CSCI 136
Data Structures &
Advanced Programming

Simple Sorting Algorithms

Introduction to Sorting

- Along with search, sorting is among the most ubiquitous computations
- Simple Examples
  - Reordering a list of integers so that they are in increasing order
  - Reordering a list of strings so that they are in alphabetical (*lexicographic*) order
  - Ordering the results of a Google search by decreasing relevance

Introduction to Sorting

- More Examples
  - Reordering a list of strings so that they are in *lexicographical order by length*
    - Shorter strings come before longer strings and
    - Equal-length strings are in lexicographic order
  - Ordering tracks in a music collection by artist
    - Breaking ties by title
      - Breaking remaining ties by release date
- If you can order it, you can sort it!
- So, let's look at some simple sorting methods

Introduction to Sorting

In this video we will

- Introduce three simple sorting algorithms
  - Bubble Sort, Insertion Sort, and Selection Sort
- Discuss their implementation
- Discuss their time and space complexity
  - Identify some sorting-specific measures of complexity
- Introduce some notation for describing *lower bounds*

Bubble Sort

- First Pass:
  - (**5** 1 3 2 9) → (1 **5** 3 2 9)
  - (1 **5** 3 2 9) → (1 3 **5** 2 9)
  - (1 3 **5** 2 9) → (1 3 2 **5** 9)
  - (1 3 2 **5** 9) → (1 3 2 5 **9**)
- Second Pass:
  - (**1** 3 2 5 9) → (**1** 3 2 5 9)
  - (1 **3** 2 5 9) → (1 2 **3** 5 9)
  - (1 2 **3** 5 9) → (1 2 3 **5** 9)
- Third Pass:
  - (**1** 2 3 5 9) -> (**1** 2 3 5 9)
  - (1 **2** 3 5 9) -> (1 **2** 3 5 9)
- Fourth Pass:
  - (**1** 2 3 5 9) -> (**1** 2 3 5 9)
- Finished!

http://www.visualgo.net/sorting
http://www.youtube.com/watch?v=lyZQPjUT5B4

Bubble Sort

Bubble sort uses a utility method swap

```
private static void swap(int[]A, int i, int j) {
        int temp = a[i];
        A[i] = A[j];
        A[j] = temp;
}
public static void bubbleSort(int[] A) {
        for(int i = 1; i < A.length; i++)
                // Process all but last i-1 elements
                for(int j = 0; j < A.length -i; j++)
                        if( A[j] > A[j+1] ) swap(A,j,j+1);
}
```

The only subtlety: Do loops start and end at reasonable points!

Bubble Sort

- Repeatedly scans through the list to be sorted, comparing two items at a time and swapping them if they are in the wrong order
  - Works on smaller initial slice each time
  - Can be improved to stop after a "swap-free" scan
- Gets its name from the way larger elements "bubble" to the end of the list
- Time complexity?
  - $O(n^2)$: Might perform $O(n^2)$ compares *and* $O(n^2)$ swaps
- Space complexity?
  - $O(n)$ total (very little additional space is required)
- It's a *Stable* sorting method
  - Equal elements remain in same relative positions

Bubble Sort

```
public static void bubbleSort(int[] A) {
        for(int i = 1; i < A.length; i++)
                // Process all but last i-1 elements
                for(int j = 0; j < A.length -i; j++)
                        if( A[j] < A[j+1] ) swap(A,j,j+1);
}
```

Counting Operations (where n = A.length)

- Outer loop executes n-1 times
    - for i^{th} iteration: inner loop executes n-i times
        - Performing, say at most, say, 5 operations each time

$$\sum_{i=1}^{n-1} 5(n-i) = \sum_{i=1}^{n-1} 5n - \sum_{i=1}^{n-1} 5i = 5n(n-1) - 5\sum_{i=1}^{n-1} i = 5n(n-1) - 5n(n-1)/2$$

- This equals $\frac{5n(n-1)}{2}$ which is $O(n^2)$

Aside: Lower Bound Notation

There are situations in which bubble sort must necessarily perform a quadratic number of operations.

- Any "almost reverse-sorted" list will cause this

This observation describes a lower bound on the (worst-case) running time of the algorithm

- It's useful to have notation for lower-bound claims, similar to the Big-O notation for upper bound
- It exists: It's called "Big-Ω" (Big Omega) notation

Aside: Lower Bound Notation

Definition: A function *f(n)* is *Ω(g(n))* if for some constant $c > 0$ and all $n \geq n_0$

$$f(n) \geq c\, g(n)$$

So, *f(n)* is *Ω(g(n))* exactly when *g(n)* is O*(f(n))*

All three sorting algorithms have time complexity

- $O(n^2)$: Never use more than cn^2 operations
- $\Omega(n^2)$: Sometimes use at least cn^2 operations

When f(n) is O(g(n)) and f(n) is **Ω**(g(n)) we write:

f(n) is **Θ**(g(n))

(pronounced "Big-Theta")

Bubble Sort Complexity

Time complexity?

- $\Theta(n^2)$: That is, both $O(n^2)$ and $\Omega(n^2)$
  - $O(n^2)$: Never performs more than c n^2 operations
  - $\Omega(n^2)$: Sometimes uses at least cn^2 operations
    - Might perform $O(n^2)$ compares *and* $O(n^2)$ swaps

Space complexity?

- $\Theta(n)$: That is, both $O(n)$ and $\Omega(n)$
  - $\Omega(n)$: *Needs to store* an n-element array of constant-sized values
  - $O(n)$: *Only stores* the array *plus a few* other constant-sized variables

Insertion Sort

5	7	0	3	4	2	6	1
5	7	0	3	4	2	6	1
0	5	7	3	4	2	6	1
0	3	5	7	4	2	6	1
0	3	4	5	7	2	6	1
0	2	3	4	5	7	6	1
0	2	3	4	5	6	7	1
0	1	2	3	4	5	6	7

http://www.visualgo.net/sorting

Insertion Sort

- Simple sorting algorithm that works by building a sorted list one entry at a time
- Less efficient on large lists than more advanced algorithms
- Advantages:
  - Simple to implement and efficient on small lists
  - Efficient on data sets which are already mostly sorted
- Time complexity (Worst Case): $\Theta(n^2)$
  - $O(n^2)$: Only perform $O(n^2)$ compares and $O(n^2)$ moves
  - $\Omega(n^2)$: Could perform $\Omega(n^2)$ compares and $\Omega(n^2)$ moves
- Space complexity : $\Theta(n)$
- Stable: Yes

Selection Sort

- 11 3 27 5 16 — Swap 27 with 16
- 11 3 16 5 27 — Swap 16 with 5
- 11 3 5 16 27 — Swap 11 with 5
- 5 3 11 16 27 — Swap 5 with 3
- 3 5 11 16 27 — Done!

- The algorithm works as follows:
  - Find the maximum value in the list
  - Swap it with the value in the last position
  - Repeat the steps above for remainder of the list (ending at the second to last position)
  - Continue on progressively smaller portions of array

Selection Sort

- Similar to insertion sort
- Noted for its simplicity and performance advantages when compared to complicated algorithms
- Time complexity (Worst Case): $\theta(n^2)$
  - $O(n^2)$: Only perform $O(n^2)$ compares and $O(n)$ moves
  - $\Omega(n^2)$: Could perform $\Omega(n^2)$ compares and $\Omega(n)$ moves
- Space Complexity : $\theta(n)$
- Stable : Yes

Implementation Details

Selection sort uses two utility methods

Uses a swap method

```
private static void swap(int[]A, int i, int j) {
        int temp = a[i];
        A[i] = A[j];
        A[j] = temp;
}
```

And a max-finding method

```
// Find position of largest value in A[0 .. last]
public static int findPosOfMax(int[] A, int last) {
        int maxPos = 0;      // A wild guess
        for(int i = 1; i <= last; i++)
                if (A[maxPos] < A[i]) maxPos= i;
        return maxPos;
}
```

Iterative & Recursive Selection Sort

An Iterative Selection Sort

```
public static void selectionSort(int[] A) {
       for(int i = A.length - 1; i>0; i--)
           int big= findPosOfMax(A,i);
           swap(A, i, big);
       }
}
```

A Recursive Selection Sort (just the helper method)

```
public static void recSSHelper(int[] A, int last) {
       if(last == 0) return; // base case

       int big= findPosOfMax(A, last);
       swap(A,big,last);
       recSSHelper(A, last-1);
}
```

Notes

Three simple algorithms: Bubble, Insertion, Selection

- All perform two basic operations: *comparisons* and *swaps/moves*

Comparisons vs Swaps/Moves (worst case)

- Bubble Sort performs (naïve version)
    - quadratic number of comparisons in all cases
    - quadratic number of swaps for almost reverse-sorted data
- Insertion Sort performs
    - linear number of comparisons on almost-sorted data and quadratic number on almost reverse-sorted data
    - quadratic number of moves on almost reverse-sorted data
- Selection Sort
    - quadratic number of comparisons in all cases
    - at most (and sometimes) a linear number of swaps

Coming Up Next

How can we adapt our sorting algorithms to work on non-primitive data types?

Can we break the $\Omega(n^2)$ performance bottleneck of our sorting algorithms?

- Spoiler: Yes!

Stay tuned….