

CSCI 136
Data Structures &
Advanced Programming

Traversing Trees using Iterators

Designing Tree Iterators

- **Goal:** design iterators to dispense items in the same order that the different tree traversal algorithms visit nodes.
- Methods provided by `BinaryTree` class:
 - `preorderIterator()`
 - `inorderIterator()`
 - `postorderIterator()`
 - `levelorderIterator()`

Implementing the Iterators

- Iterators should dispense values in same order as corresponding the traversal method
- **Challenge:** We must phrase algorithm steps in terms of `next ()` and `hasNext ()`
 - Recursive methods don't convert as easily, so, let's start with the most "straightforward" traversal order: level-order!

Level-Order Iterator

- Should return elements in same order as processed by level-order traversal method
 - Visit all nodes at depth i before visiting any node at depth $i+1$
- Must phrase in terms of `next ()` and `hasNext ()`
 - Basic Idea: We “capture” our traversal in a queue
 - The queue holds “to be visited” nodes

Level-Order Iterator

```
public BTLevelorderIterator(BinaryTree<E> root) {
    todo = new QueueList<BinaryTree<E>>();
    this.root = root; // needed for reset
    reset();
}

public void reset() {
    todo.clear();

    // empty queue, add root
    if (!root.isEmpty()) todo.enqueue(root);
}
```

Level-Order Iterator

```
public boolean hasNext() {  
    return !todo.isEmpty();  
}
```

```
public E next() {  
    BinaryTreeNode<E> current = todo.dequeue();  
    E result = current.value();  
    if (!current.left().isEmpty())  
        todo.enqueue(current.left());  
    if (!current.right().isEmpty())  
        todo.enqueue(current.right());  
    return result;  
}
```

Pre-Order Iterator

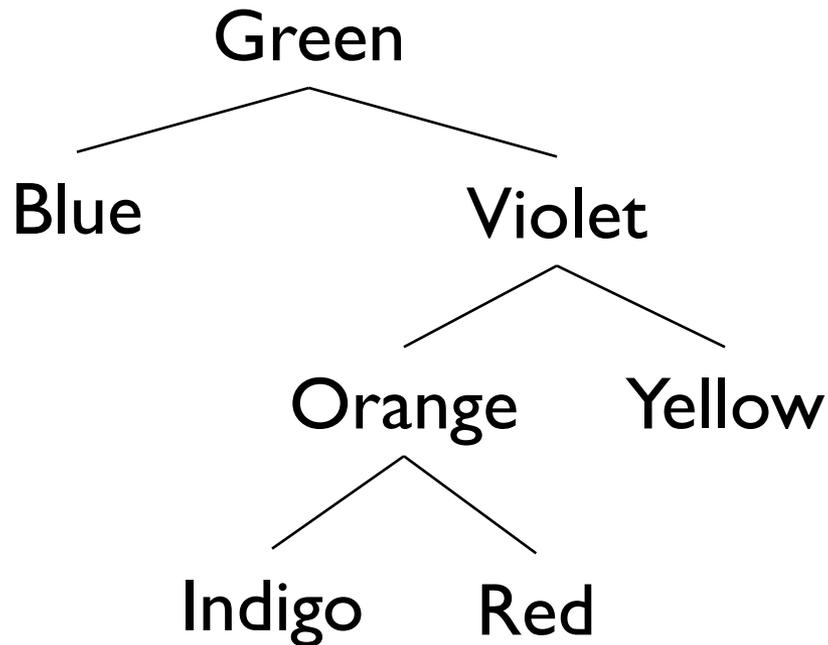
- Should return elements in same order as processed by pre-order traversal method:
 - Visit node, then left subtree, then right subtree
- Must phrase in terms of `next ()` and `hasNext ()`
- Basic idea: We “simulate recursion” with stack
 - The stack holds “partially processed” nodes

Pre-Order Iterator

- Order: node -> left subtree -> right subtree
 1. Constructor: Push root onto **TODO** stack
 2. On call to `next ()`:
 - Pop node from TODO stack
 - Push right and then left nodes of popped node onto TODO stack
 - Return popped node's value
 3. On call to `hasNext ()`:
 - `return !stack.isEmpty ()`

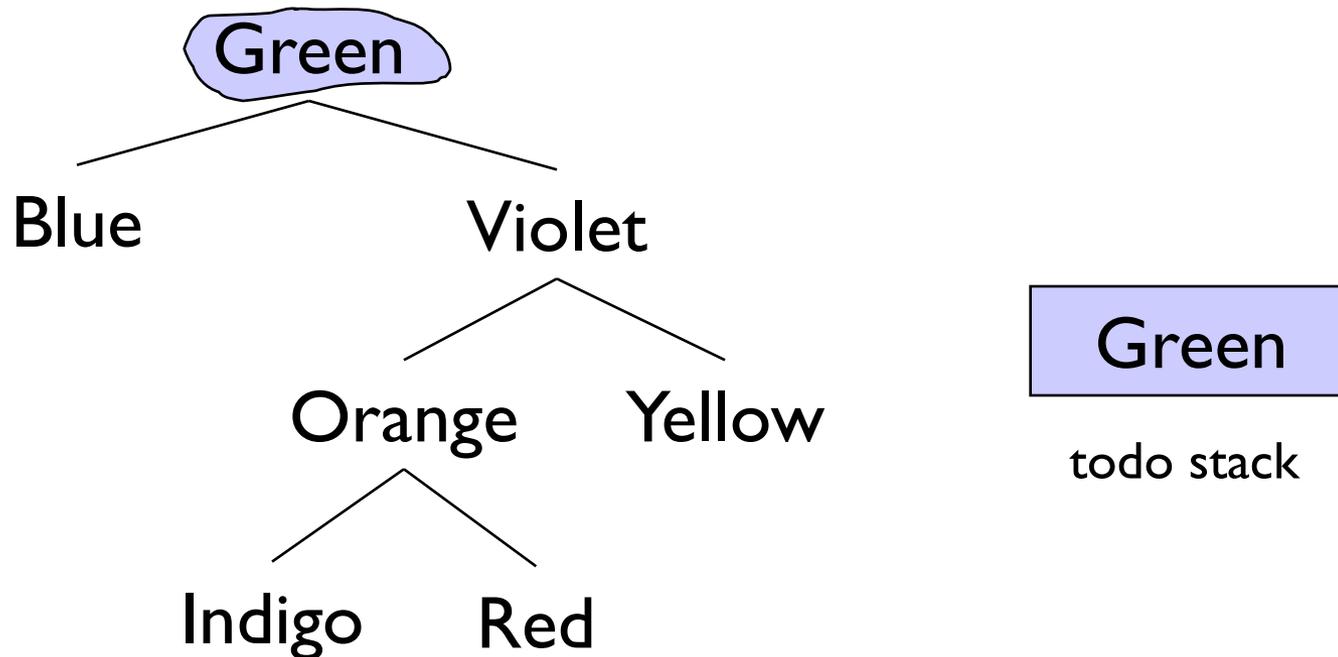
Pre-Order Iterator

Visit node, then each node in left subtree, then each node in right subtree.



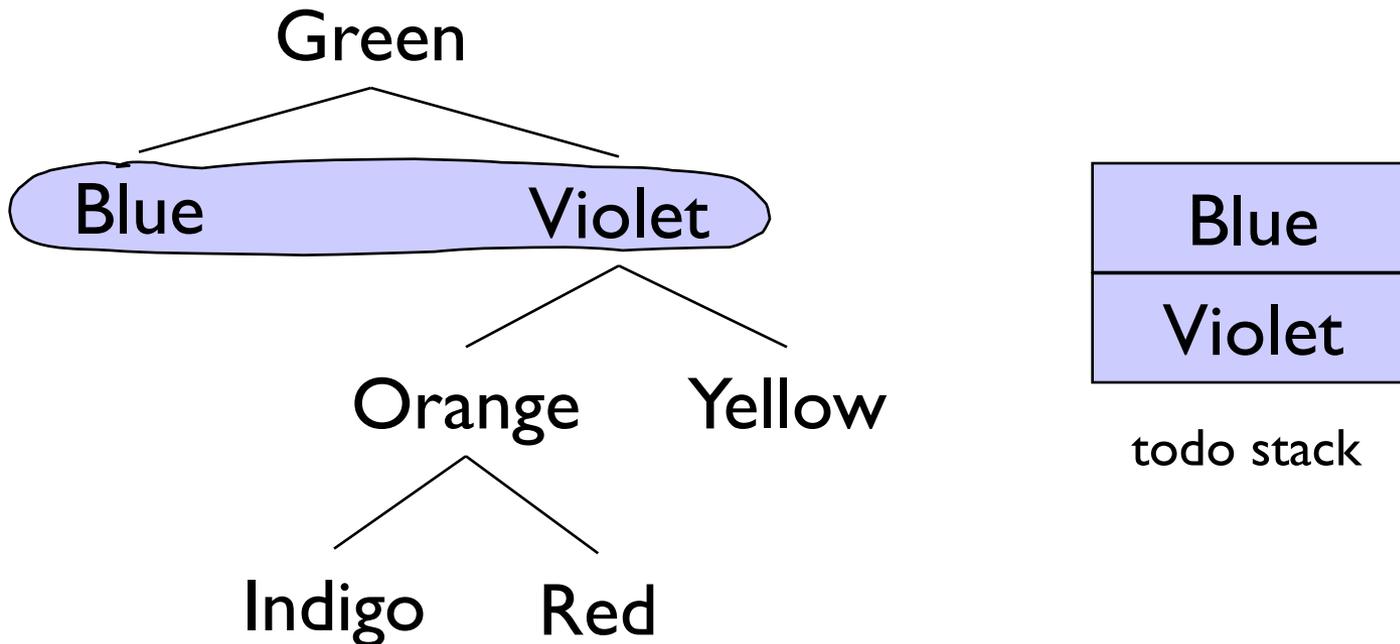
Pre-Order Iterator

Visit node, then each node in left subtree, then each node in right subtree.



Pre-Order Iterator

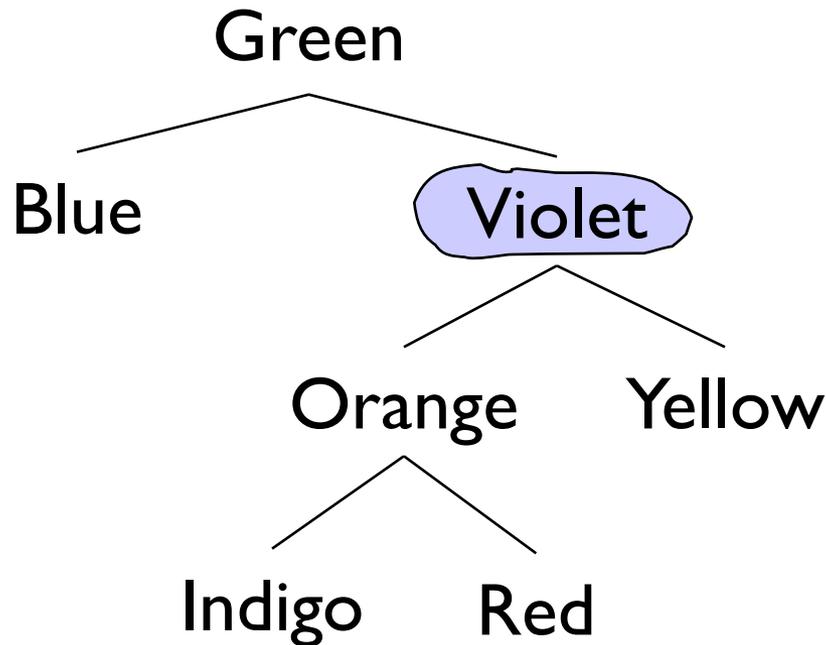
Visit node, then each node in left subtree, then each node in right subtree.



G

Pre-Order Iterator

Visit node, then each node in left subtree, then each node in right subtree.

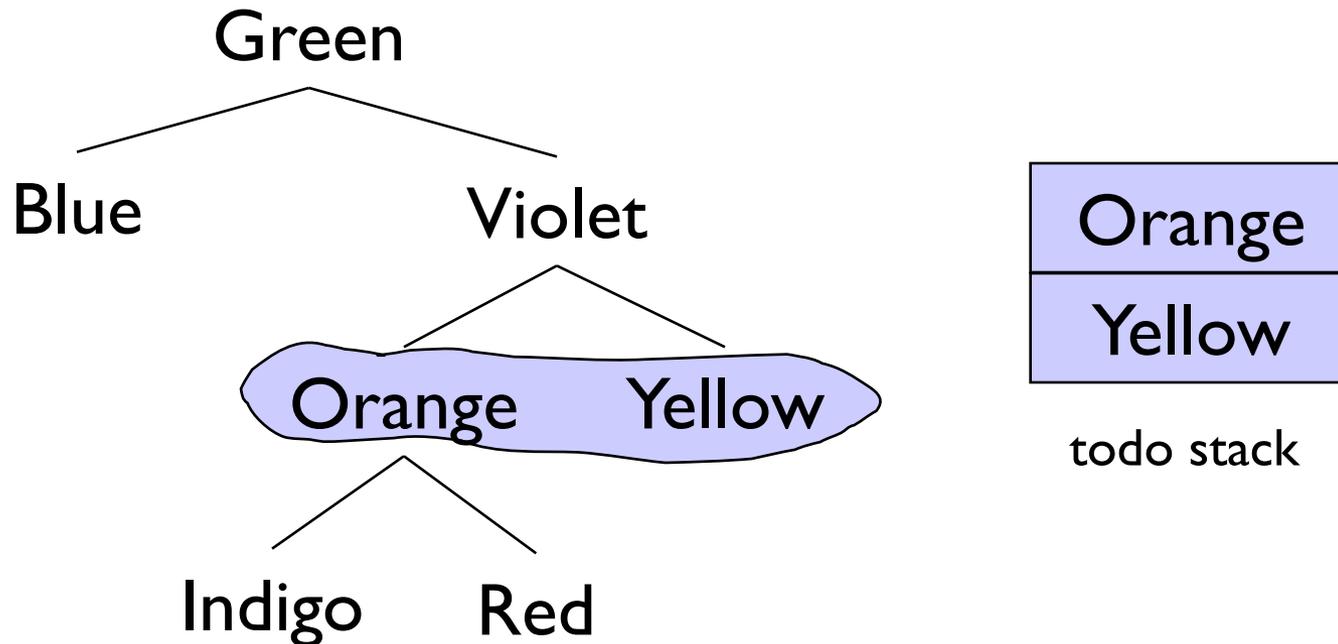


todo stack

G B

Pre-Order Iterator

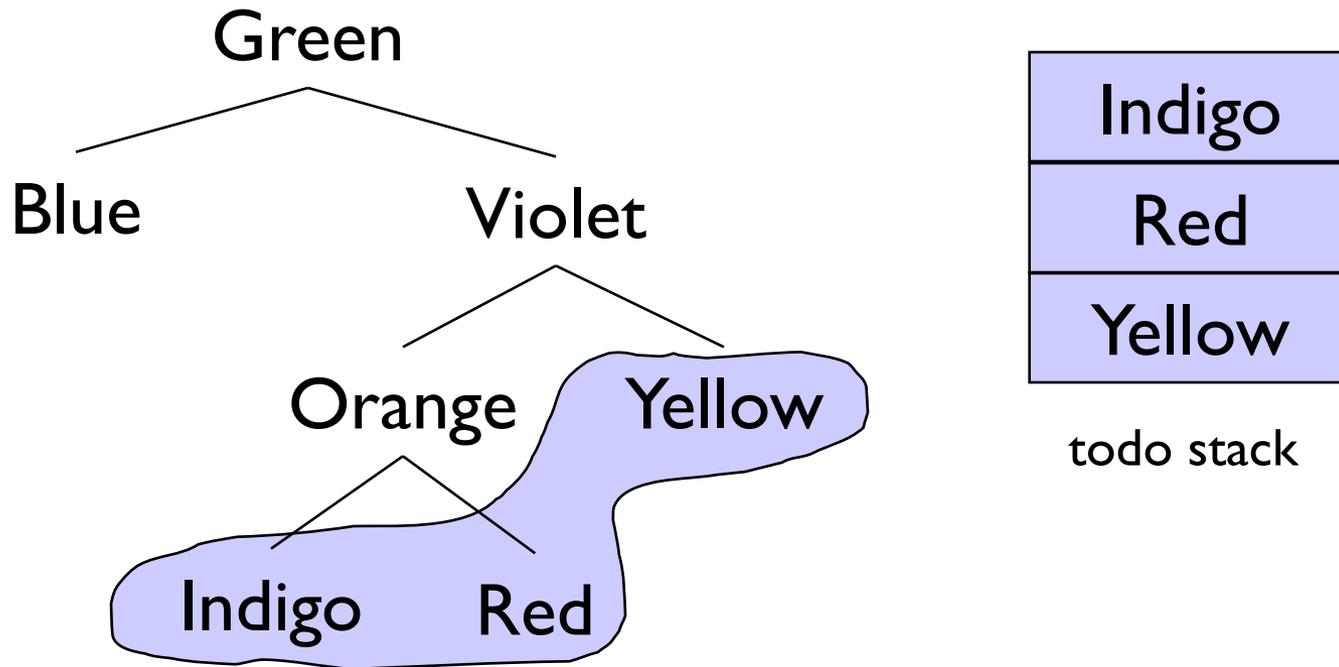
Visit node, then each node in left subtree, then each node in right subtree.



G B V

Pre-Order Iterator

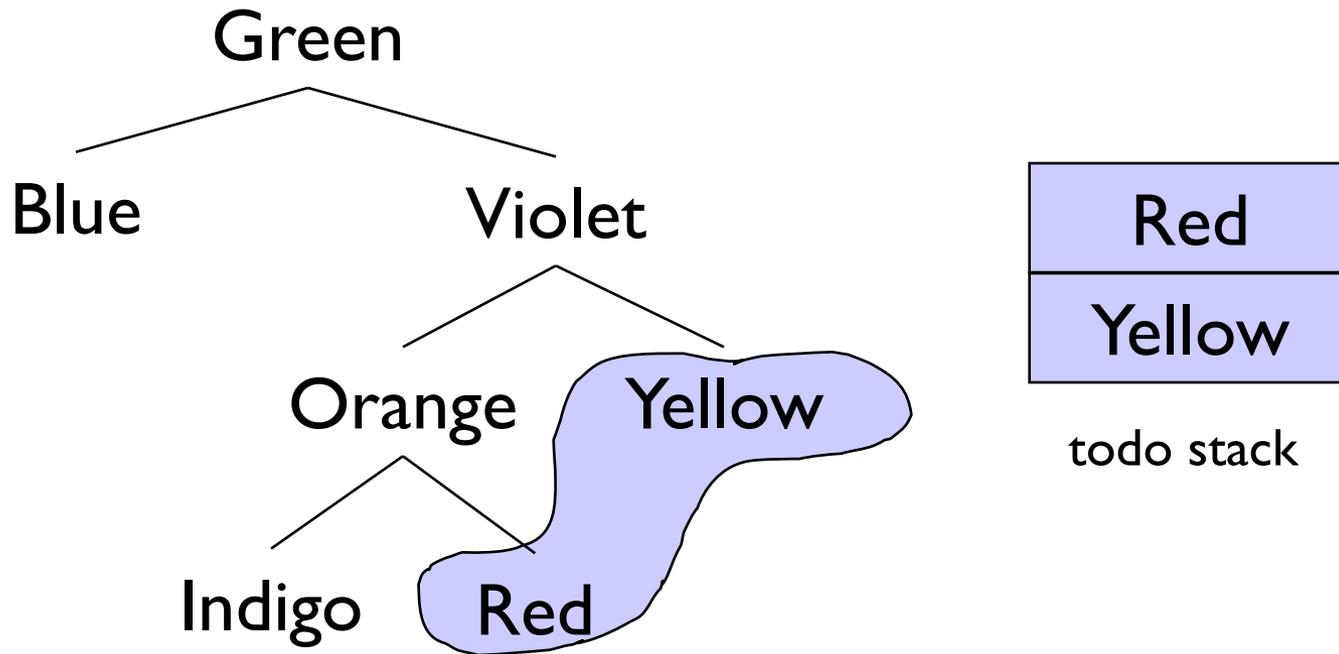
Visit node, then each node in left subtree, then each node in right subtree.



G B V O

Pre-Order Iterator

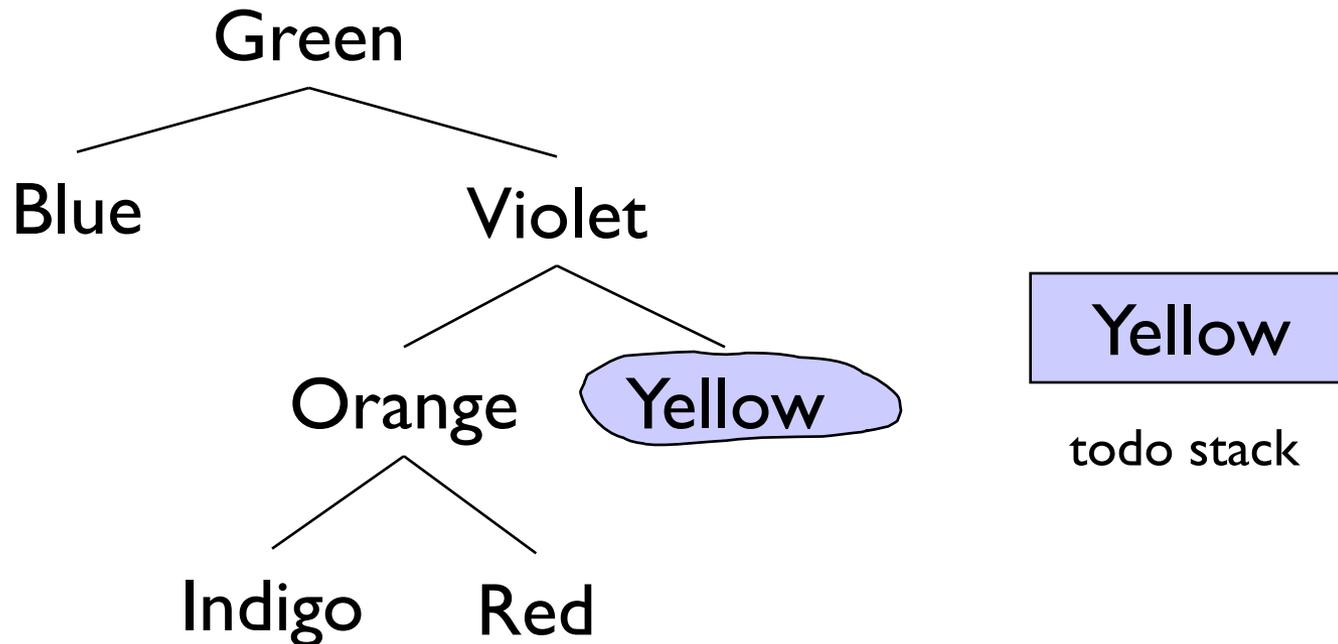
Visit node, then each node in left subtree, then each node in right subtree.



G B V O I

Pre-Order Iterator

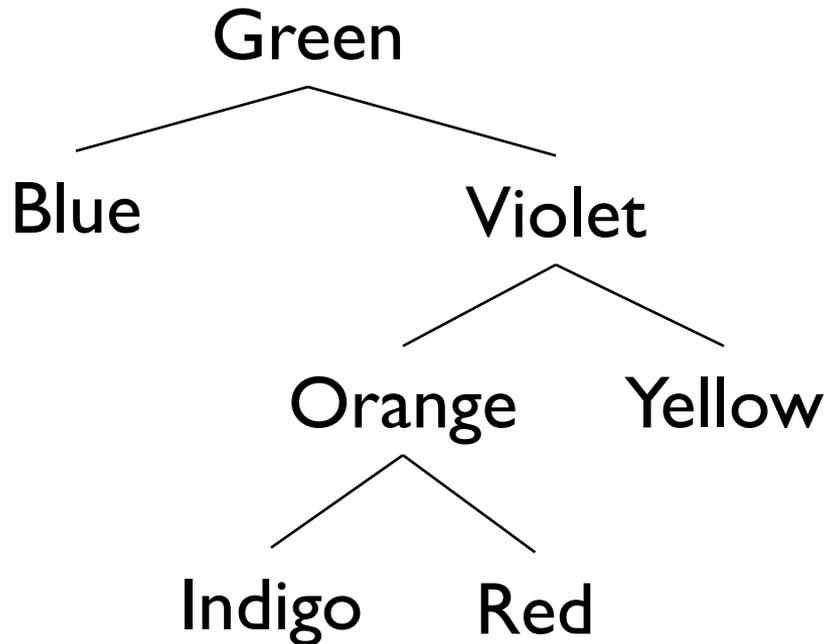
Visit node, then each node in left subtree, then each node in right subtree.



G B V O I R

Pre-Order Iterator

Visit node, then each node in left subtree, then each node in right subtree.



todo stack

G B V O I R Y

Pre-Order Iterator

```
public BTPreorderIterator(BinaryTree<E> root) {
    todo = new StackList<BinaryTree<E>>();
    this.root = root;
    reset();
}

public void reset() {
    todo.clear();

    // stack is now empty; push root on TODO stack
    if ((!root.isEmpty()))
        todo.push(root);
}
```

Pre-Order Iterator

```
public boolean hasNext() {
    return !todo.isEmpty();
}

public E next() {
    BinaryTree<E> old = todo.pop();
    E result = old.value();

    if (!old.right().isEmpty())
        todo.push(old.right());

    if (!old.left().isEmpty())
        todo.push(old.left());

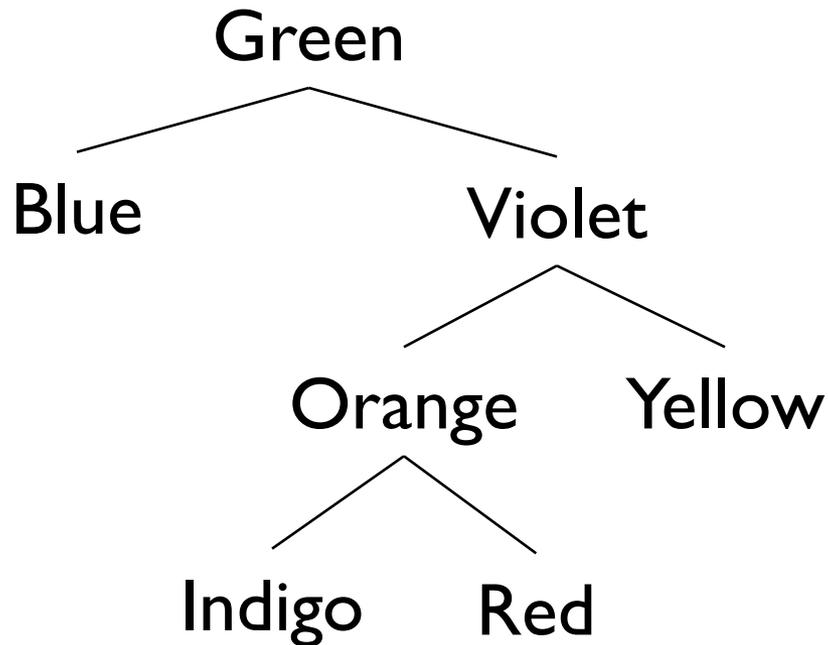
    return result;
}
```

Tree Traversal Practice Problems

- Prove that `levelOrder()` is correct: that is, that it touches the nodes of the tree in the correct order (*Hint: induction by level*)
- Prove that `levelOrder()` takes $O(n)$ time, where ***n*** is the size of the tree
- Prove that the PreOrder (or LevelOrder) Iterator visits the nodes in the same order as the PreOrder (or LevelOrder) traversal method

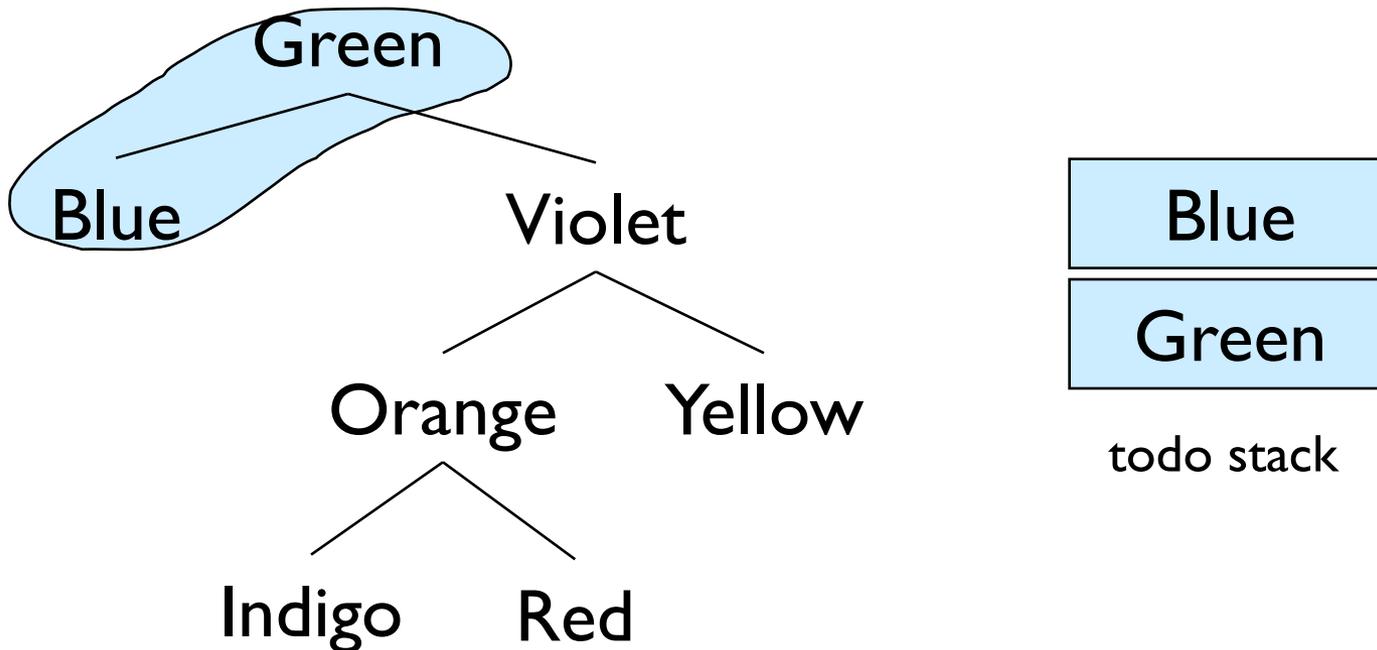
In-Order Iterator

Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree.



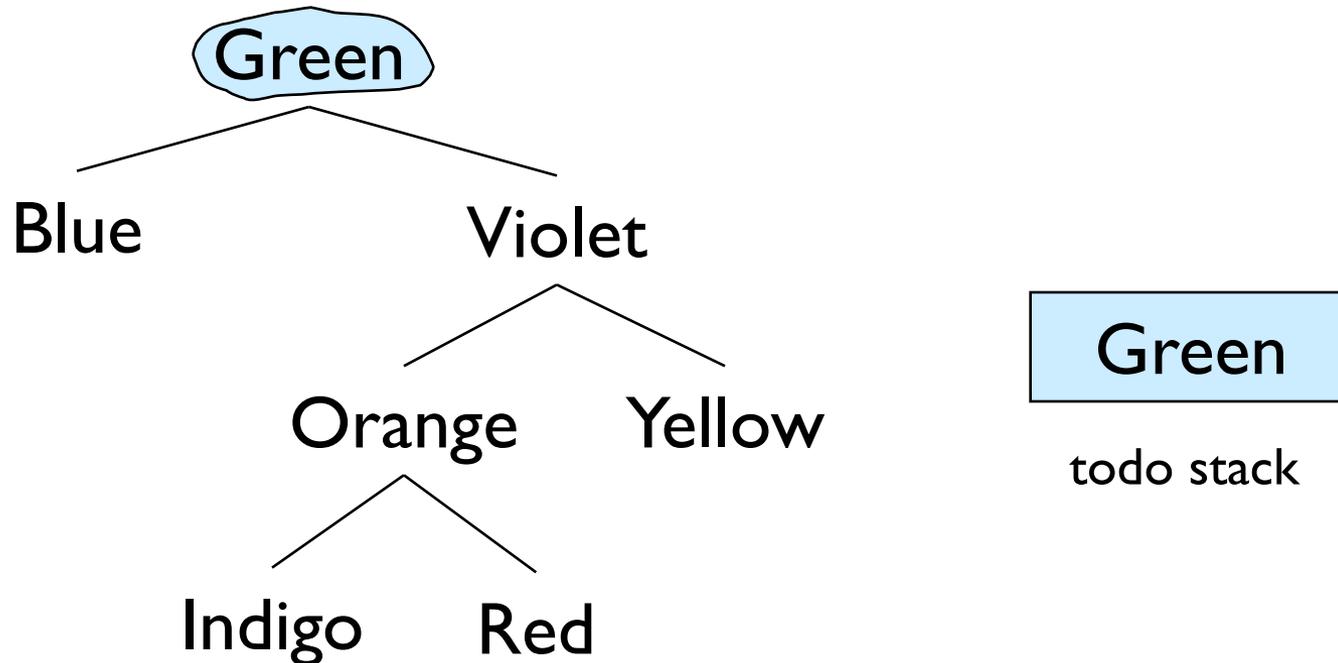
In-Order Iterator

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In-Order Iterator

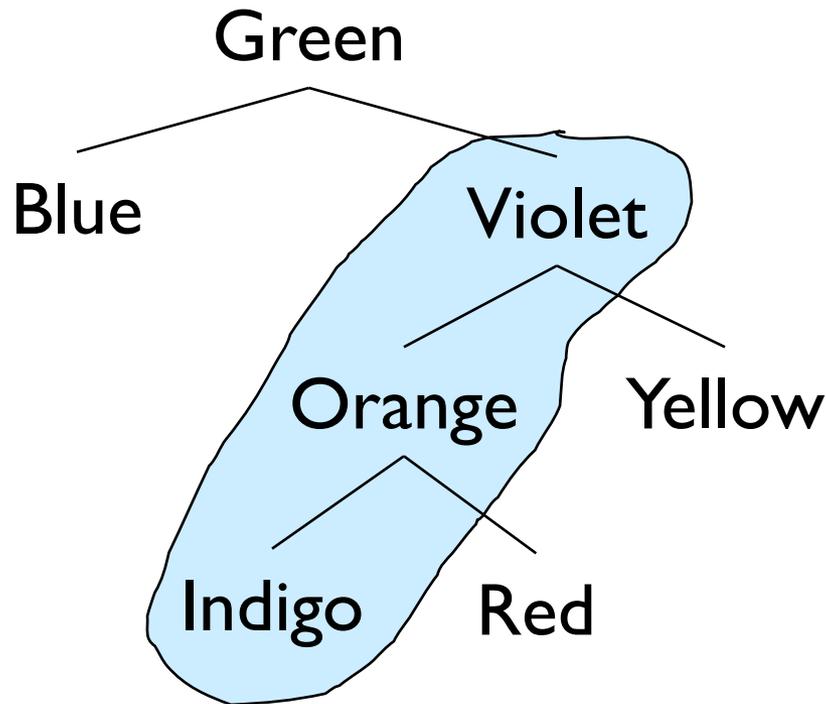
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B

In-Order Iterator

Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree.

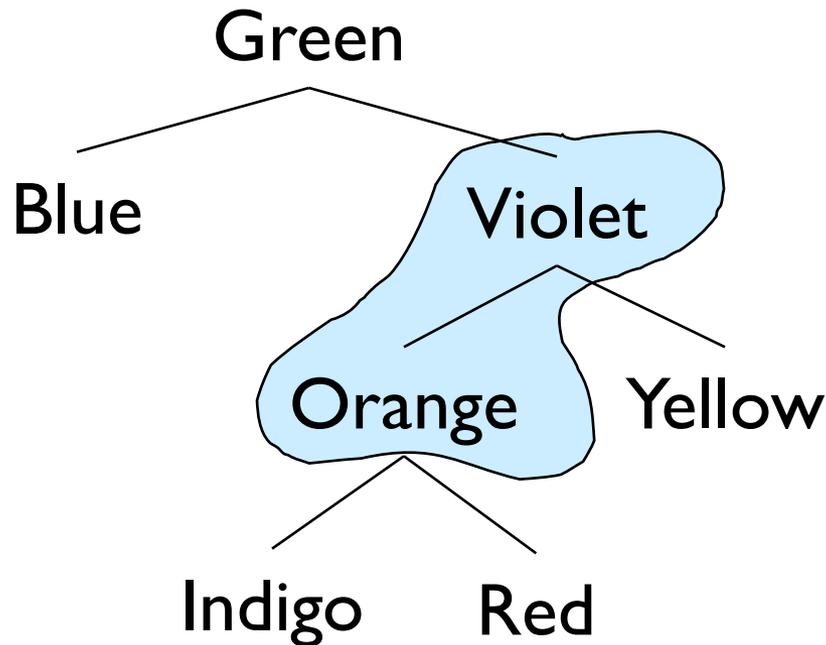


todo stack

B G

In-Order Iterator

Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree.

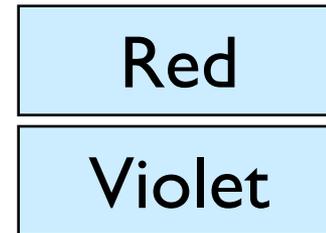
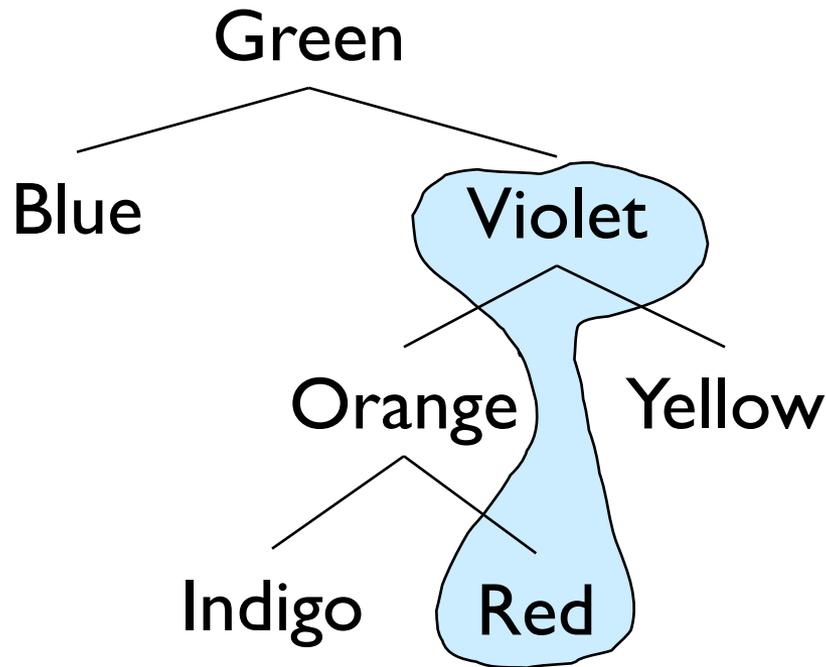


todo stack

B G I

In-Order Iterator

Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree.

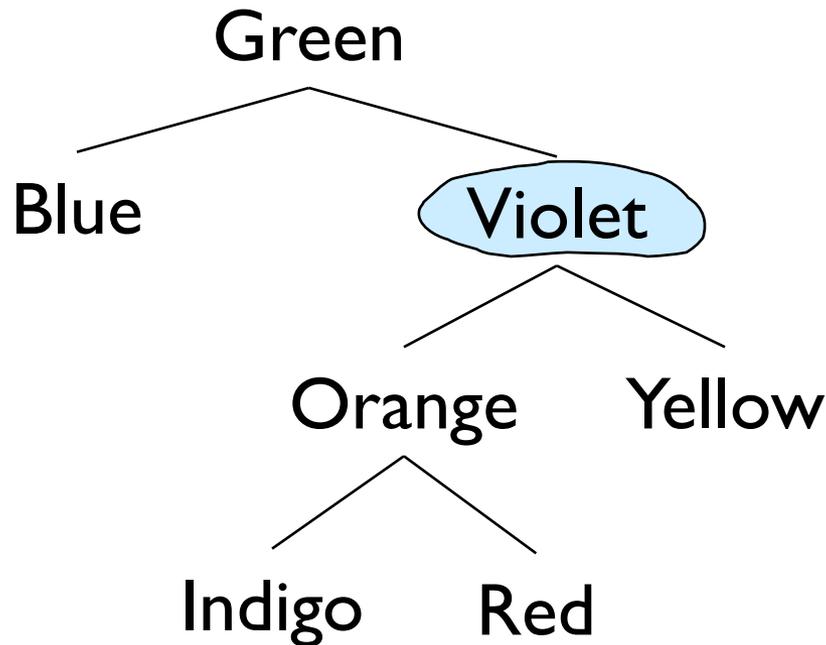


todo stack

B G I O

In-Order Iterator

Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree.

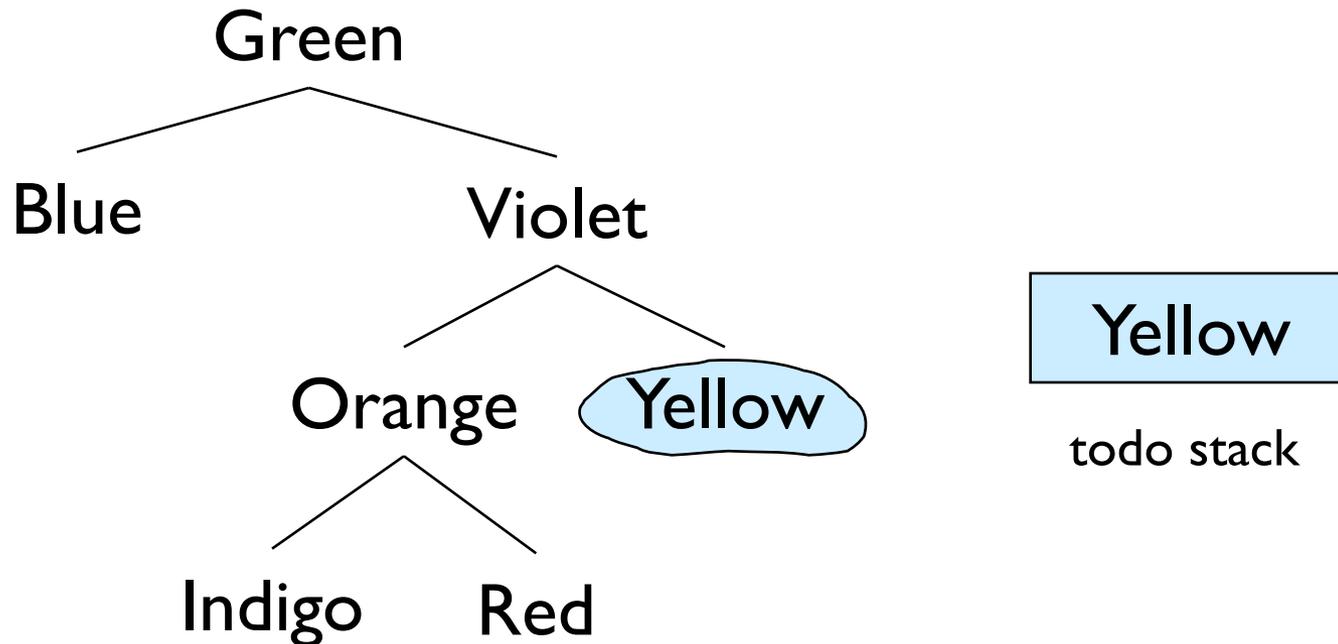


todo stack

B G I O R

In-Order Iterator

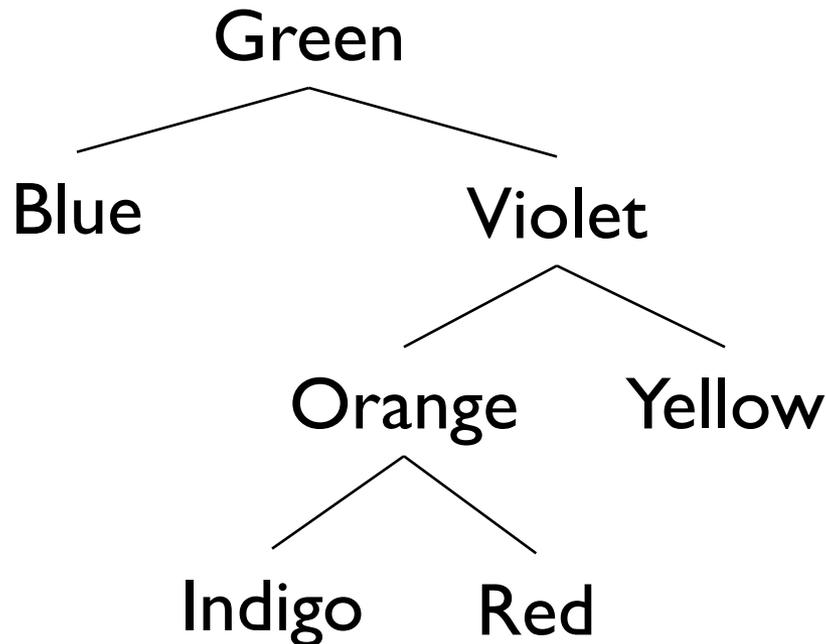
Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree.



B G I O R V

In-Order Iterator

Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree.



todo stack

B G I O R V Y

In-Order Iterator

- Should return elements in same order as processed by in-order traversal method:
 - Traverse left subtree, then node, then right subtree
- Must phrase in terms of `next ()` and `hasNext ()`
- Basic idea: We again “simulate recursion” with stack

In-Order Iterator

- Outline: left -> node -> right
 1. Push left children (as far as possible) onto stack
 2. On call to `next ()`:
 - Pop node from stack
 - Push right child and follow left children as far as possible
 - Return node's value
 3. On call to `hasNext ()`:
 - return `!stack.isEmpty ()`

In-Order Iterator

```
public BTInorderIterator(BinaryTree<E> root) {
    todo = new StackList<BinaryTree<E>>();
    this.root = root;
    reset();
}

public void reset() {
    todo.clear();
    // stack is empty. Push on nodes from root along
    // longest "left-only" path
    BinaryTree<E> current = root;
    while (!current.isEmpty()) {
        todo.push(current);
        current = current.left();
    }
}
```

In-Order Iterator

```
public E next() {
    BinaryTree<E> old = todo.pop();
    E result = old.value();
    // we know this node has no unvisited left children;
    // if this node has a right child,
    //   we push right child and longest “left-only” path
    // else
    //   top element of stack is next node to be visited
    if (!old.right().isEmpty()) {
        BinaryTree<E> current = old.right();
        do {
            todo.push(current);
            current = current.left();
        } while (!current.isEmpty());
    }
    return result;
}
```

Post-Order Iterator

- Outline: left -> right -> node
 1. Push path to leftmost leaf onto stack
 2. On call to `next ()`:
 - Pop node from stack
 - Determine whether it was the left or right node of its parent
 - If left child, push parent's right child and the entire path to leftmost leaf parent's right subtree
 - Return node's value
 3. On call to `hasNext ()`:
 - return `!stack.isEmpty ()`

Post-Order Iterator

```
public BTPostorderIterator(BinaryTree<E> root) {
    todo = new StackList<BinaryTree<E>>();
    this.root = root;
    reset();
}

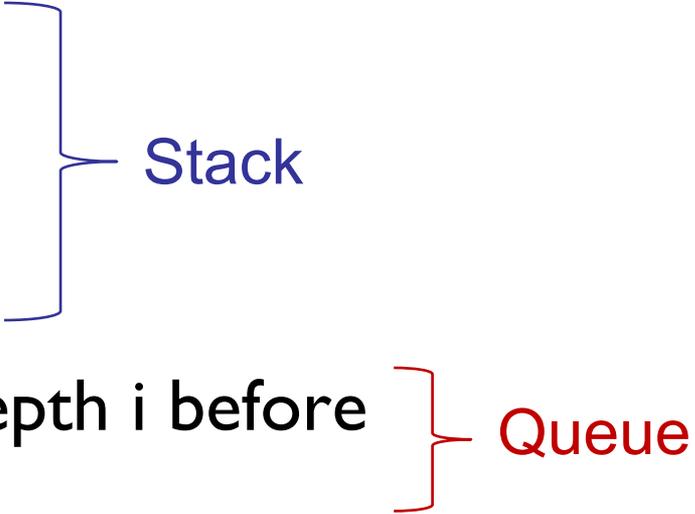
public void reset() {
    todo.clear();
    BinaryTree<E> current = root;
    while (!current.isEmpty()) {
        todo.push(current); // current now 'below' children
        if (!current.left().isEmpty())
            current = current.left();
        else
            current = current.right();
    } // Top of stack is now left-most unvisited leaf
}
```

Post-Order Iterator

```
public E next() {
    BinaryTree<E> current = todo.pop();
    E result = current.value();
    if (!todo.isEmpty()) {
        BinaryTree<E> parent = todo.get();
        if (current == parent.left()) {
            current = parent.right();
            while (!current.isEmpty()) {
                todo.push(current);
                if (!current.left().isEmpty())
                    current = current.left();
                else current = current.right();
            }
        }
    }
    return result;
}
```

Tree Traversals

In summary:

- In-order: “left, node, right”
 - Pre-order: “node, left, right”
 - Post-order: “left, right, node”
 - Level-order: visit all nodes at depth i before depth $i+1$
- Stack
- Queue
- 

Traversals & Searching

- We can use traversals for searching trees
- How might we search a tree for a value?
 - **Breadth-First**: Explore nodes near the root before nodes far away (level-order traversal)
 - **Depth-First**: Search until leaves are reached
 - (post-order traversal; but halt when solution found)
- Which is better?
 - Depends on the situation!
 - Does the tree structure represent a concept, e.g., distance or relationship between items?
 - Is the tree “sparse” or “dense”?

Final Thoughts

- Iterators continue to provide a useful service: common structure to enumerate the contents of a data structures
- We have defined four iterators that let us traverse the nodes of a tree in a variety of principled ways
- The best iterator for the task at hand will depend on our problem and our goals. So think critically!