CSCI 136
Data Structures &
Advanced Programming

Recursion

Fall 2020

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Recursion

Recursion

- General problem-solving strategy
  - Decompose problem into sub-problems
    - Sub-problem : Simpler version of same problem
  - Solve sub-problems
    - Either by further decomposition
    - Or directly, if sub-problem is easy
  - Combine sub-problem solutions to build problem solution

Recursion

- Many algorithms are recursive
  - Recursive algorithms are often easier to
    - Understand and implement
    - Prove correct
- In this presentation, we
  - Review recursion
  - Introduce techniques for reasoning about recursive algorithms

A Simple Example : Factorial

- $n!$ (pronounced "n factorial") denotes the product of the first n positive integers

$$n! = n \cdot (n-1) \cdot (n-2) \cdot \cdots \cdot 2 \cdot 1$$

- Note: $n!$ is only defined for $n \geq 1$, although by convention we define $0! = 1$
- We could compute $n!$ with a for loop…

```
int product = 1;
for(int i = 1;i <= n; i++)
    product *= i;
```

- But we could also write it recursively….

Factorial

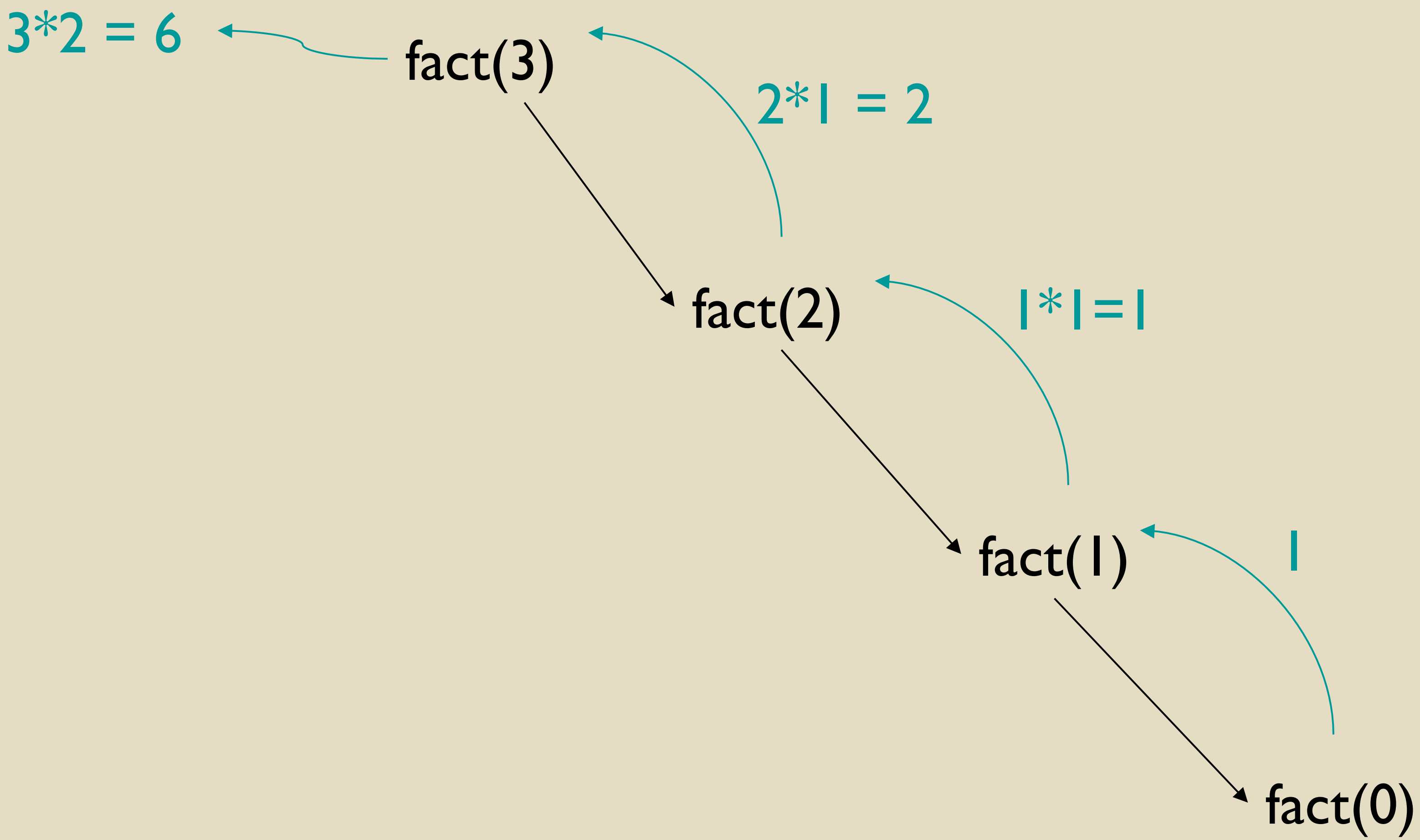
$$n! = n \cdot (n-1) \cdot (n-2) \cdot \cdots \cdot 2 \cdot 1$$

- Recursive definition (what "…" really means!)
  - $n! = n \cdot (n-1)!$
  - 1! = 1 (or 0! = 1)

```
// Pre: n >= 0
public static int fact(int n) {
    if (n==0) return 1;
    else return n*fact(n-1);
}
```

Factorial

```
int k = fact(3);
```



Factorial

- In recursion, we always use the same basic approach
- What's our easy case? [Sometimes "cases"]
  - `n=0 : fact(0) = 1`
- What's the recursive relationship?
  - `n>0 : fact(n) = n·fact(n-1)`

Recursive Method Structure

(simple version)

my_recursive_method(data) {

if (data is simple) // typically called "the base case"

solve directly

else {

divide data into $data_1$, …, $data_k$

call my_recursive_method on each of $data_1$, …, $data_k$
and combine the results as needed

Example : Fibonacci Numbers

- 1, 1, 2, 3, 5, 8, 13, ...
- Definition

$$F_0 = 1, F_1 = 1$$

$$F_n = F_{n-1} + F_{n-2} \; (for \; n > 1)$$

- Inherently recursive!
- It appears almost everywhere
  - Growth: Populations, plant features
  - Architecture
  - Data Structures!

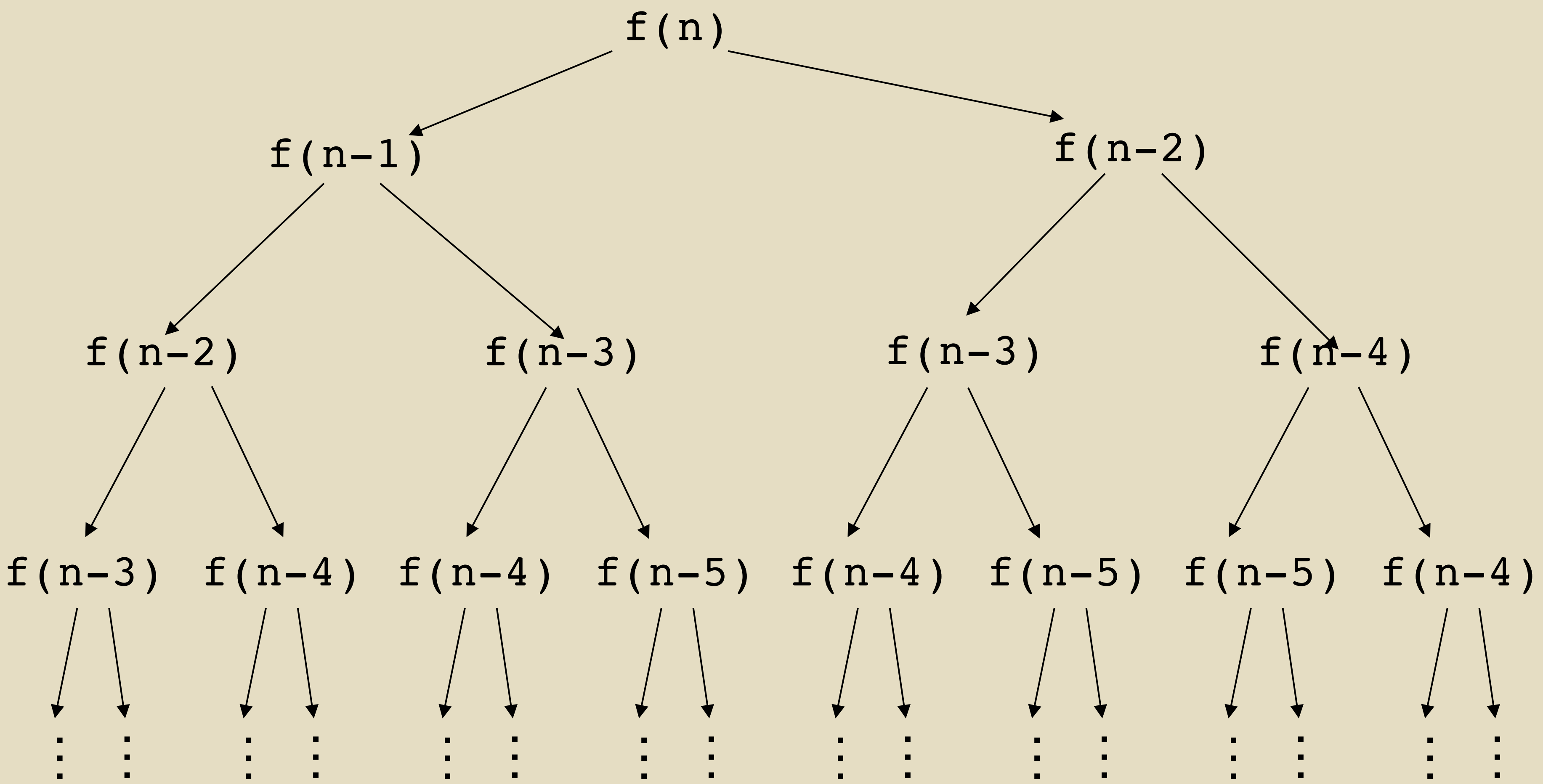
fib.java

```
public class fib{
   // pre: n is non-negative
   public static int fib(int n) {

       if (n==0 || n == 1)              // Note: Two base cases!
               return 1;
       else
               return fib(n – 1) + fib(n – 2);
   }

   public static void main(String args[]) {
        // This method could be a single line but: readability!
        int n = Integer.valueOf(args[0]);
        int result = fib(n);
        System.out.println(result);
   }
}
```

Demo: RecursiveMethods.java….

Question: Why is fib so slow?!

Recursive Fibonacci Method



Yikes!

Recursion Tradeoffs

- Advantages
  - Often easier to construct recursive solution
  - Code is usually cleaner
  - Some problems do not have obvious non-recursive solutions
- Disadvantages
  - Overhead of recursive calls
  - Can use lots of memory (need to store state for each recursive call until base case is reached)
    - E.g. recursive fibonacci method

Recursive Contains

Consider a slightly rewritten contains method for SinglyLinkedList

```
public static boolean contains(Node<String> n, String v) {
   if( n == null || v == null ) return false;

   while(n != null) {
        if( v.equals(n.value())) return true;
        n = n.next();
   }
   return false;
}
```

Now let's try a recursive approach

```
public static boolean contains(Node<String> n, String v) {
   if( n == null || v == null ) return false;

   return v.equals(n.value()) || contains(n.next(), v);
}
```

Nice!

Recursive Contains for Vector

Replace loop in contains method with recursive helper method

- Helper method will work on any *slice* of the array
  - Like python array slice, but only when fromIndex ≤ toIndex

```
public boolean contains(E elt) {
   return contains(elt, 0, size()-1); }

// Helper method: returns true if elt is stored in range from..to
public boolean contains(E elt, int from, int to) {
   if (from > to)
        return false; // Base case: empty range
   else
        return elt.equals(elementData[from]) ||
               contains(elt, from+1, to);
}
```

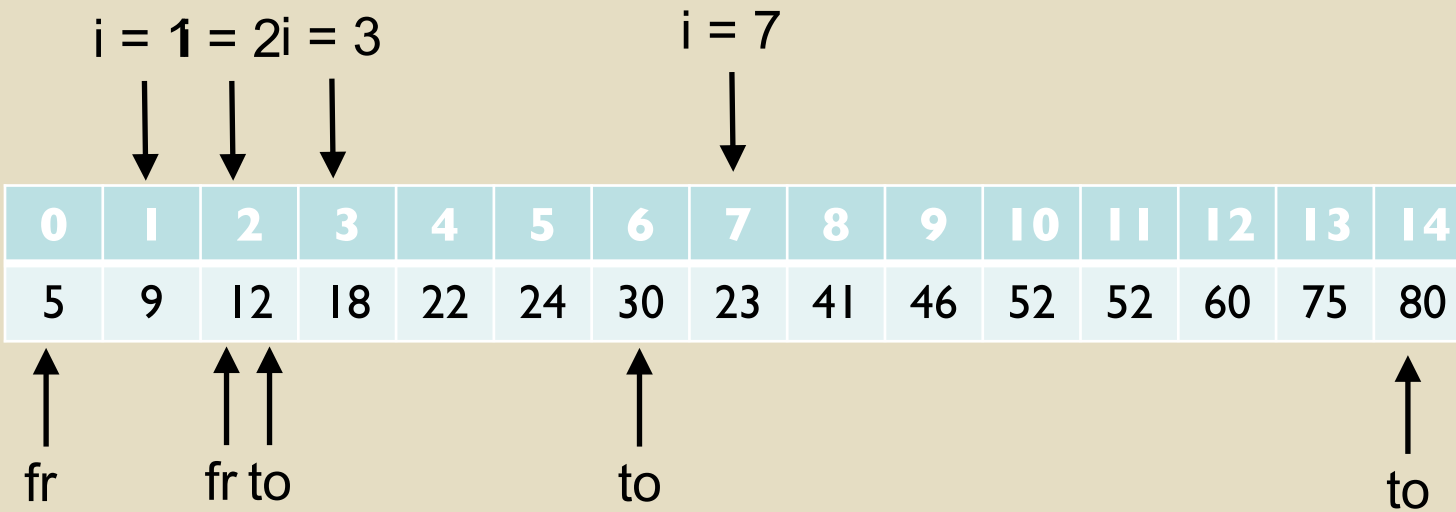
Binary Search in Sorted Array

- Given an array a[] of positive integers in increasing order, and an integer x, find location of x in a[].
  - Take "indexOf" approach: return -1 if x is not in a[]

- Idea
  - Compare x to "middle" element of array
    - If they are equal, return index of middle element
  - If x is smaller, recursively check " left half" of array
  - Otherwise, recursively check "right half" of array

Binary Search in Sorted Array

Let's picture the idea: Search for 12



0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
5	9	12	18	22	24	30	23	41	46	52	52	60	75	80

Notes

- Need to keep track of current *search range*: `fromLoc..toLoc`
- Need to know when search has failed
  - Search for 11 : Same sequence until failure

Binary Search Pseudo-Code

```
// Search arr[from..to] for value
// Interpret from > to as an empty range
int recBinSearch(int[] arr, int from, int to, value)
        if (from > to) return -1

        int mid = (to + from)/2
        if (value equals arr[mid])
                return mid
        elseif (value < arr[mid])
                return recBinSearch(arr, from, mid-1, value)
        else
                return recBinSearch(arr, mid+1, to, value)
```

Binary Search : Java Version

- Given an array a[] of positive integers in increasing order, and an integer x, find location of x in a[].
  - Take "indexOf" approach: return -1 if x is not in a[]

```
protected static int recBinarySearch(int a[], int value,
              int low, int high) {
   if (low > high) return -1;
   else {
        int mid = (low + high) / 2;       //find midpoint
        if (a[mid] == value) return mid;  //first comparison
                                          //second comparison
        else if (a[mid] < value)          //search upper half
        return recBinarySearch(a, value, mid + 1, high);
         else                             //search lower half
              return recBinarySearch(a, value, low, mid - 1);
   }
```

Alternative Pseudo-Code

```
// Pre: from ≤ to
boolean recBinSearch(int[] arr, int from, int to, value)
        if (from == to)
                if (value equals arr[from]) return true
                else return false

        int mid = ⌊(to + from)/2⌋          // round down
        if (value <= arr[mid])
                return recBinSearch(arr, from, mid, value)
        else
                return recBinSearch(arr, mid+1, to, value)
```

Only two tests in all but base case. Modestly better on average!

Next Episode : Mathematical Induction