CSCI 136
Data Structures &
Advanced Programming

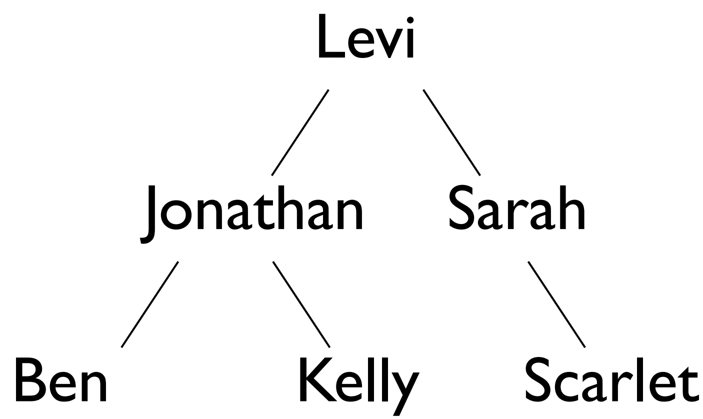
Tree Traversals

Tree Traversal Methods

Tree Traversals

- In linear structures, there are only a few basic ways to traverse (visit) the elements of the data structure
  - Start at one end and visit each element
  - Start at the other end and visit each element
- How do we traverse binary trees?
  - (At least) four reasonable mechanisms
- We imagine that we want to do some work at each node
  - We call that work *processing* the node

Tree Traversals



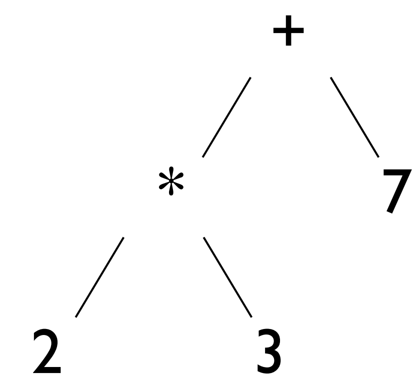
In-order: Ben, Jonathan, Kelly, Levi, Sarah, Scarlet

Pre-order: Levi, Jonathan, Ben, Kelly, Sarah, Scarlet

Post-order: Ben, Kelly, Jonathan, Scarlet, Sarah, Levi,

Level-order: Levi, Jonathan, Sarah, Ben, Kelly, Scarlet

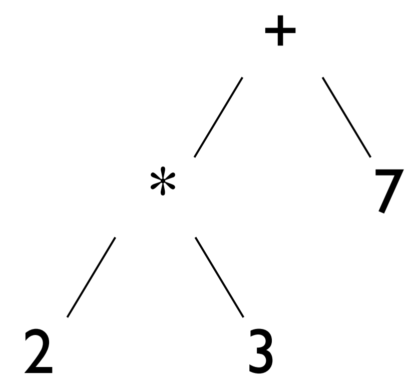
Tree Traversals



- Pre-order
  - Each node is processed before any children. Process node, process left subtree, then process right subtree. (node, left, right)
    - +*237
- In-order
  - Each node is processed after all nodes in left subtree are processed and before any nodes in right subtree. (left, node, right)
    - 2*3+7

("pseudocode")

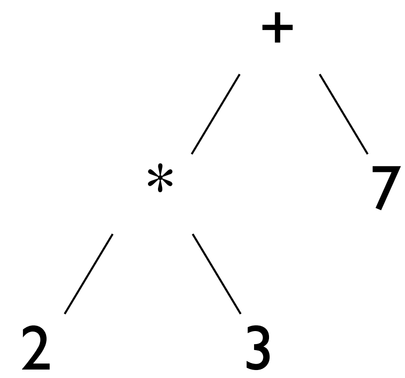
Tree Traversals



- In-order
  - Each node is processed after all nodes in left subtree are processed and before any nodes in right subtree. (left, node, right)
    - 2*3+7
- Aside: If processing means *printing*, we could also print a "(" before we process a subtree and a ")" after we process the subtree (skip leaves)
  - ((2 * 3) + 7)

("pseudocode")

Tree Traversals

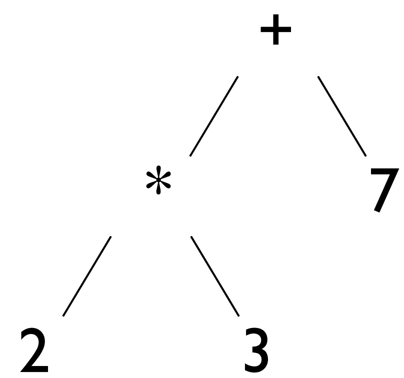


- Post-order
  - Each node is processed after its children. Process all nodes in left subtree, then all nodes in right subtree, then node itself. (left, right, node)
    - 23*7+
    - Post-order = PostScript order = RPN
- Level-order (not obviously recursive!)
  - Nodes at level i are processed before nodes at level i+1. (process nodes left to right on each level)
    - +*723

("pseudocode")

Tree Traversals

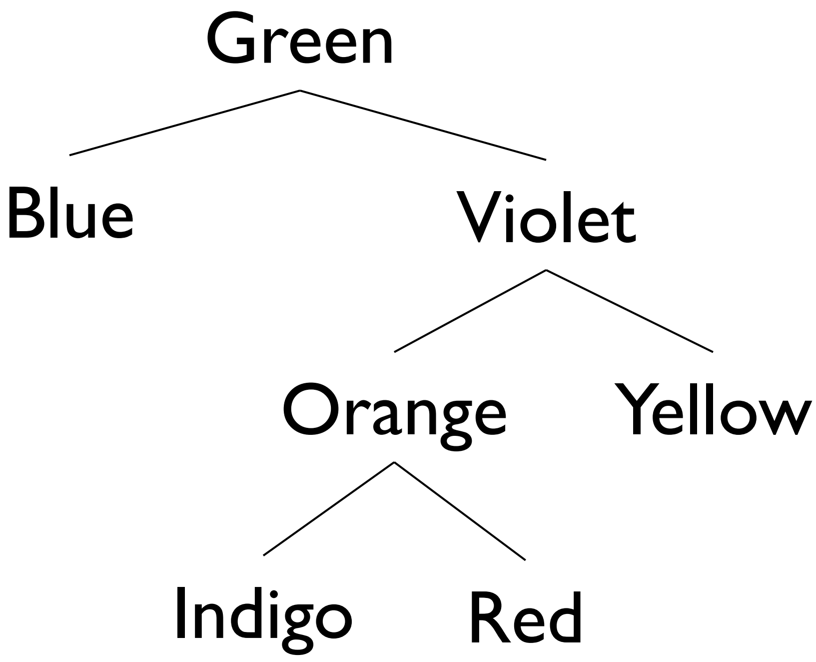
```
public void pre-order(BinaryTree t) {
      if(t.isEmpty()) return;
      process(t); // some method
      preOrder(t.left());
      preOrder(t.right());
}
```



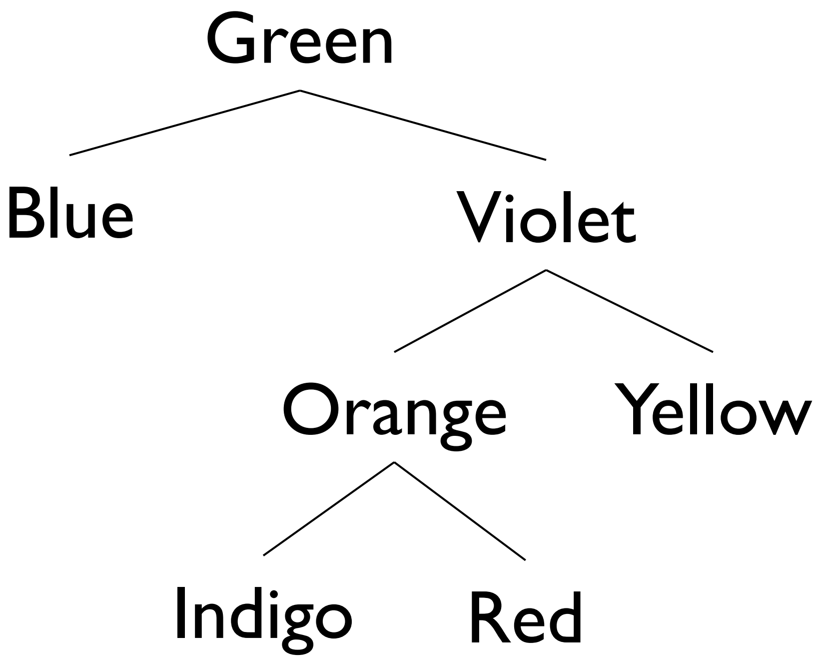
For in-order and post-order: just move `process`(t)!

But what about level-order???

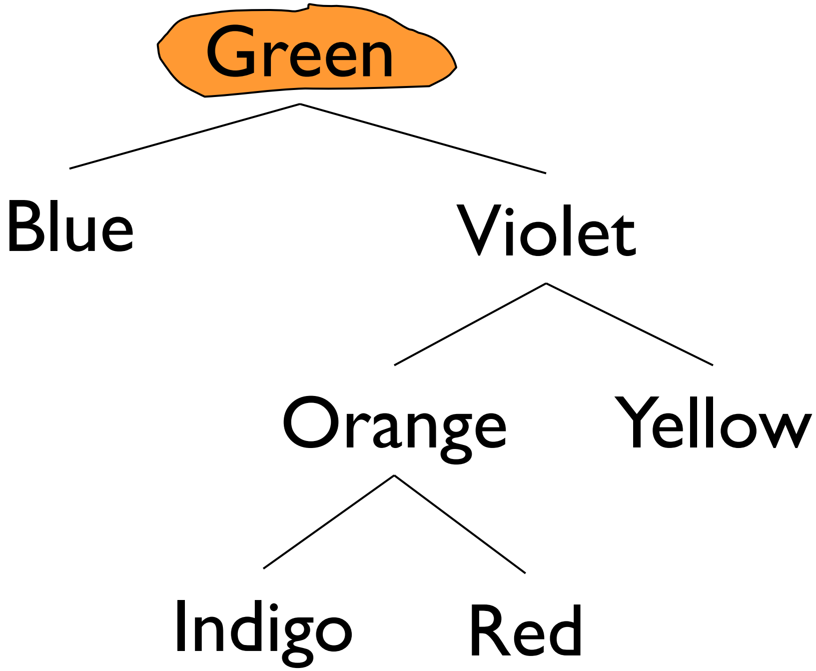
Level-Order Traversal



Level-Order Traversal



Level-Order Traversal



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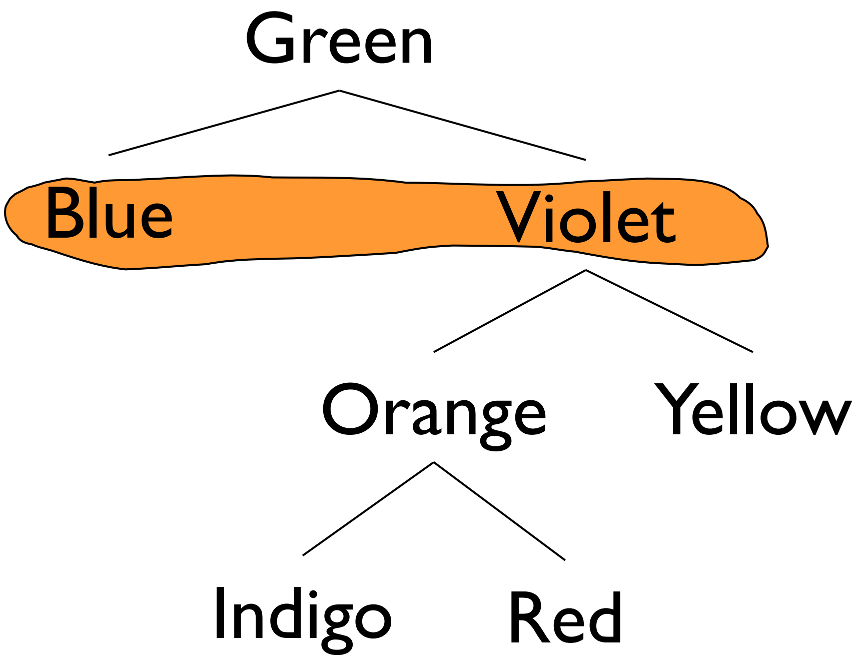
Level-Order Traversal



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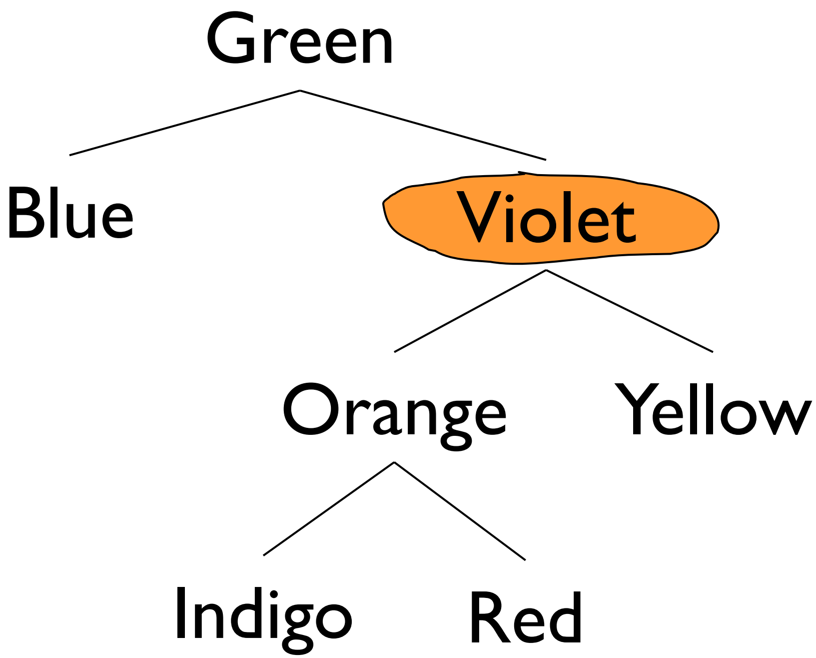
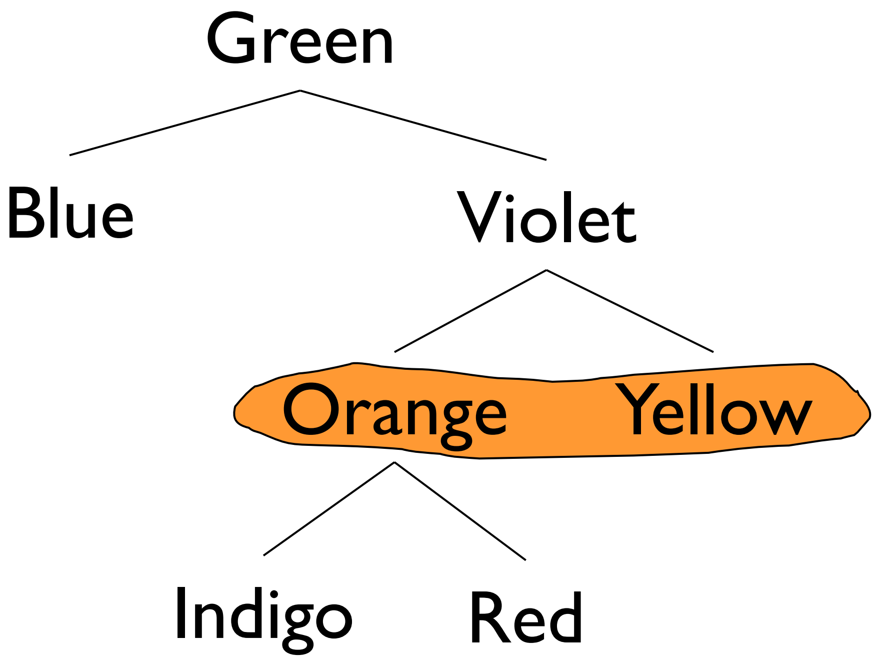
Level-Order Traversal



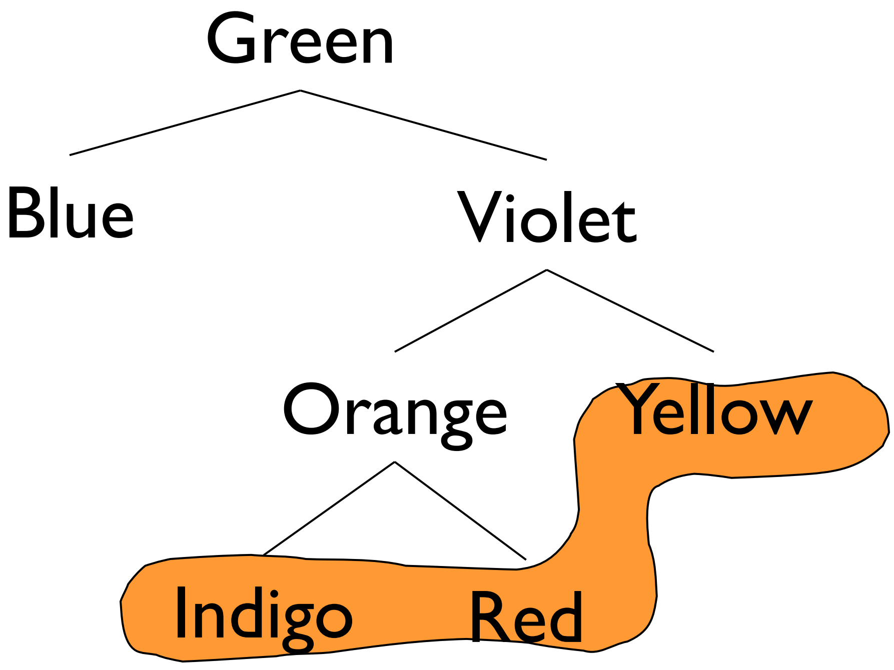
G B

Level-Order Traversal



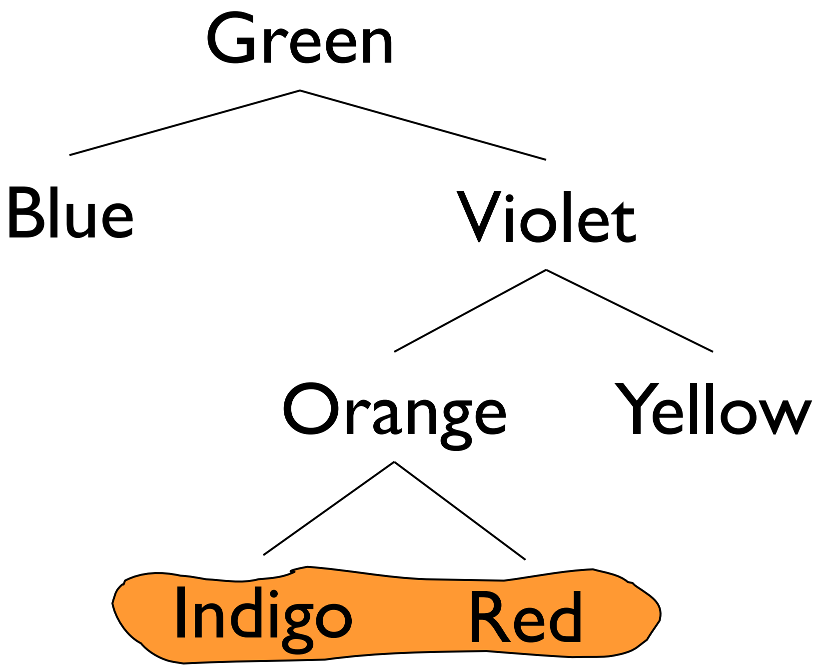
G B V

Level-Order Traversal



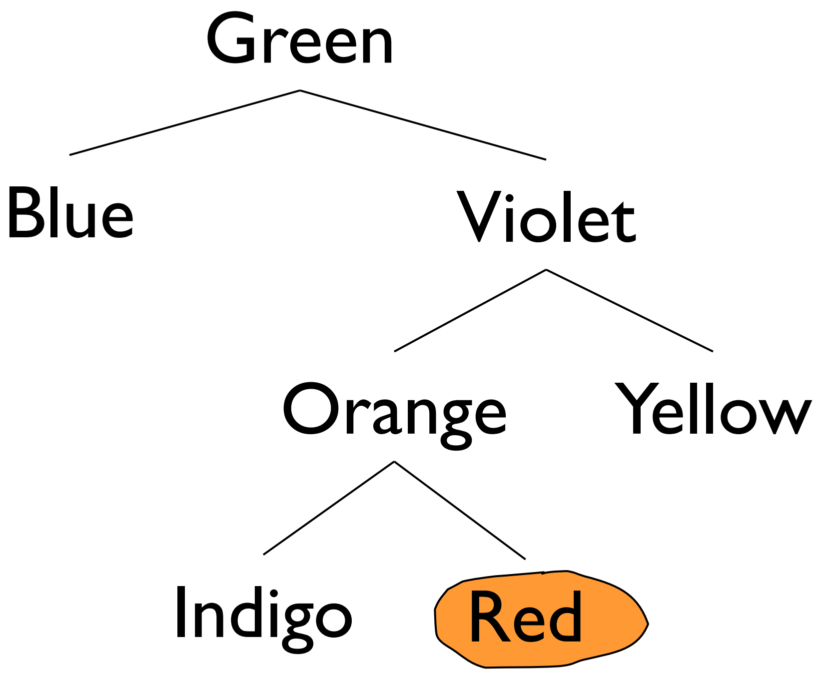
G B V O

Level-Order Traversal



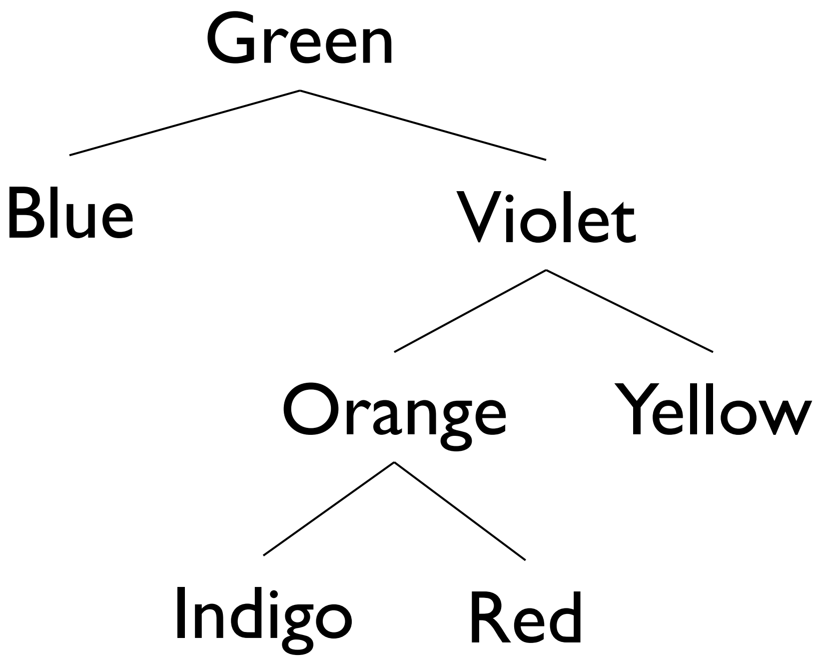
G B V O Y

Level-Order Traversal



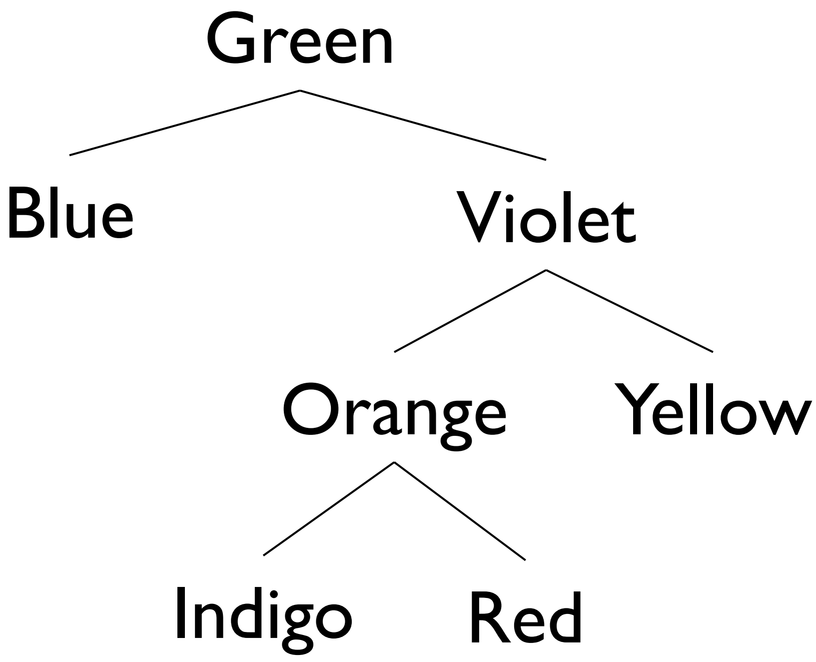
G B V O Y I

Level-Order Traversal



G B V O Y I R

Level-Order Traversal



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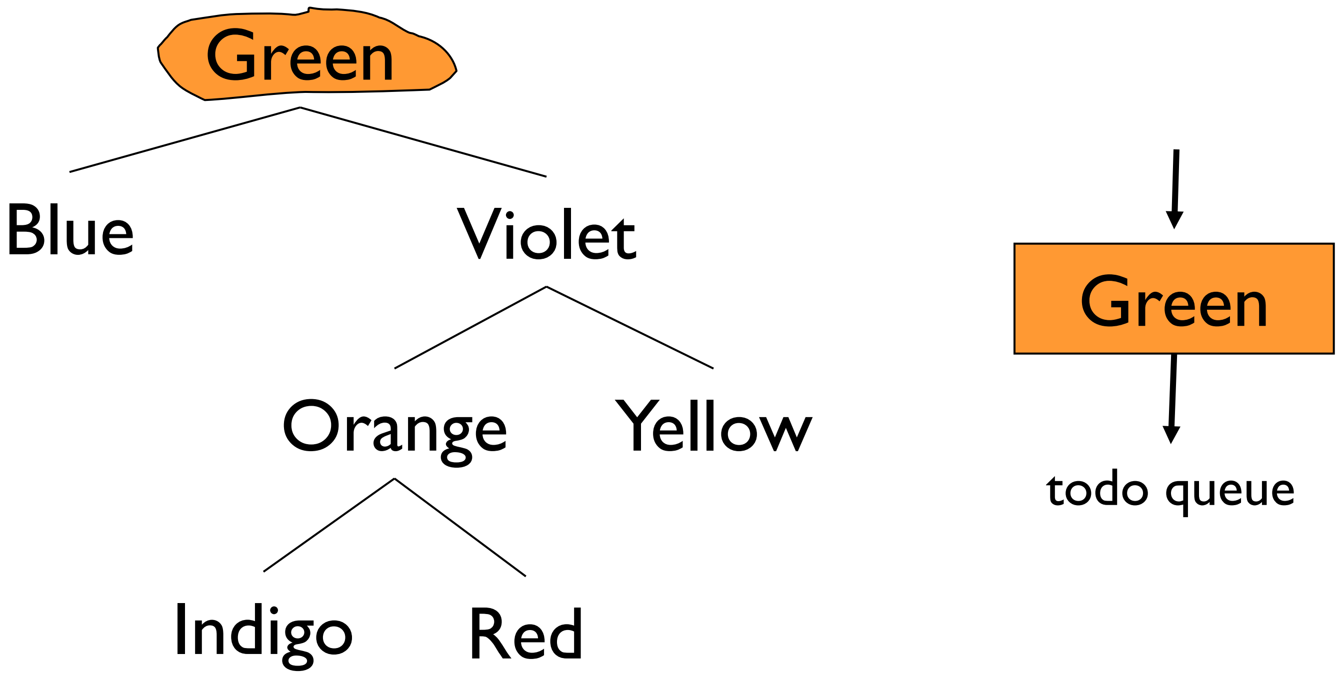
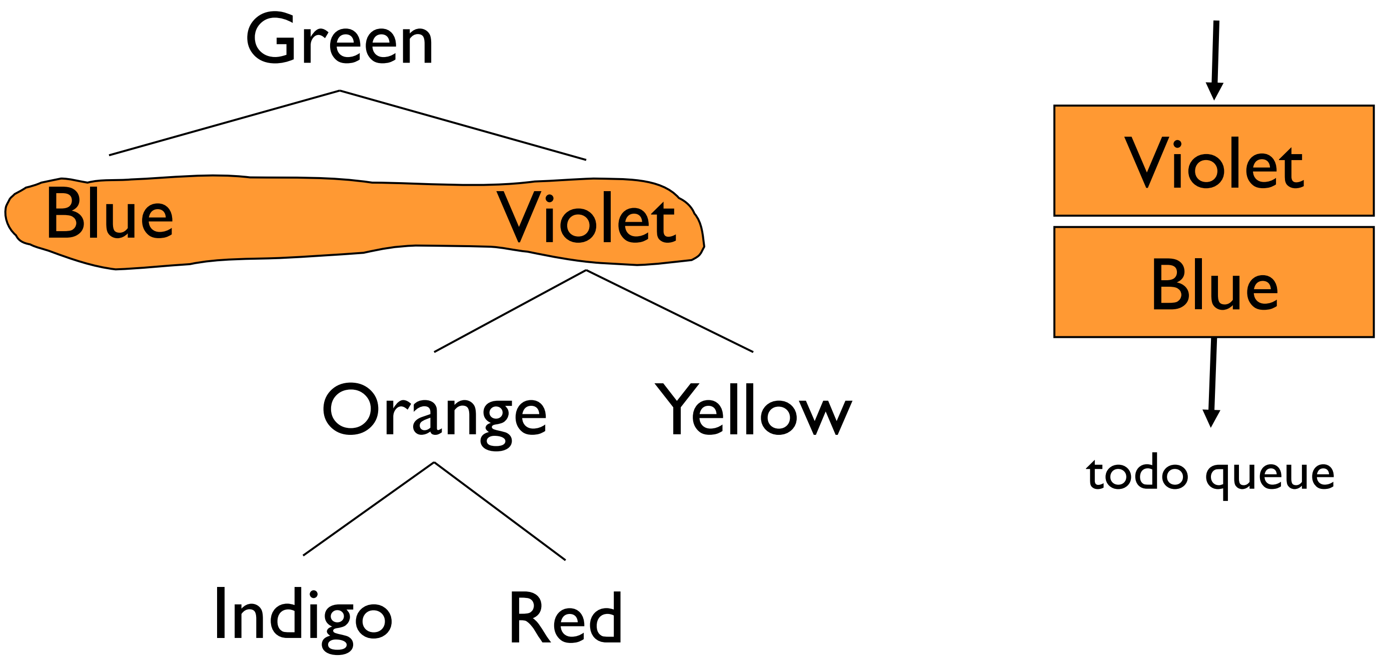


Level-Order Traversal



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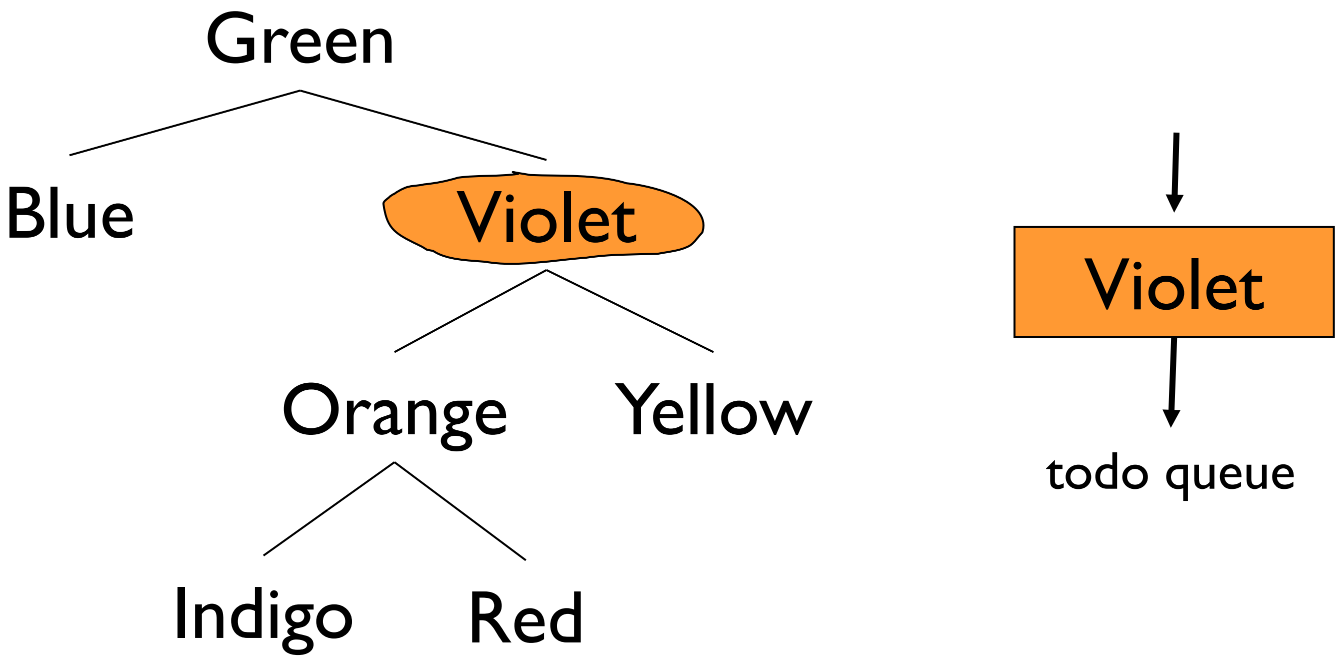
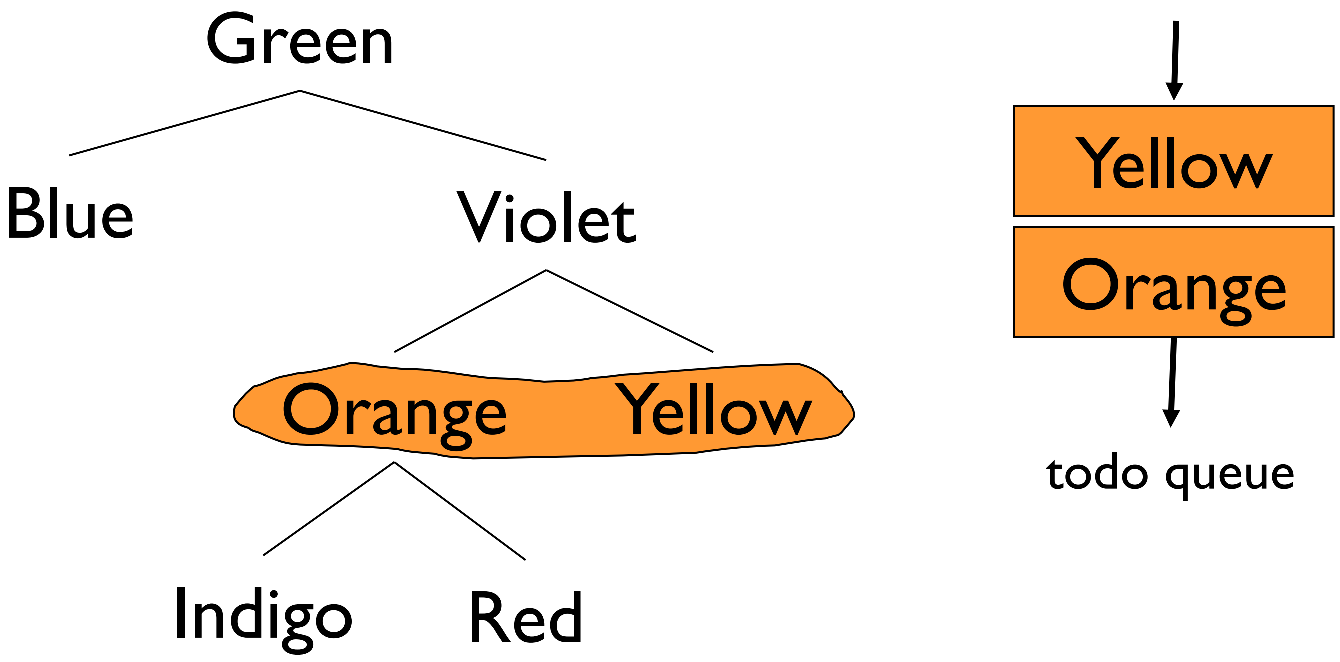
Level-Order Traversal



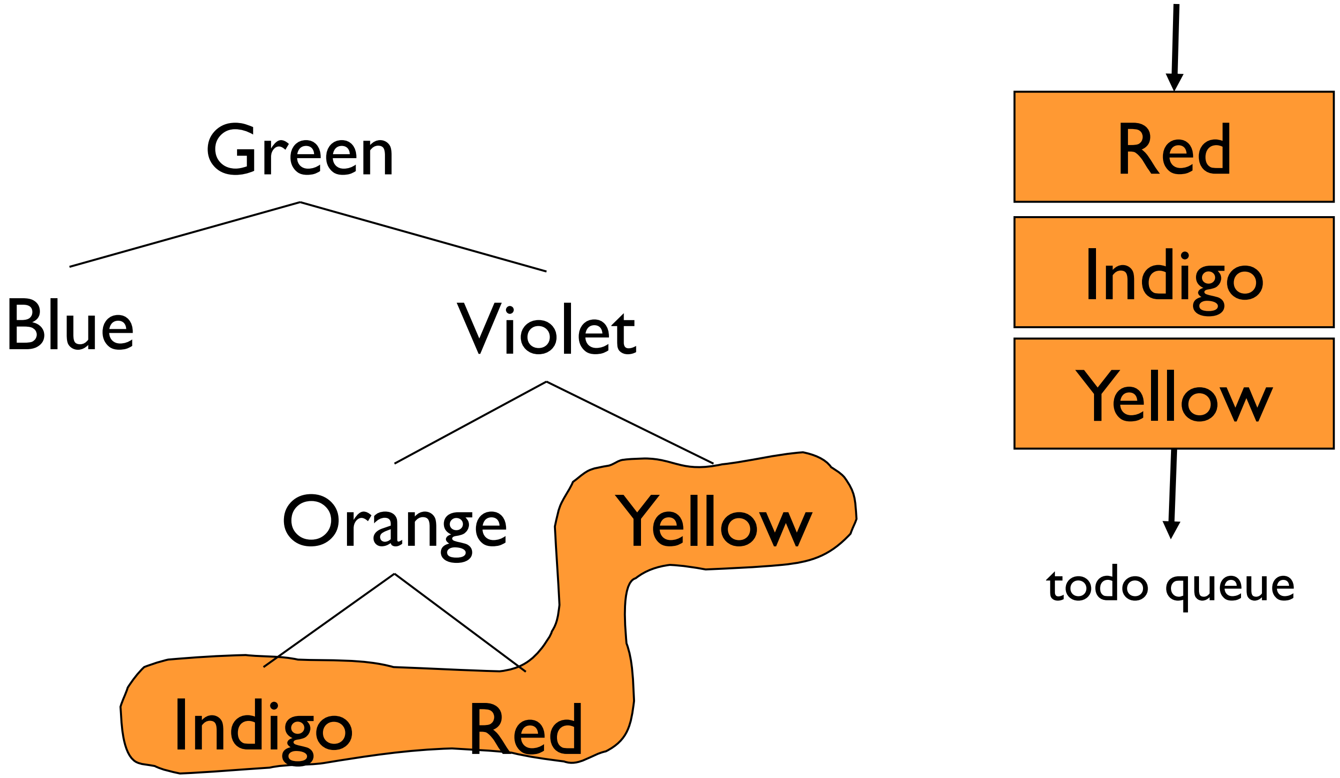
G B

Level-Order Traversal



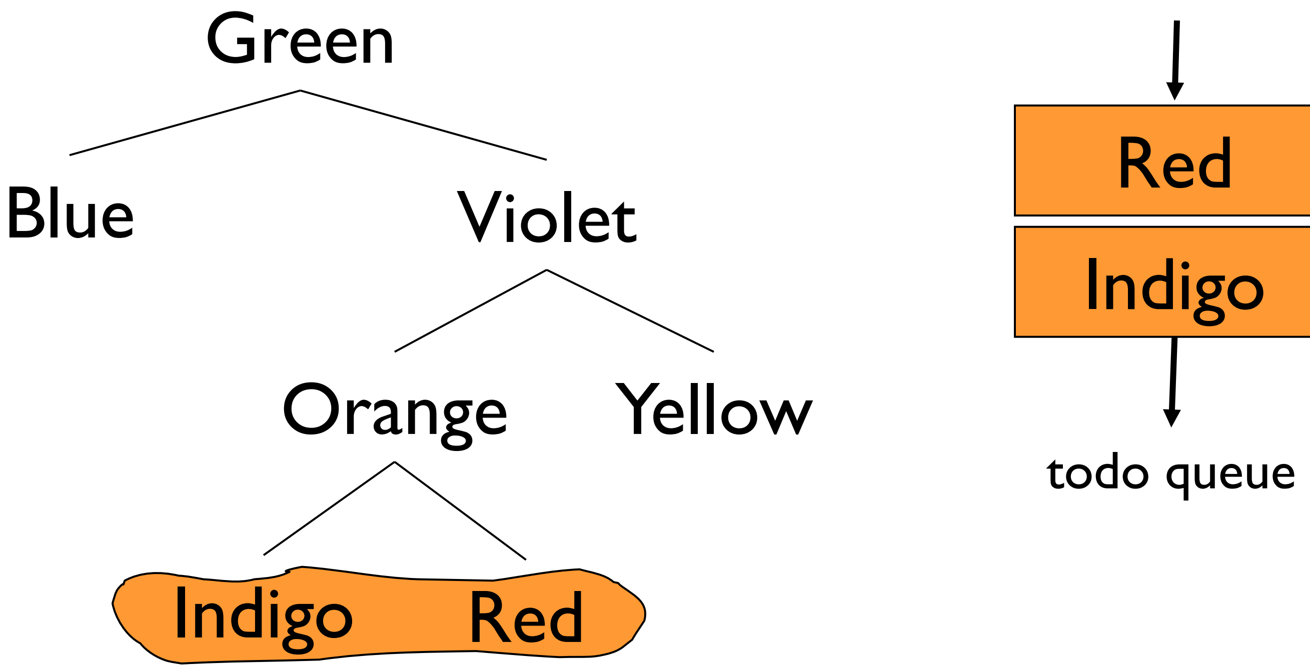
G B V

Level-Order Traversal



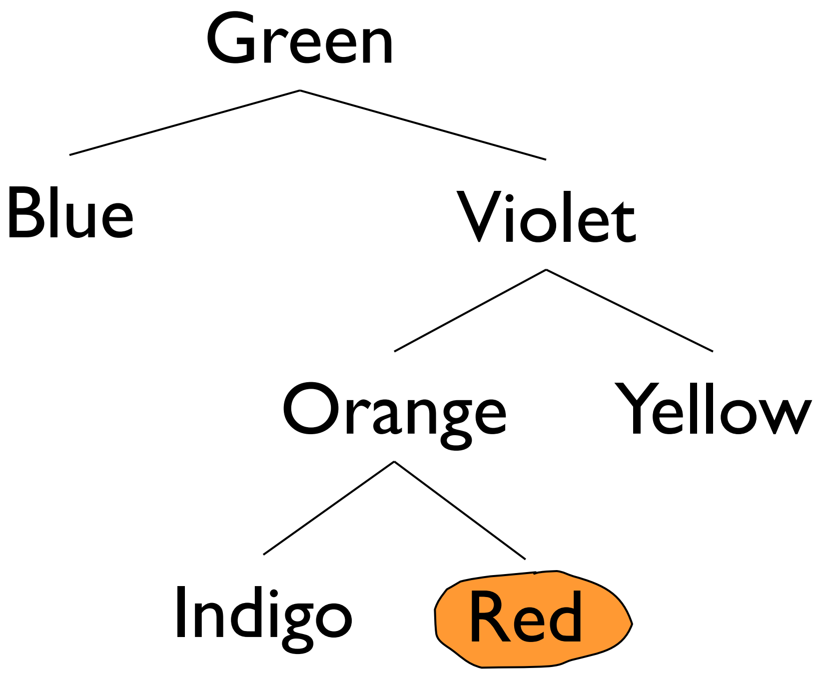
G B V O

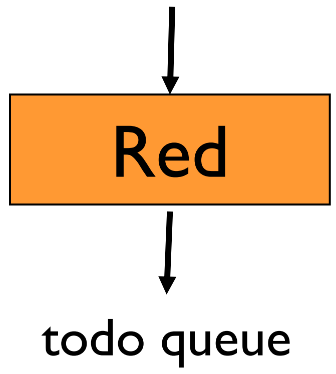
Level-Order Traversal



G B V O Y

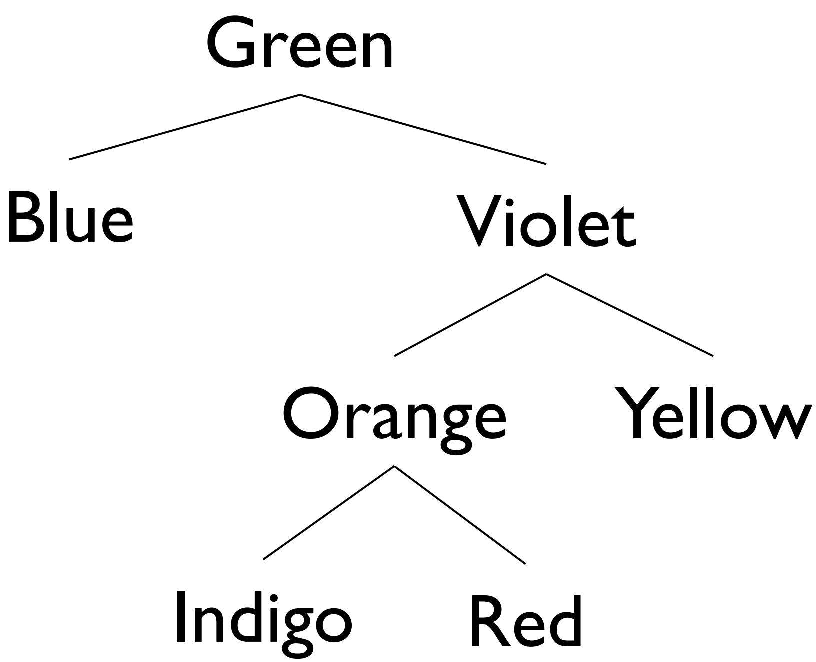
Level-Order Traversal

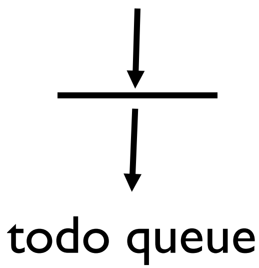




G B V O Y I

Level-Order Traversal





G B V O Y I R

Level-Order Tree Traversal

```
public static <E> void levelOrder(BinaryTree<E> t) {
   if (t.isEmpty()) return;

   // The queue holds nodes for in-order processing
   Queue<BinaryTree<E>> q = new QueueList<BinaryTree<E>>();
   q.enqueue(t); // put root of tree in queue

   while(!q.isEmpty()) {
      BinaryTree<E> next = q.dequeue();
      process(next);
      if(!next.left().isEmpty()  ) q.enqueue( next.left() );
      if(!next.right().isEmpty() ) q.enqueue(next.right());
   }
}
```

Pre-Order Tree Traversal

```
public static <E> void preOrder(BinaryTree<E> t) {
   if (t.isEmpty()) return;

   // The stack holds nodes for in-order processing
   Stack<BinaryTree<E>> st = new StackList<BinaryTree<E>>();
   st.push(t); // put root of tree in stack

   while(!st.isEmpty()) {
      BinaryTree<E> next = st.pop();
      process(next);
      if(!next.right().isEmpty() ) st.push(next.right());
      if(!next.left().isEmpty()  ) st.push( next.left() );
   }
}
```

Pre-Order Tree Traversal

Is this really a pre-order traversal?

How could we convince ourselves?

Let's prove it by induction!

Claim: Stack-based preOrder(t) processes the nodes of the tree rooted at t in the same order as the recursive preOrder(t) method

Idea: Induction on size of t

Base Case: t.size() = 0

- Both methods return, doing no other work. ✓

Pre-Order Tree Traversal

Induction Hypothesis

For some $n > 0$, iterative preOrder(t) processes the nodes of t in the same order as recursive preOrder(t) for all trees t having fewer than n nodes

Inductive Step

Now show that iterative preOrder(t) processes the nodes of t in the same order as recursive preOrder(t) for all trees t having n nodes

- Both methods process the root t first

Pre-Order Tree Traversal

- recursive preOrder(t) then processes the left sub-tree of t before the right subtree of t
- Iterative preOrder(t) will then pop t.left off the stack
- But now both methods are working with t.left, which has fewer nodes than t, and so both methods, by induction, process nodes in the same order
  - Note that iterative preOrder() will not pop t.right off its stack until all of the descendants of t.left have been processes
- Then they both process t.right in the same order (again, by induction) ✓

Summary & Observations

We've seen 4 reasonable traversal methods for trees

They can be efficiently implemented using

- A queue to guide a level-order traversal, or
- A stack to guide a pre-order traversal
  - By storing different information on the stack, we can turn our pre-order traversal into either a post-order or an in-order traversal.
  - We'll explore this in the next video….