CSCI 136
Data Structures &
Advanced Programming

Lecture 6

Fall 2019

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Last Time

- Finished Java overview/review
- Introduction to Vectors
  - Example: Word Frequencies
  - Vector instance variable and method declarations
  - First details of implementation

Today' s Outline

From Previous Lecture Slides

- Vector Implementation
- Miscellany: Wrappers
- Lab 2 Design and Strategies

Today's Slides

- Generic Data Types
- Condition Checking
  - Pre- and post-conditions, Assertions
- Measuring Program Resource Use

Recall: Vectors

- Vectors are collections of Objects
- Methods include:
  - `add(Object o), remove(Object o)`
  - `contains(Object o)`
  - `indexOf(Object o)`
  - `get(int index), set(int index, Object o)`
  - `remove(int index)`
  - `add(int index, Object o)`
  - `size(), isEmpty()`
- Remove methods preserve order, close “gap”

Using Generic (Parameterized) Types

- What limitations are associated with casting Objects as they are added and removed from Associations?
  - Errors cannot be detected by compiler
  - Must rely on runtime checks
- Instead of casting Objects, Java supports using generic or parameterized data types (Read Ch 4)
- Instead of:

```
Association a = new Association("Bill",(Integer) 97);
Integer grade = (Integer) a.getValue();  //Cast to Integer
```

- Use:

```
Association<String, Integer> a =
    new Association<String, Integer>("Bill", (Integer) 97);
    Integer grade = a.getValue();  //no cast!
```

Generic Association<K,V> Class

```
class Association<K,V> {
   protected K theKey;
   protected V theValue;

   //pre: key != null
   public Association (K key, V value) {
        Assert.pre (key != null, “Null key”);
        theKey = key;
        theValue = value;
   }

   public K getKey() {return theKey;}
   public V getValue() {return theValue;}
   public V setValue(V value) {
        V old = theValue;
        theValue = value;
        return old;
   }
}
```

Making Vector Generic

- Structure5 provides a generic version of Vector
  - And of all of the data structures in the structure package
  - Instead of:

```
Vector v = new Vector();  //Vector of Objects
String word = (String)v.get(index);  //Cast to String
```

  - Use:

```
Vector<String> v = new Vector<String>(); //Vector of Strings
String word = v.get(index);  //no cast!
```

  - Or:

```
Vector<Association<String, Integer>> v =
  new Vector<Association<String, Integer>>();
int count = v.get(index).getValue(); //no cast!
```

  - See GenWordFreq.java…

(Look at WordFreq.java with gen)

Class Vector<E>

```
public class Vector<E> {
private Object[] elementData;     // Underlying array
protected int elementCount;       // Number of elts in Vector
protected final static int defaultCapacity;
protected int capacityIncrement;  // How much to grow by
protected E initialValue;         // A default elt value
}
```

- Why (still!) Object[]?
  - Java restriction: Can't use a type variable for an array declaration, only a concrete type

Basic Vector<E> Methods

```
public class Vector<E> {
public Vector()              // Make a small Vector
public Vector(int initCap) // Make Vector of given capacity
public void add(E elt)      // Add elt to (high) end of Vector
public void add(int i, E elt)     // Add elt at position i
public E remove(E elt)      // Remove (and return) elt
public E remove(int i)      // Remove (and return) elt at pos i
public int capacity()       // Return capacity
public int size()           // Return current size
public boolean isEmpty()    // Is size == 0?
public boolean contains(E elt) // Is elt in Vector?
public E get(int i)         // Return elt at position i
public E set(int i, E elt) // Change value at position i
public int indexOf(E elt)  // Return earliest position of elt
}
```

Lab 2 Preview

- Three classes:
  - FrequencyList.java
  - Table.java
  - WordGen.java
- Two Vectors of Associations
- toString() in Table and FrequencyList for debugging
- What are the key stages of execution?
  - Test code thoroughly before moving on to next stage
- Use GenWordFreq as example

Lab 2: Core Tasks

- FreqencyList
  - A Vector of Associations of String and Integer
  - Add a letter
    - Is it a new letter or not?
    - Use indexOf from Vector class
- Pick a random letter based on frequencies
  - Let total = sum of frequencies in FL
  - generate random int r in range [0…total]
  - Find smallest k s.t. r <= sum of first k frequencies

Lab 2: Core Tasks

- Table
  - A Vector of Associations of String and FrequencyList
  - Add a letter to a k-gram
    - Is it a new k-gram or not?
  - Pick a random letter given a k-gram
    - Find the k-gram then ask its FrequencyList to pick
- WordGen
- Convert input into (very long) String
  - Use a StringBuffer---see handout

Pre and Post Conditions

- Recall `charAt(int index)` in Java String class
- What are the pre-conditions for charAt?
  - 0 <= index < length()
- What are the post-conditions?
  - Method returns char at position index in string
- We put pre and post conditions in comments above most methods

```
/* pre: 0 ≤ index < length
 * post: returns char at position index
 */
public char charAt(int index) { … }
```

Pre and Post Conditions

- Pre and post conditions "form a contract"
- Post-condition is guaranteed if method is called when pre-condition is true
- Examples:
  - `s.charAt(s.length() - 1)`: index < length, so valid
  - `s.charAt(s.length() + 1)`: index > length, not valid
- These conditions document requirements that user of method should satisfy
- But, as comments, they are not enforced

Other Examples

- Other places pre and post conditions are useful

```
// Pre: other is of type Card
// Post: Returns true if suits and ranks match
public boolean equals(Object other) {
   Card oc = (Card) other;
   return this.getRank() == oc.getRank() &&
          this.getSuit() == oc.getSuit();
}
```

Assert Class

- Pre- and post-condition comments are important for *documenting* code.
- BUT: It would be *even better* in some cases that a pre-condition was violated.
- Program could catch error and gracefully halt
- The Assert class (in structure5 package) allows us to programmatically check for pre- and post-conditions

Assert Class

The Assert class contains the methods

```
public static void pre(boolean test, String message);
public static void post(boolean test, String message);
public static void condition(boolean test, String message);
public static void fail(String message);
```

If the boolean test is NOT satisfied, an exception is raised, the message is printed and the program halts

Assert Examples

The Vector class uses Assert in many places

```
// Pre: initialCapacity >= 0
public Vector(int initialCapacity) {
      Assert.pre(initialCapacity >= 0,"Capacity
            must not be negative");

// Pre: 0 <= index && index < size()
public E elementAt(int index) {
      Assert.pre(0 <= index && index < size(),"index
            is within bounds");
```

General Rules about Assert

1. State pre/post conditions in comments
2. Check conditions in code using "Assert"
3. Use Fail in unexpected cases (such as the default block of a switch statement)

- Any questions?
- You should start using Assertions in Lab 2

The Java assert keyword

- An alternative to Duane's Assert class
- Added in Java 1.4
- Two variants
  - assert boolean_expression
    - Throws an AssertionError if the expression is false
  - assert boolean_expression : other_expression
    - In addition, prints value of other_expression\

Measuring Computational Cost

Consider these two code fragments…

```
for (int i=0; i < arr.length; i++)
   if (arr[i] == x) return "Found it!";
```

…and…

```
for (int i=0; i < arr.length; i++)
   for (int j=0; j < arr.length; j++)
     if( i !=j && arr[i] == arr[j]) return "Match!";
```

How long does it take to execute each block?

Measuring Computational Cost

- How can we measure the amount of work needed by a computation?
  - Absolute clock time
    - Problems?
      - Different machines have different clocks
      - Too much other stuff happening (network, OS, etc)
      - Not consistent. Need lots of tests to predict future behavior

Measuring Computational Cost

- Counting computations
  - Count *all* computational steps?
  - Count how many "expensive" operations were performed?
  - Count number of times "x" happens?
    - For a specific event or action "x"
    - i.e., How many times a certain variable changes
- Question: How accurate do we need to be?
  - 64 vs 65? 100 vs 105? Does it really matter??

An Example

```
// Pre: array length n > 0
public static int findPosOfMax(int[] arr) {
      int maxPos = 0 // A wild guess
      for(int i = 1; i < arr.length; i++)
            if (arr[maxPos] < arr[i]) maxPos = i;
      return maxPos;
}
```

- Can we count steps exactly?
  - ”if” makes it hard
- Idea: Overcount: assume “if” block always runs
- Overcounting gives *upper bound* on run time
- Can also undercount for lower bound
- Overcount: 4(n-1) + 4; undercount: 3(n-1) + 4

Measuring Computational Cost

- Rather than keeping exact counts, we want to know the *order of magnitude* of occurrences
  - 60 vs 600 vs 6000, *not* 65 vs 68
  - n, *not* 4(n-1) + 4
- We want to make comparisons without looking at details and without running tests
- Avoid using specific numbers or values
- Look for overall trends

Measuring Computational Cost

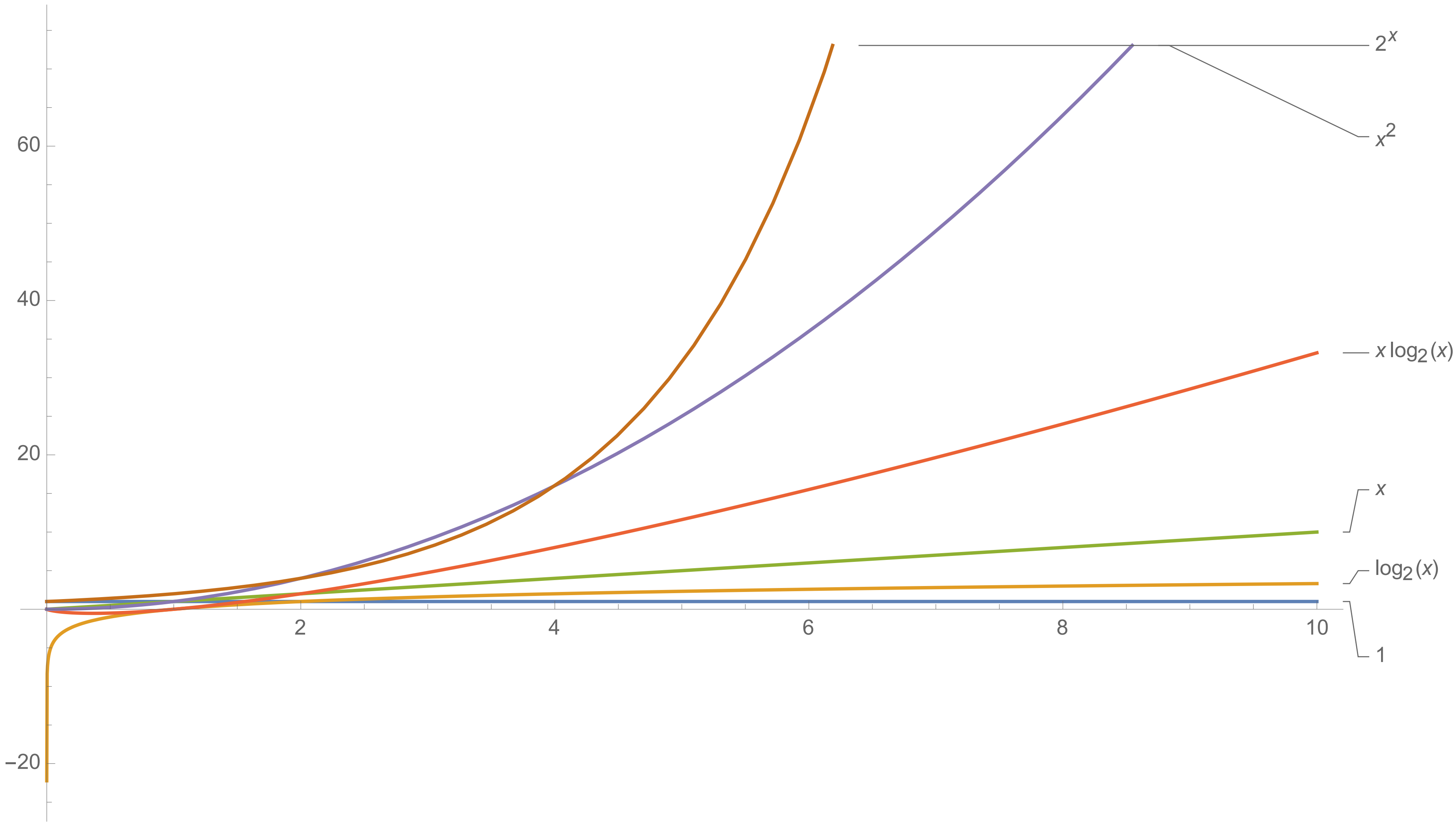
- How does algorithm scale with problem size?
  - E.g.: If I double the size of the problem instance, how much longer will it take to solve:
    - Find maximum: $n - 1 \rightarrow (2n) - 1$ (≈ twice as long)
    - Bubble sort: $n(n-1)/2 \rightarrow 2n(2n - 1)/2$ (≈ 4 times as long)
    - Subset sum: $2^{n-1} \rightarrow 2^{2n-1}$ (2^n times as long!!!)
    - Etc.
- We will also measure amount of space used by an algorithm using the same ideas….

Function Growth

Consider the following functions, for $x \geq 1$

- $f(x) = 1$
- $g(x) = \log_2(x)$ // Reminder: if x=2^n, $\log_2(x) = n$
- $h(x) = x$
- $m(x) = x \log_2(x)$
- $n(x) = x^2$
- $p(x) = x^3$
- $r(x) = 2^x$

Function Growth



Function Growth & Big-O

- Rule of thumb: ignore multiplicative constants
- Examples:
  - Treat n and n/2 as same order of magnitude
  - $n^2/1000$, $2n^2$, and $1000n^2$ are "pretty much" just n^2
  - $a_0 n^k + a_1 n^{k-1} + a_2 n^{k-2} + \cdots a_k$ is roughly n^k
- The key is to find the most *significant* or *dominant* term
- Ex: $\lim_{x\to\infty} (3x^4 - 10x^3 - 1)/x^4 = 3$ (Why?)
  - So $3x^4 - 10x^3 - 1$ grows "like" x^4

Asymptotic Bounds (Big-O Analysis)

- A function f(n) is *O(g(n))* if and only if there exist positive constants c and n_0 such that

  $$|f(n)| \le c \cdot g(n) \text{ for all } n \ge n_0$$

- $c \cdot g$ is "at least as big as" f **for large n**
  - Up to a multaplicative constant c!
- Example:
  - $f(n) = n^2/2$ is $O(n^2)$
  - $f(n) = 1000n^3$ is $O(n^3)$
  - $f(n) = n/2$ is $O(n)$

Determining "Best" Upper Bounds

- We typically want the *most conservative* upper bound when we estimate running time
  - And among those, the *simplest*
- Example: Let $f(n) = 3n^2$
  - $f(n)$ is $O(n^2)$
  - $f(n)$ is $O(n^3)$
  - $f(n)$ is $O(2^n)$ (see next slide)
  - $f(n)$ is NOT $O(n)$ (!!)
- "Best" upper bound is $O(n^2)$
- We care about $\mathbf{c}$ and $\mathbf{n_0}$ in practice, but focus on size of $\mathbf{g}$ when designing algorithms and data structures

What's n_0? Messy Functions

- Example: Let $f(n) = 3n^2 - 4n + 1$. f(n) is $O(n^2)$
  - Well, $3n^2 - 4n + 1 \leq 3n^2 + 1 \leq 4n^2$, for $n \geq 1$
  - So, for $c = 4$ and $n_0 = 1$, we satisfy Big-O definition
- Example: Let $f(n) = n^k$, for any fixed $k \geq 1$. f(n) is $O(2^n)$
  - Harder to show: Is $n^k \leq c\, 2^n$ for some $c > 0$ and large enough n?
  - It is if and only if $\log_2(n^k) \leq \log_2(2^n)$, that is, iff $k \log_2(n) \leq n$.
  - That is iff $k \leq n/\log_2(n)$. But $n/\log_2(n) \rightarrow \infty$ as $n \rightarrow \infty$
  - This implies that for some n_0 on $n/\log_2(n) \geq k$ if $n \geq n_0$
  - Thus $n \geq k \log_2(n)$ for $n \geq n_0$ and so $2^n \geq n^k$

Input-dependent Running Times

- Algorithms may have different running times for different input values
- Best case (typically not useful)
  - BubbleSort already sorted array: O(n)
  - Find item in first place that we look: O(1)
- Worst case (generally useful, sometimes misleading)
  - Don't find item in list: O(n)
  - BubbleSort array that's in reverse order: O(n^2)
- Average case (useful, but often hard to compute)
  - Linear search O(n)
  - QuickSort random array O(n log n) ← We'll sort soon

Vector Operations : Worst-Case

For n = Vector size (*not* capacity!):

- O(1): size(), capacity(), isEmpty(), get(i), set(i), firstElement(), lastElement()
- O(n): indexOf(), contains(), remove(elt), remove(i)
- What about add methods?
  - If Vector doesn't need to grow
    - add(elt) is O(1) but add(elt, i) is O(n)
  - Otherwise, depends on ensureCapacity() time
    - Time to compute newLength : O($\log_2(n)$)
    - Time to copy array: O(n)
    - $O(\log_2(n)) + O(n)$ is O(n)

Vector: Add Method Complexity

Suppose we grow the Vector's array by a fixed abount d. How long does it take to add n items to an empty Vector?

- The array will be copied each time its capacity needs to exceed a multiple of d
  - At sized 0, d, 2d, …, n/d
- Copying an array of size kd takes ckd steps for some constant c, giving a total of

$$\sum_{k=1}^{n/d} c \cdot k \cdot d = c \cdot d \sum_{k=1}^{n/d} k = c \cdot d \cdot \frac{\left(n/d\right)\left(n/d + 1\right)}{2} = O(n^2)$$

Vector: Add Method Complexity

Suppose we want to grow the Vector's array by doubling. How long does it take to add *n* items to an empty Vector?

- The array will be copied each time it's capacity needs to exceed a power of 2.
  - At sizes 0, 1, 2, 4, 8, …, $2^{\log_2 n}$
- Copying an array of size 2^k takes $c2^k$ steps for some constant c, giving a total of:

$$\sum_{k=1}^{\log_2 n} c \cdot 2^k = c \sum_{k=1}^{\log_2 n} 2^k = c \cdot (2^{1+\log_2 n} - 1) = O(n)$$

Common Complexities

For n = measure of problem size:

- $O(1)$: constant time and space
- $O(\log n)$: divide and conquer algorithms, binary search
- $O(n)$: linear dependence, simple list lookup
- $O(n \log n)$: divide and conquer sorting algorithms
- $O(n^2)$: matrix addition, selection sort
- $O(n^3)$: matrix multiplication
- $O(n^{12})$: Original AKS primality test for n-bit integers
- $O(2^n)$: subset sum, graph 3-coloring, satisfiability, ...
- $O(n!)$: traveling salesman problem (in fact $O(n^2 2^n)$)