

CSCI 136
Data Structures &
Advanced Programming

Williams College

Linear Structures

- **General idea:** we impose *access restrictions* on our data structure, disallowing add/remove/access at arbitrary indices
 - **No** `get(int i), set(int i, E value)`
 - **No** `add(int i), remove(int i)`
- **Insight:** By limiting access, we can actually gain some utility—linear structures are useful building blocks with important use cases!

Examples: Dining Hall

- FIFO: First In – First Out (**Queue**)
 - Line at dining hall
- LIFO: Last In – First Out (**Stack**)
 - Pile of plates or cups at dining hall

Examples: Computer Science

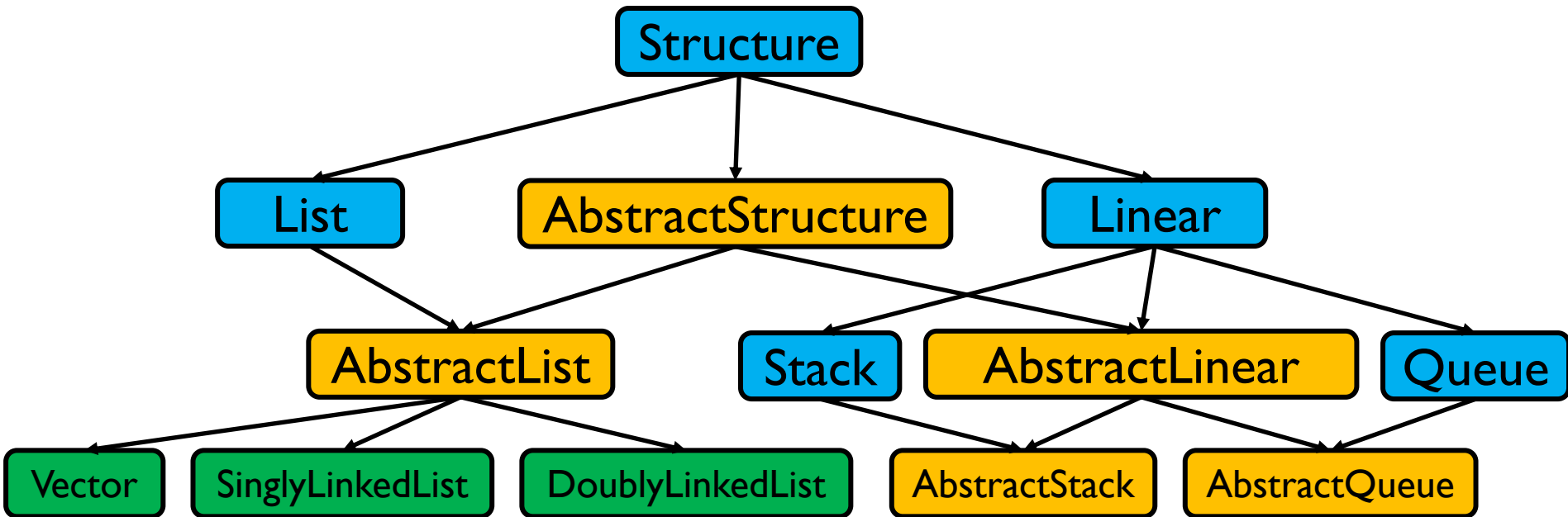
- FIFO: First In – First Out (**Queue**)
 - Data packets arriving at a router
- LIFO: Last In – First Out (**Stack**)
 - Java Virtual Machine stack

The Structure5 Universe (+ Linear!)

Interface

Abstract Class

Class



Quick Note about Terminology

- Note: Stack interface *extends* Linear interface
 - Interfaces *extend* other interfaces
 - Classes *implement* interfaces
- If you look at the structure5 [documentation for Linear](#), you will see:
 - A list of superinterfaces
 - A list of subinterfaces
 - A list of implementing classes

Linear Interface

- How should `Linear` interface differ from `List`?
 - Should have fewer methods than `List` interface since we are **limiting access** ...
- **Methods:**
 - Inherits all of the `Structure` interface methods
 - `add(E value)` – Add `value` to the structure.
 - `E remove(E o)` – Remove `value o` from the structure.
 - `size()`, `isEmpty()`, `clear()`, `contains(E val)`, ...
 - Adds new methods
 - `E get()` – Preview the *next* object to be removed.
 - `E remove()` – Remove the *next* value from the structure.
 - `boolean empty()` – same as `isEmpty()`

AbstractStack

- What methods do we *need* to define?
 - Stack interface methods
- Stack introduces new terms: push, pop, peek
 - Only use push, pop, peek when talking about stacks (not queues)
 - push = add to top of stack
 - pop = remove from top of stack
 - peek = look at top of stack (do not remove)

Linear Structure Philosophy

- Why no “random access”? (i.e., no access to middle of list)
 - Supporting/Providing less functionality can yield:
 - Simpler implementations of our algorithms
 - Greater algorithmic efficiency
- What should be our Data structure implementation approach?
 - Use existing structures (Vector, LinkedList), or
 - Reimplement “stripped down” versions of those structures (same underlying organization) simplified

Stack Implementations

- Array-based stack
 - `int top, Object data[]`
 - Add/remove from index `top`

+ all operations are $O(1)$
– wasted/run out of space
- Vector-based stack
 - Vector data
 - Add/remove from tail

+/- most ops are $O(1)$ (add is $O(n)$ in worst case)
– potentially wasted space
- List-based stack
 - SLL data
 - Add/remove from *head*

+ all operations are $O(1)$
+/- $O(n)$ space overhead
(no “wasted” space)

Stack Implementations

- `structure5.StackArray`
 - `int top, Object data[]`
 - Add/remove from index `top`

+ all operations are $O(1)$
– wasted/run out of space
- `structure5.StackVector`
 - Vector data
 - Add/remove from tail

+/- most ops are $O(1)$ (add is $O(n)$ in worst case but amortized $O(1)$)
– potentially wasted space
- `structure5.StackList`
 - SLL data
 - Add/remove from head

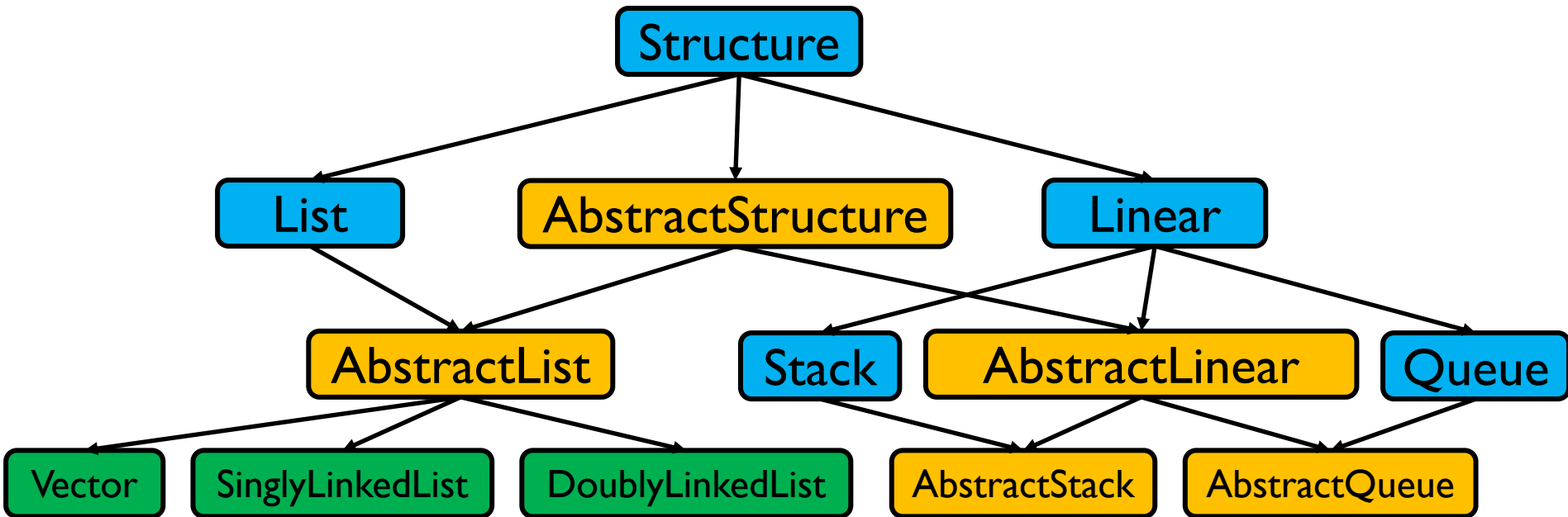
+ all operations are $O(1)$
+/- $O(n)$ space overhead
(no “wasted” space)

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Summary Notes on The Hierarchy

- `Linear` interface *extends* `Structure`
 - `add(E val)`
 - `empty()`
 - `get()`
 - `remove()`
 - `size()`
- `AbstractLinear` (partially) *implements* `Linear`
- `AbstractStack` class (partially) *extends* `AbstractLinear`
 - Essentially introduces “stack-ish” names for linear methods
 - `push(E val)` is `add(E val)`
 - `pop()` is `remove()`
 - `peek()` is `get()`

Rounding Out The Hierarchy

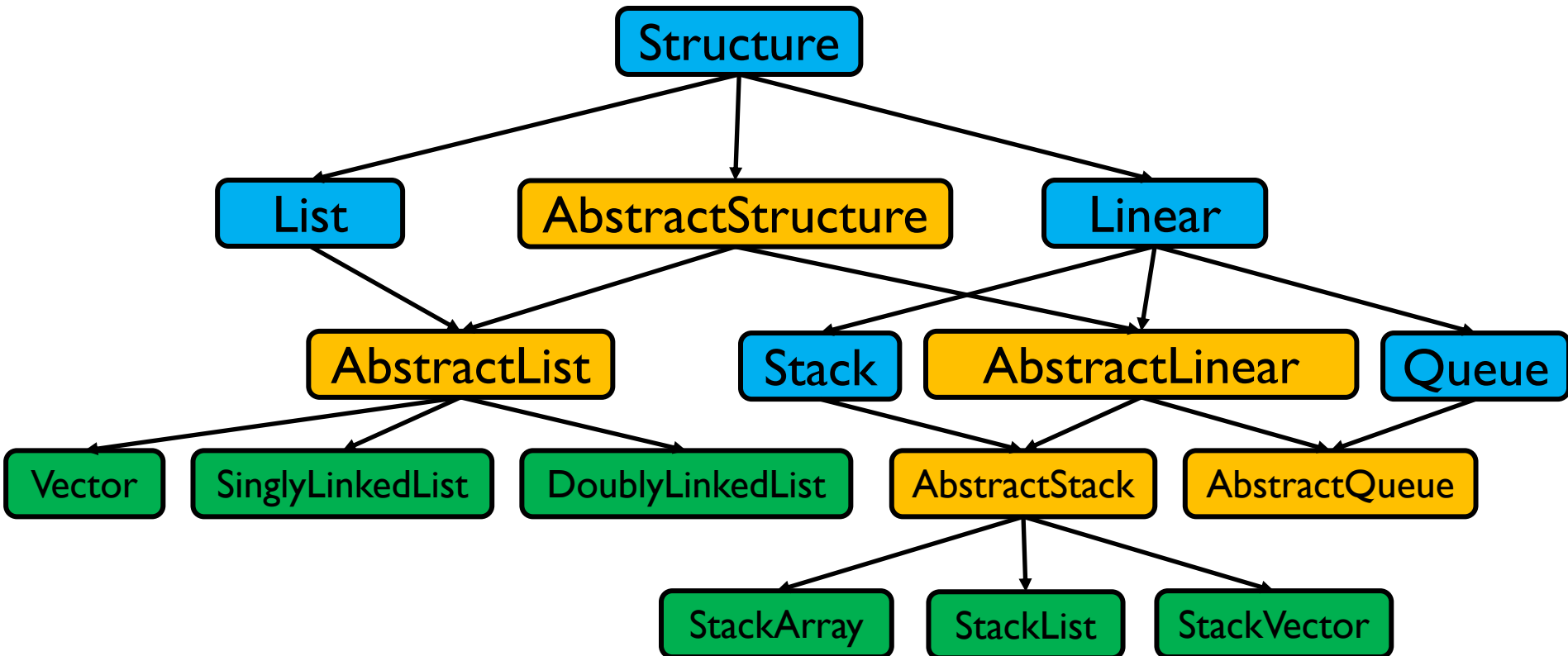
- Rundown of classes that extend `AbstractStack`:
 - `StackArray<E>`
 - holds an array of type E
 - add/remove at high end
 - Can't add once the array fills
 - `StackVector<E>`
 - Similar to `StackArray<E>`, but with a vector for dynamic growth
 - `StackList<E>`
 - A singly-linked list with add/remove at head
 - For each, we implement `add`, `empty`, `get`, `remove`, `size` directly
 - `push`, `pop`, `peek` are indirectly implemented by abstract class

The Structure5 Universe (+ Stacks!)

Interface

Abstract Class

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Video Goals

- Describe a few real-world problems
- Describe how to map one of those problems to the stack abstract data type
- Work through some examples to give us experience with (and appreciated of) stacks

Stack Applications

- The Stack implementation is simple, but there are *many* applications, including:
 - Evaluating mathematical expressions
 - Searching (**Depth-first search**)
 - Removing recursion for optimization
 - ...



See textbook for details
because this is VERY useful!

Evaluating Arithmetic Expressions

- Computer programs regularly use stacks to evaluate arithmetic expressions (as does the HP-12C calculator if you want to be a CFA...)
- Example: $x*y+z$
 - First rewrite as $xy*z+$
 - *we'll look at this rewriting process in more detail soon*
 - Then:
 - push x
 - push y
 - * (*pop twice, multiply popped items, push result*)
 - push z
 - + (*pop twice, add popped items, push result*)

Converting Expressions

- We (humans) primarily use **infix** notation to evaluate expressions
 - $(x+y)*z$
- Computers traditionally used **postfix** (also called Reverse Polish) notation
 - $xy+z*$
 - Operators appear after operands, parentheses are not necessary
- How do we convert between the two?
 - (Compilers do this for us)

Converting Expressions

- Example: $x*y+z*w$
- Conversion
 - 1) Add full parentheses to preserve order of operations
 $((x*y)+(z*w))$
 - 2) Move all operators (+-*/) after operands
 $((xy*)(zw*)+)$
 - 3) Remove parentheses
 $xy*zw*+$

Use Stack to Evaluate Postfix Exp

- While there are input “tokens” (i.e., symbols) left:
 - Read the next token from input.
 - If the token is a value, push it onto the stack.
 - Else, the token is an operator that takes n arguments. (It is known that an operator takes n arguments by its definition.)
 - If there are fewer than n values on the stack → **error**.
 - Else, pop the top n values from the stack and:
 - Evaluate the operator, with the values as arguments.
 - Push the returned result, if any, back onto the stack.
 - The top value on the stack is the result of the calculation.
 - Note that results can be left on stack to be used in future computations:
 - Eg: $3\ 2\ *\ 4\ +$ followed by $5\ /$ yields 2 on top of stack

Symbolic Example: Converting then Evaluating

- $(x*y)+(z*w) \rightarrow xy*zw*+$
- Evaluate $xy*zw*+$:
 - Push x
 - Push y
 - Mult: Pop y, Pop x, Push $x*y$
 - Push z
 - Push w
 - Mult: Pop w, Pop z, Push $z*w$
 - Add: Pop $x*y$, Pop $z*w$, Push $(x*y)+(z*w)$
 - Result is now on top of stack

Concrete Example: Converting then Evaluating

- $(x*y)+(z*w) \rightarrow xy*zw*+$
- Evaluate $xy*zw*+$:
 - Push x
 - Push y
 - Mult: Pop y, Pop x, Push $x*y$
 - Push z
 - Push w
 - Mult: Pop w, Pop z, Push $z*w$
 - Add: Pop $x*y$, Pop $z*w$, Push $(x*y)+(z*w)$
 - Result is now on top of stack
- Try with: $w=3, x=4, y=5, z=6$

PostScript

- PostScript is a programming language used for generating vector graphics
 - Best-known application: describing pages to printers
- It is a stack-based language
 - Values are put on stack
 - Operators pop values from stack, put result back on
 - There are numeric, logic, string values
 - Many operators
- Let's try it: The 'gs' command runs a PostScript interpreter....
- Implementing a tiny part of gs is something we will do in lab... it's a lot of fun!