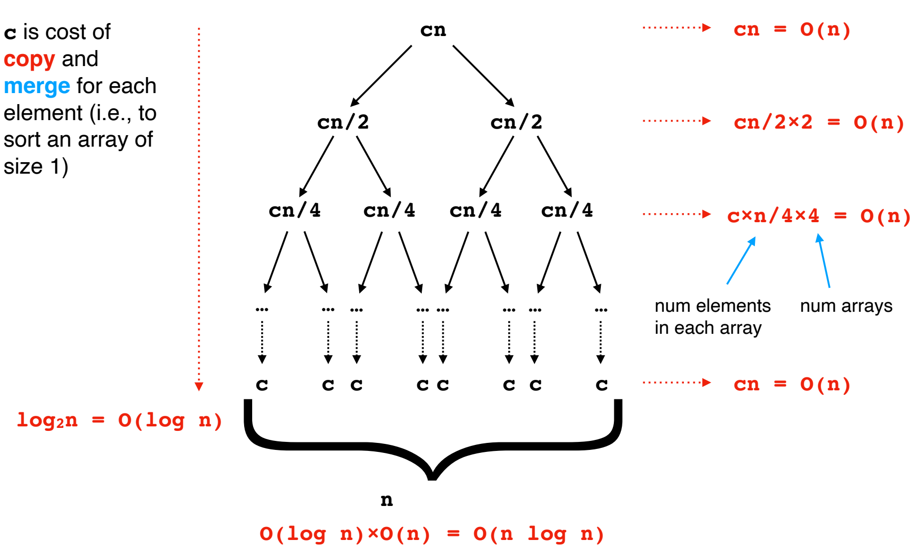
Merge takes **O(n)** because we have to copy **n/2** elements into an array of size **n** twice.

We divide **O(log n)** times and merge **O(log n)** times.

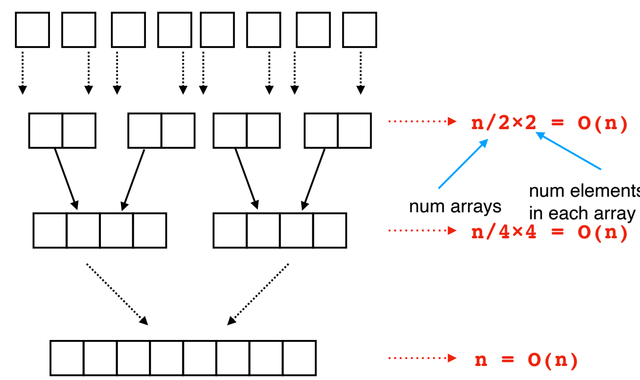
Therefore, the algorithm is **O(n log n)**.

Time complexity

c is cost of **copy** and **merge** for each element (i.e., to sort an array of size 1)



Space complexity

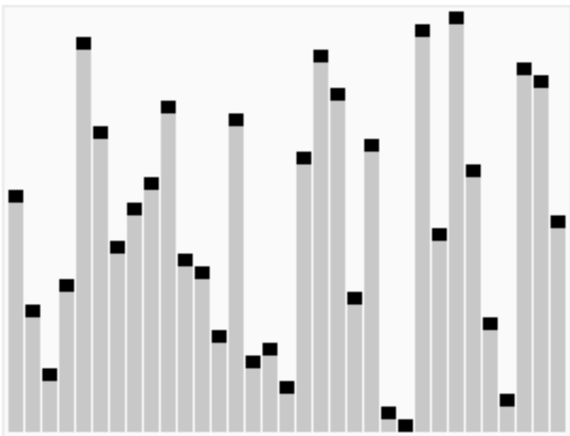


At first glance, this looks like **O(n log n)** space!

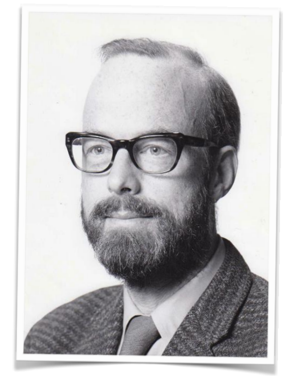
Why isn't it?

Because after merging, we can **discard old arrays** (i.e., garbage collect) and **reuse that space**.

Quicksort



Quicksort



Invented by Tony Hoare in 1959.

One of my all-time favorite algorithms.

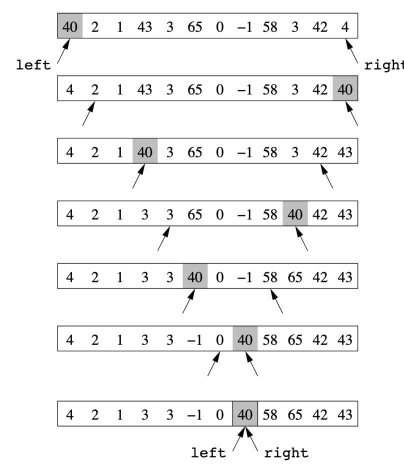
Quicksort

Quicksort is a **sorting algorithm** that uses the **divide and conquer** technique. It works by partitioning the data into two arrays around a **pivot** (a fixed element, like the first element).

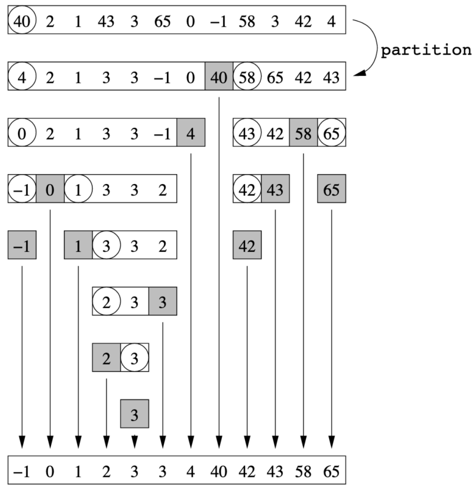
It **swaps data** so that **one array contains elements smaller than the pivot** and the **other array contains elements larger than the pivot**. This ensures that, at each step, the pivot is in the correct position in the array.

Performing this procedure **recursively** on the left and right subarrays until there is nothing left to partition **guarantees a sorted array**.

Quicksort partition step



Quicksort recursive steps



Quicksort

Unlike merge sort, quick sort does not need to combine sub arrays after splitting—**the entire array is guaranteed to be sorted upon reaching the base case**, and since the sort is done in-place no copying is required.

Base case (array of size 1): the pivot is **trivially sorted**.

Inductive case: Assume that the left and right subarrays are sorted. Since the pivot is the **middlemost element**, then everything to the left is smaller and everything to the right is bigger. Therefore, the entire array is sorted.

Quicksort

Quicksort takes **$O(n^2)$** time in the **worst case**. This case is improbable, and highly improbable as **$n \to \infty$**.

Quicksort takes **O(n log n)** time in the **best case**.

Quicksort takes **O(n log n)** time in the **average case**.

I.e., quicksort is an **in-place sort**. Therefore it needs no auxiliary space. As a result, **quicksort is almost always chosen over merge sort** in any application where all the data can fit into RAM.

Quicksort time proof sketch

In the **worst case**, we repeatedly choose the worst pivot (either the min or max value in the array). This means that we need to do **n-1** swaps.

Since there are n worst case choices of pivots, in the worst case, we do **n-1** swaps **n** times. **$O(n^2)$**.

In the **best case**, **we always happen to choose the middlemost value** as a pivot. I.e., the two subarrays are the same size. The rest of the proof looks just like the proof for merge sort where we intentionally choose two subarrays of the same size.

If you're thinking that quicksort's best case is the same as merge sort's worst case, remember that quicksort is **in-place**.

Sorting Wrapup

	Time	Space
Bubble	Worst: $O(n^2)$ Best: $O(n)$ - if "optimized"	$O(n)$: $n + c$
Insertion	Worst: $O(n^2)$ Best: $O(n)$	$O(n)$: $n + c$
Selection	Worst = Best: $O(n^2)$	$O(n)$: $n + c$
Merge	Worst = Best: $O(n \log n)$	$O(n)$: $2n + c$
Quick	Average = Best: $O(n \log n)$ Worst: $O(n^2)$	$O(n)$: $n + c$

Recap & Next Class

Today:

- Sort stability
- Merge sort
- Quick sort

Next class:

- Linear structures