CSCI 136
Data Structures &
Advanced Programming

Lecture 30

Fall 2019

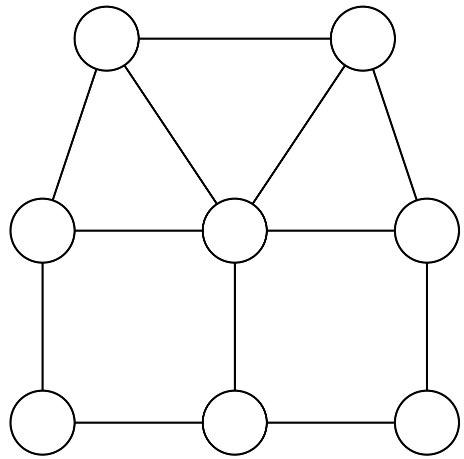
Instructors: B&S

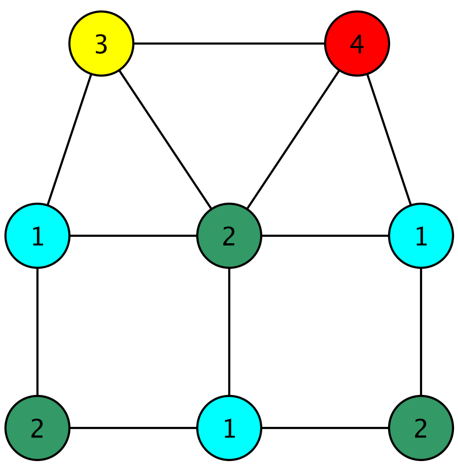
Lab 10 Overview:
Graph Algorithms using structure5

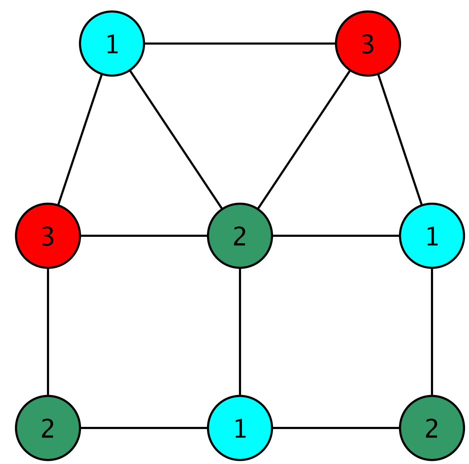
Greedy Algorithms

- A *greedy algorithm* attempts to find a globally optimum solution to a problem by making locally optimum (greedy) choices
- Example: Graph Coloring
  - A *(proper) coloring* of a graph G = (V,E) is an assignment of a value (color) to each vertex so that adjacent vertices get different values (colors)
  - Typically one strives to minimize the number of colors used

Greedy Coloring







Greedy Coloring : Math

Here's a greedy coloring algorithm

Build a collection $C = \{C_1, \ldots, C_k\}$ *of sets of vertices*

$i = 0$; $C_i = \{\}$ *// empty set*

while G is has more vertices

    for each vertex u in G

        if u is not adjacent to any vertex of C_i

            remove u from G and add u to C_i

    add C_i *to C*

    i++;

Return C as the coloring

Greedy Coloring : CS

Here's a greedy coloring algorithm

Create a structure C to hold a collection of lists

while G is not empty

pick a vertex v in G; create an empty list L; add v to L

for each vertex u ≠ v in G

if u is not adjacent to any vertex of L

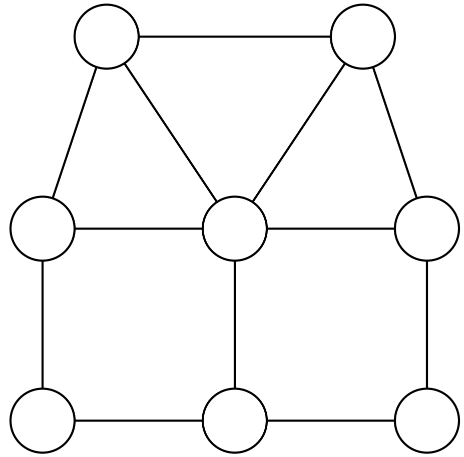
add u to L

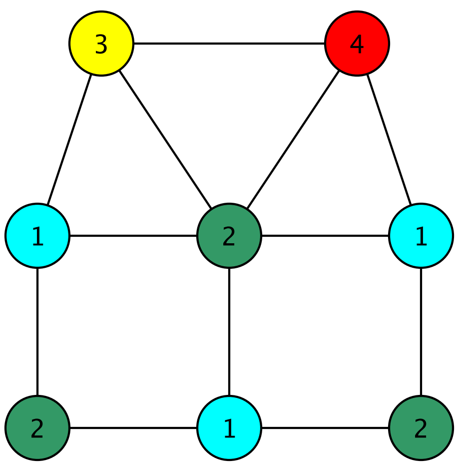
remove all vertices of L from G

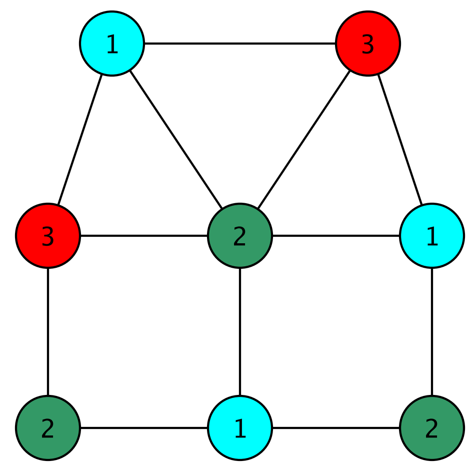
add L to C

Return C as the coloring

Greedy Coloring







Greedy Coloring

Some observations

- Each list (color class) L is a set of vertices no two of which are adjacent (an *independent set*)
- Each color class is maximal: cannot be made any larger
  - The hope is that this results in fewer colors being needed
  - But the solution is not always optimum!
  - This is a *very hard problem*
- The coloring problem is the same as finding a *partition* of the vertex set into independent sets
  - Partition means union of disjoint sets

Lab 10 : Exam Scheduling

Find a schedule (set of time slots) for exams so that

- No student has two exams in the same slot
- Every course is in a slot
- The number of slots is as small as possible

This is just the graph coloring problem in disguise!

- Each course is a vertex
- Two vertices are adjacent if the courses share students
- A slot must be an independent set of vertices (that is, a color class)

Lab 10 Notes: Using Graphs

- Create a new graph in structure5
  - GraphListDirected, GraphListUndirected,
  - GraphMatrixDirected, GraphMatrixUndirected

- Graph<V,E> conflictGraph = new GraphListUndirected<V,E>();

Lab 10 : Useful Graph Methods

- `void add(V label)`
  - add vertex to graph
- `void addEdge(V vtx1, V vtx2, E label)`
  - add edge between vtx1 and vtx2
- `Iterator<V> neighbors(V vtx1)`
  - Get iterator for all neighbors to vtx1
- `boolean isEmpty()`
  - Returns true iff graph is empty
- `Iterator<V> iterator()`
  - Get vertex iterator
- `V remove(V label)`
  - Remove a vertex from the graph
- `E removeEdge(V vLabel1, V vLabel2)`
  - Remove an edge from graph

Last Time

- Lab 10 Overview: Exam Scheduling
- Array-Based Graph implementations

This Time

- Array-Based Graph Efficiency
- List-Based Graph Implementations
- Iterators Everywhere….

GraphMatrix Efficiency

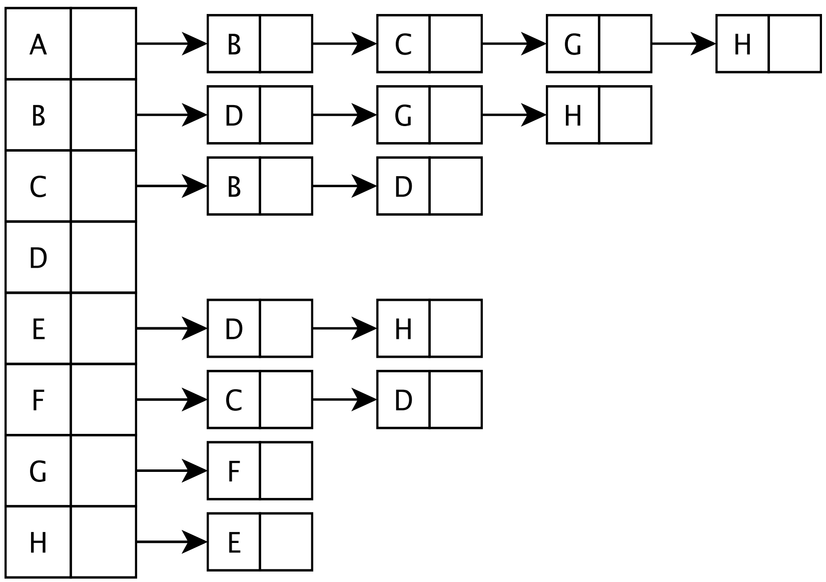
- Assume Map operations are O(1)

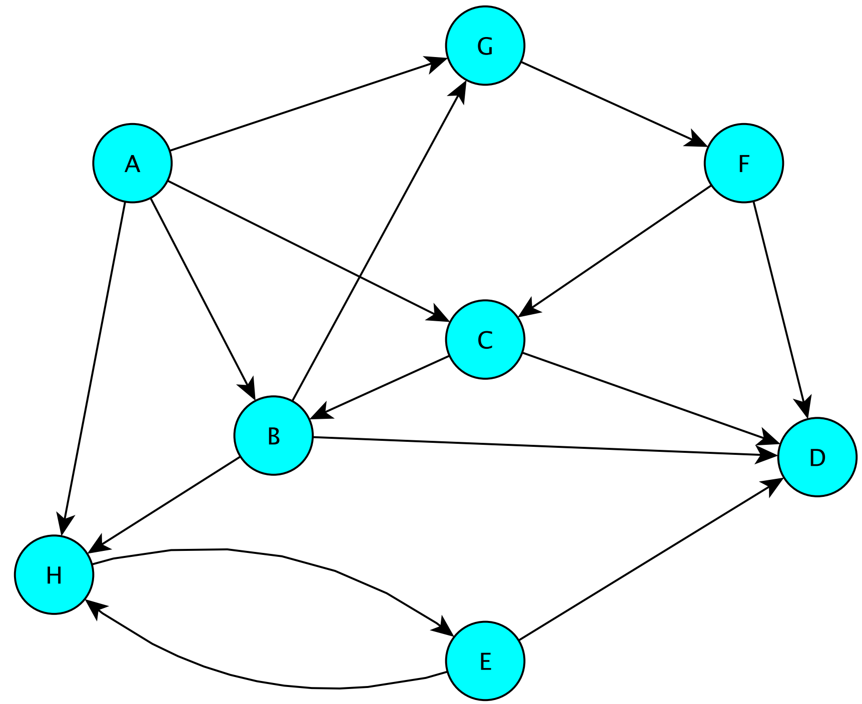
  (For now---even though they are not!)
  - |E| = number of edges (often folks write m = |E|)
  - |V| = number of vertices (often folks write n = |V|)
- Runtime of add, addEdge, getEdge, removeEdge, remove?
- Space usage?
- Conclusions
  - Matrix is good for dense graphs
  - Have to commit to maximum # of vertices in advance

Efficiency : Assuming O(1) Map Ops

	GraphMatrix
add	O(1)
addEdge	O(1)
getEdge	O(1)
removeEdge	O(1)
remove	O(|V|)
space	O(|V|^2)

Adjacency List : Directed Graph

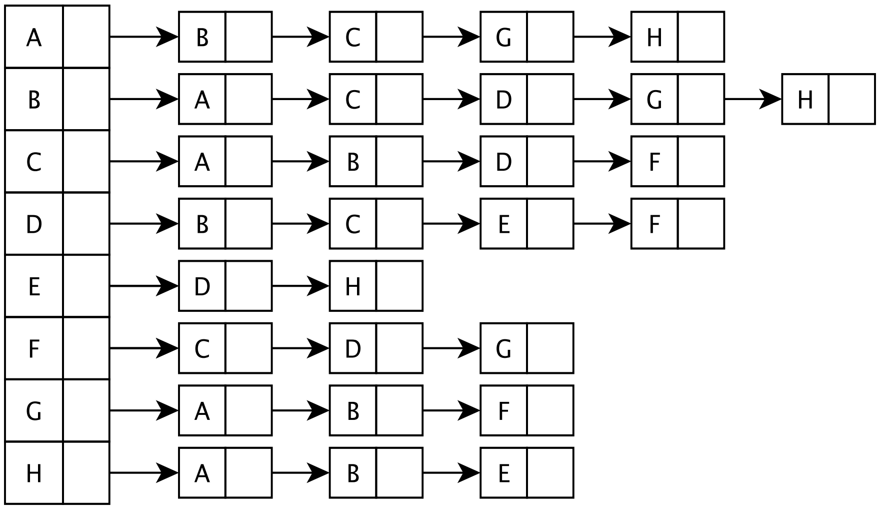


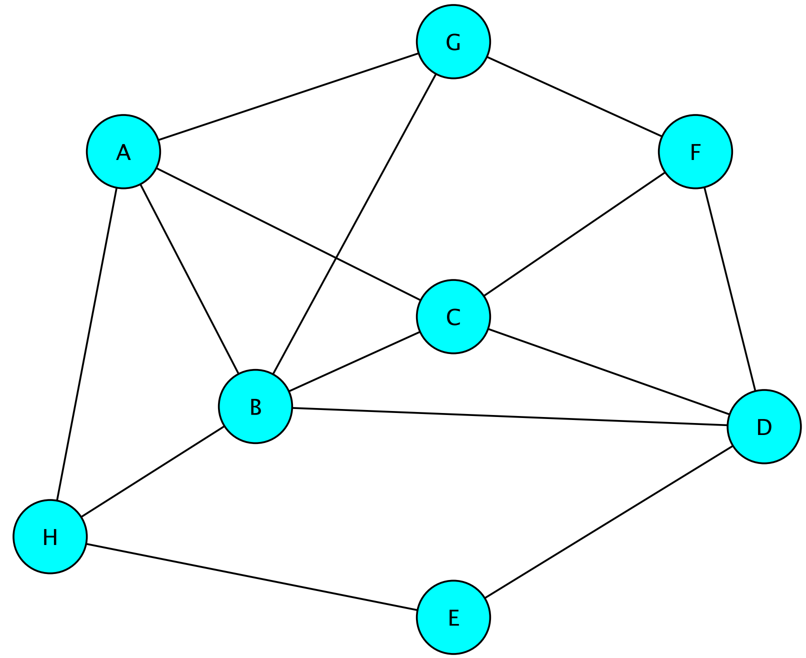


The vertices are stored in an array V[]

V[] contains a linked list of edges having a given source

Adjacency List : Undirected Graph





The vertices are stored in an array V[]
V[] contains a linked list of edges incident to a given vertex

GraphList

- Maintain an *adjacency list of edges* at each vertex (no adjacency matrix)
  - Keep only outgoing edges for directed graphs
- Support both directed and undirected graphs (GraphListDirected, GraphListUndirected)

Vertex and GraphListVertex

- We use the same Edge class for all graph types
- We extend Vertex to include an Edge list
- GraphListVertex class adds to Vertex class
  - A Structure to store edges adjacent to the vertex

    protected Structure<Edge<V,E>> adjacencies; // adjacent edges

    – adjacencies is created as a SinglyLinkedList of edges
  - Several methods

```
public void addEdge(Edge<V,E> e)
public boolean containsEdge(Edge<V,E> e)
public Edge<V,E> removeEdge(Edge<V,E> e)
public Edge<V,E> getEdge(Edge<V,E> e)
public int degree()
// and methods to produce Iterators...
```

GraphListVertex

```
public GraphListVertex(V key){
        super(key); // init Vertex fields
        adjacencies = new SinglyLinkedList<Edge<V,E>>();
}

public void addEdge(Edge<V,E> e){
        if (!containsEdge(e)) adjacencies.add(e);
}

public boolean containsEdge(Edge<V,E> e){
        return adjacencies.contains(e);
}

public Edge<V,E> removeEdge(Edge<V,E> e) {
        return adjacencies.remove(e);
}
```

GraphListVertex Iterators

```
// Iterator for incident edges
public Iterator<Edge<V,E>> adjacentEdges() {
        return adjacencies.iterator();
}

// Iterator for adjacent vertices
public Iterator<V> adjacentVertices() {
        return new GraphListAIterator<V,E>
            (adjacentEdges(), label());
}
```

GraphListAIterator creates an Iterator over *vertices* based on the Iterator over *edges* produced by `adjacentEdges()`

GraphListAIterator

GraphListAIterator uses two instance variables

```
    protected AbstractIterator<Edge<V,E>> edges;
    protected V vertex;

public GraphListAIterator(Iterator<Edge<V,E>> i, V v) {
        edges = (AbstractIterator<Edge<V,E>>)i;
        vertex = v;
}

public V next() {
        Edge<V,E> e = edges.next();
        if (vertex.equals(e.here()))
            return e.there();
        else { // could be an undirected edge!
            return e.here();
}
```

GraphListEIterator

GraphListEIterator uses one instance variable

```
protected AbstractIterator<Edge<V,E>> edges;
```

GraphListEIterator
- Takes the Map storing the vertices
- Uses it to build a linked list of all edges
- Gets an iterator for this linked list and stores it, using it in its own methods

GraphList

- To implement GraphList, we use the GraphListVertex (GLV) class
- GraphListVertex class
  - Maintain linked list of edges at each vertex
  - Instance vars: label, visited flag, linked list of edges
- GraphList abstract class
  - Instance vars:
    - Map<V,GraphListVertex<V,E>> dict; // label -> vertex
    - boolean directed; // is graph directed?
- How do we implement key GL methods?
  - GraphList(), add(), getEdge(), …

```java
protected GraphList(boolean dir){
      dict = new Hashtable<V,GraphListVertex<V,E>>();
      directed = dir;
}

public void add(V label) {
      if (dict.containsKey(label)) return;
      GraphListVertex<V,E> v = new
            GraphListVertex<V,E>(label);
      dict.put(label,v);
}

public Edge<V,E> getEdge(V label1, V label2) {
      Edge<V,E> e = new Edge<V,E> (get(label1),
      get(label2), null, directed);
      return dict.get(label1).getEdge(e);
}
```

GraphListDirected

- GraphListDirected (GraphListUndirected) implements the methods requiring different treatment due to (un)directedness of edges
  - addEdge, remove, removeEdge, …

```java
// addEdge in GraphListDirected.java
// first vertex is source, second is destination
 public void addEdge(V vLabel1, V vLabel2, E label) {
    // first get the vertices
   GraphListVertex<V,E> v1 = dict.get(vLabel1);
   GraphListVertex<V,E> v2 = dict.get(vLabel2);
    // create the new edge
   Edge<V,E> e = new Edge<V,E>(v1.label(), v2.label(), label, true);
    // add edge only to source vertex linked list (aka adjacency list)
    v1.addEdge(e);
}
```

```
public V remove(V label) {
   //Get vertex out of map/dictionary
   GraphListVertex<V,E> v = dict.get(label);

   //Iterate over all vertex labels (called the map "keyset")
   Iterator<V> vi = iterator();
   while (vi.hasNext()) {
        //Get next vertex label in iterator
        V v2 = vi.next();

        //Skip over the vertex label we're removing
        //(Nodes don't have edges to themselves...)
        if (!label.equals(v2)) {
            //Remove all edges to "label"
            //If edge does not exist, removeEdge returns null
            removeEdge(v2,label);
        }
    }
    //Remove vertex from map
    dict.remove(label);
    return v.label();
}
```

```
public E removeEdge(V vLabel1, V vLabel2) {
    //Get vertices out of map
    GraphListVertex<V,E> v1 = dict.get(vLabel1);
    GraphListVertex<V,E> v2 = dict.get(vLabel2);

    //Create a "temporary" edge connecting two vertices
    Edge<V,E> e = new Edge<V,E>(v1.label(), v2.label(), null, true);

    //Remove edge from source vertex linked list
    e = v1.removeEdge(e);
    if (e == null) return null;
    else return e.label();
}
```

Efficiency Revisited

- Assume Map operations are O(1) (for now)
  - |E| = number of edges
  - |V| = number of vertices
- Runtime of add, addEdge, getEdge, removeEdge, remove?
- Space usage?
- Conclusions
  - Matrix is better for dense graphs
  - List is better for sparse graphs
  - For graphs "in the middle" there is no clear winner

Efficiency : Assuming Fast Map

	Matrix	GraphList
add	O(1)	O(1)
addEdge	O(1)	O(1)
getEdge	O(1)	O(|V|)
removeEdge	O(1)	O(|V|)
remove	O(|V|)	O(|E|)
space	O($\|V\|^2$)	O(|V|+|E|)

Basic Graph Properties

- A *subgraph* of a graph G=(V, E) is a graph G'=(V',E') where
  - $V' \subseteq V$
  - $E' \subseteq E$, and
  - If $e \in E'$ where $e = \{u,v\}$, then $u, v \in V'$
- If E' contains *every* edge of E having both ends in V', then G' is called the subgraph of G *induced by* V'
- If V' = V, then G' is called a *spanning subgraph of G*