CSCI 136
Data Structures &
Advanced Programming

Lecture 24

Fall 2019

Instructor: B&S

Administrative Details

- Lab 8 today!
- You can work with a partner
- Bring a design to lab
- Try to take advantage of
  - Abstract base classes/inheritance
  - Data structures you've learned

2

Last Time

- Heapsort
- Skew Heaps: A Mergeable Heap Structure
- Introduction to Binary Search Trees (BSTs)

Today's Outline

- Binary search trees (Ch 14)
  - Overview
  - Definition
  - Some Applications
  - The *locate* method
  - Further Implementation

Binary Search Trees

- Binary search trees maintain a *total* ordering among elements (assumes comparability)
- Definition: A BST T is either:
  - Empty
  - Has root r with subtrees T_L and T_R such that
    - All nodes in T_L have smaller value than r
    - All nodes in T_R have larger value than r
    - T_L and T_R are also BSTs

BST Observations

- The same data can be represented by many BST shapes
- Searching for a value in a BST takes time proportional to the height of the tree
  - Reminder: trees have height, nodes have depth
- Additions to a BST happen at nodes missing at least one child (*a constraint!*)
- Removing from a BST can involve *any* node

BST Operations

- BSTs will implement the OrderedStructure Interface
  - `add(E item)`
  - `contains(E item)`
  - `get(E item)`
  - `remove(E item)`
  - Runtime of above operations?
    - All O(h) – where h is the tree height
  - `iterator()`
    - `This will provide an in-order traversal`

BST Implementation

- The BST holds the following items
  - BinaryTree root: the root of the tree
  - BinaryTree EMPTY: a static empty BinaryTree
    - To use for all empty nodes of tree
  - int count: the number of nodes in the BST
  - Comparator<E> ordering: for comparing nodes
    - Note: E must implement Comparable
- Two constructors: One takes a Comparator
  - Other creates a NaturalComparatot

BST Implementation: locate

- Several methods search the tree
  - add, remove, contains
- We factor out common code: locate method
- *protected* locate(BinaryTree<E> *node*, E *v*)
  - Returns a BinaryTree<E> *n* in the subtree with root *node* such that either
    - *n* has its value equal to *v*, or
    - *v* is not in this subtree and *n* is the node whose child *v* should be
- How would we implement locate()?

BST Implementation: locate

```
BinaryTree locate(BinaryTree root, E val)
    if root's value equals val return root
    child ← child of root whose subtree should
            hold val
    if child is emptry tree, return root
            // val not in subtree based at root
    else //keep looking
            return locate(child, val)
```

BST Implementation: locate

- What about this line?

  child ← *child of root whose subtree should hold value*

- If the tree can have multiple nodes with same value, then we need to be careful
- Convention: During *add* operation, only move to right subtree if value to be added is *greater than* value at node
- We'll look at *add* later
- Let's look at *locate* now

The code : locate

```
protected BinaryTree<E> locate(BinaryTree<E> root, E value) {
       E rootValue = root.value();
       BinaryTree<E> child;

       // found at root: done
       if (rootValue.equals(value)) return root;

       // look left if less-than, right if greater-than
       if (ordering.compare(rootValue,value) < 0)
           child = root.right();
       else
           child = root.left();

       // no child there: not in tree, return this node,
       // else keep searching
       if (child.isEmpty()) return root;
       else
           return locate(child, value);
}
```

Other core BST methods

- locate(v) returns either a node containing v or a node where v can be added as a child
- locate() is used by
  - `public boolean contains(E value)`
  - `public E get(E value)`
  - `public void add(E value)`
  - `Public void remove(E value)`
- Some of these also use another utility method
  - `protected BT predecessor(BT root)`
- Let's look at contains() first...

Contains

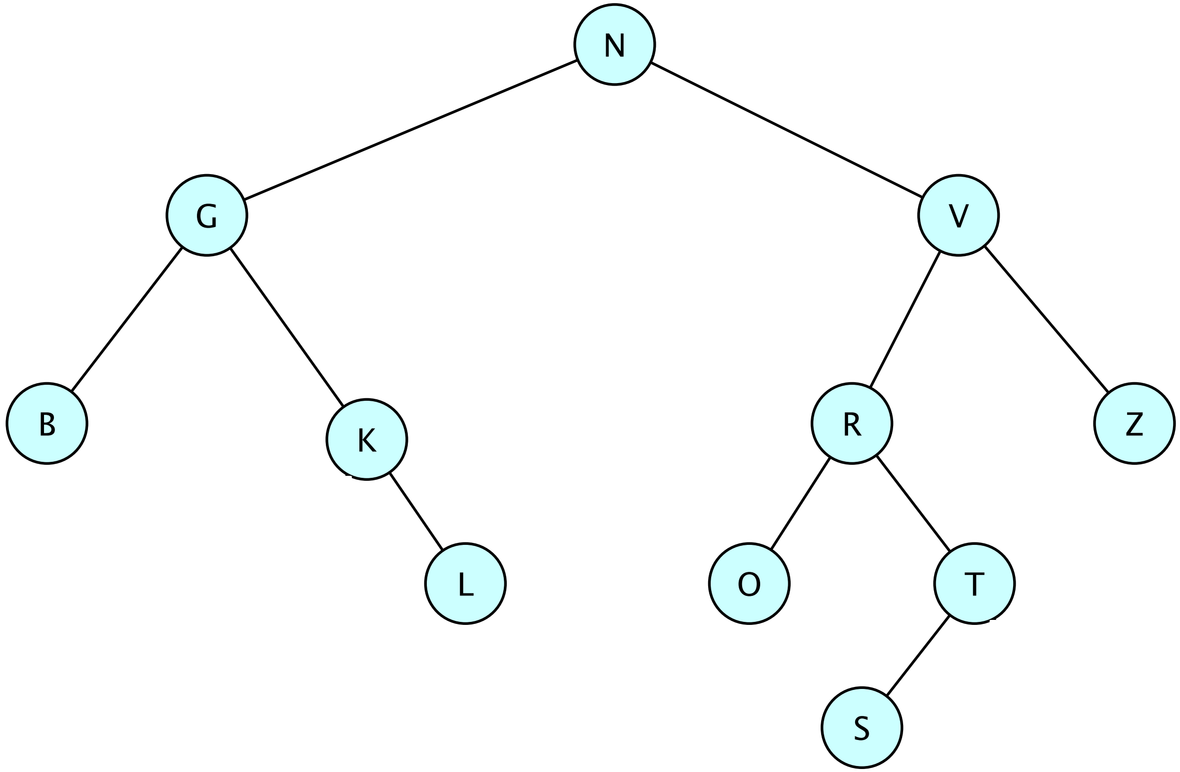
```
public boolean contains(E value){
   if (root.isEmpty()) return false;

   BinaryTree<E> possibleLocation = locate(root,value);

   return value.equals(possibleLocation.value());
}
```

First (Bad) Attempt: add(E value)

```
public void add(E value) {
        BinaryTree<E> newNode = new BinaryTree<E>(value,EMPTY,EMPTY);
        if (root.isEmpty()) root = newNode;
        else {
                BinaryTree<E> insertLocation = locate(root,value);
                E nodeValue = insertLocation.value();
        if (ordering.compare(nodeValue,value) < 0)
                insertLocation.setRight(newNode);
        else
                insertLocation.setLeft(newNode);
        }
        count++;
}
```

Problem: If repeated values are allowed, left subtree might not be empty when setLeft is called

Add: Repeated Nodes



Where would a new K be added?
A new V?

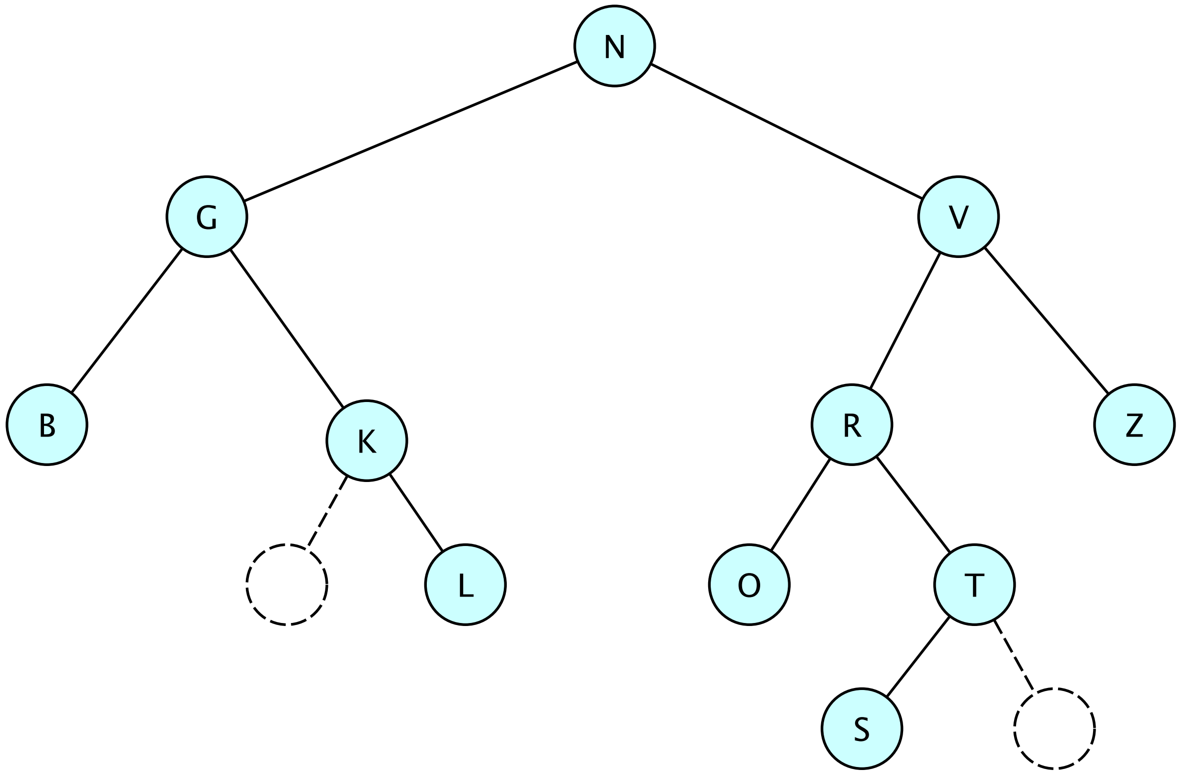
Add Duplicate to Predecessor

- If insertLocation has a left child then
  - Find insertLocation's predecessor
  - Add repeated node as right child of predecessor
  - If insertLocation has a left subtree that's where Predecessor will be
    - Do you believe me?
    - Where else could it be?

Corrected Version: add(E value)

```
BinaryTree<E> newNode = new BinaryTree<E>(value,EMPTY,EMPTY);
if (root.isEmpty()) root = newNode;
else {
   BinaryTree<E> insertLocation = locate(root,value);
   E nodeValue = insertLocation.value();
   if (ordering.compare(nodeValue,value) < 0)
      insertLocation.setRight(newNode);
   else
      if (insertLocation.left().isEmpty())
         insertLocation.setLeft(newNode);
      else
         // if value is in tree, we insert just before
         predecessor(insertLocation).setRight(newNode);
}
count++;
```

How to Find Predecessor



Where would a new K be added?
A new V?

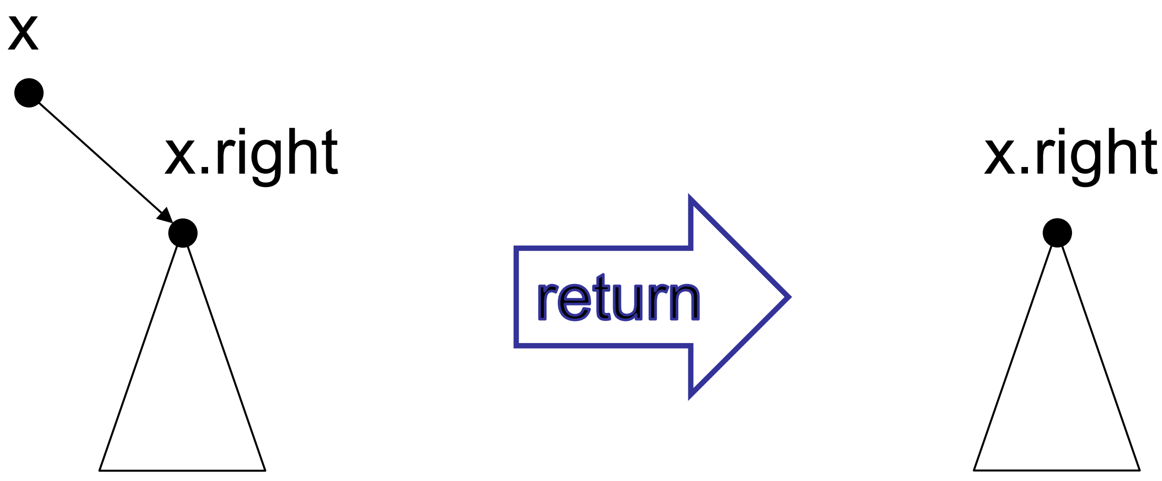
Predecessor

```
protected BinaryTree<E> predecessor(BinaryTree<E> root) {
    Assert.pre(!root.isEmpty(), ”Root has predecessor");
    Assert.pre(!root.left().isEmpty(),"Root has left child.");

    BinaryTree<E> result = root.left();

    while (!result.right().isEmpty())
        result = result.right();

    return result;
}
```

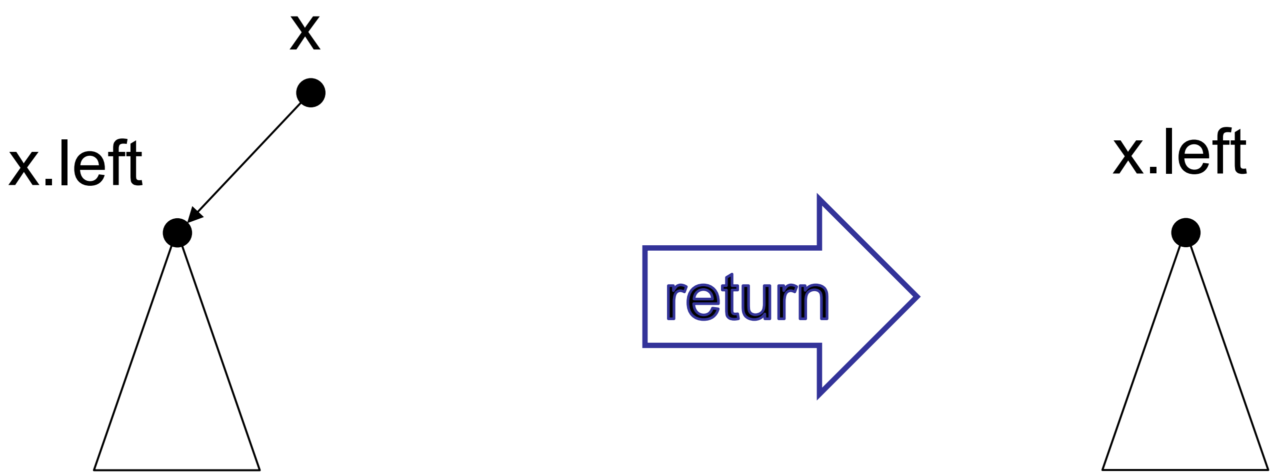
Removal

- Removing the root is a (not so) special case
- Let's figure that out first
  - If we can remove the root, we can remove any element in a BST in the same way
    - Do you believe me?
- We need to implement:
  - `public E remove(E item)`
  - `protected BT removeTop(BT top)`

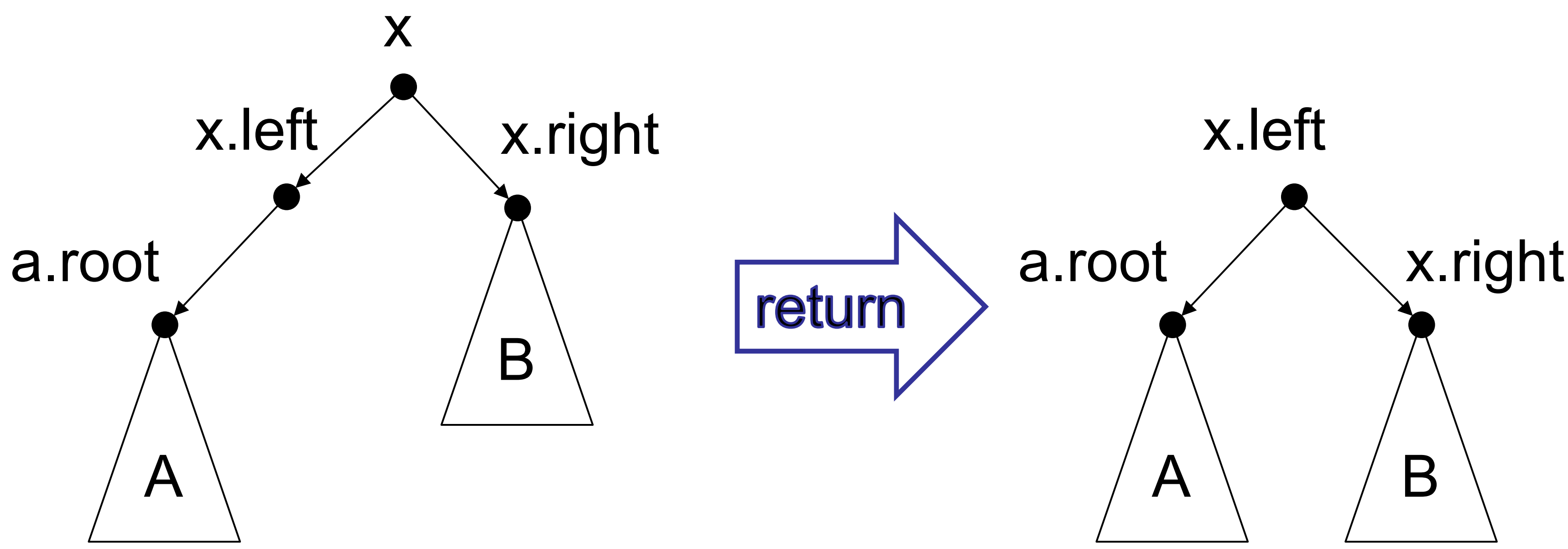
Case 1: No left binary tree



Case 2: No right binary tree



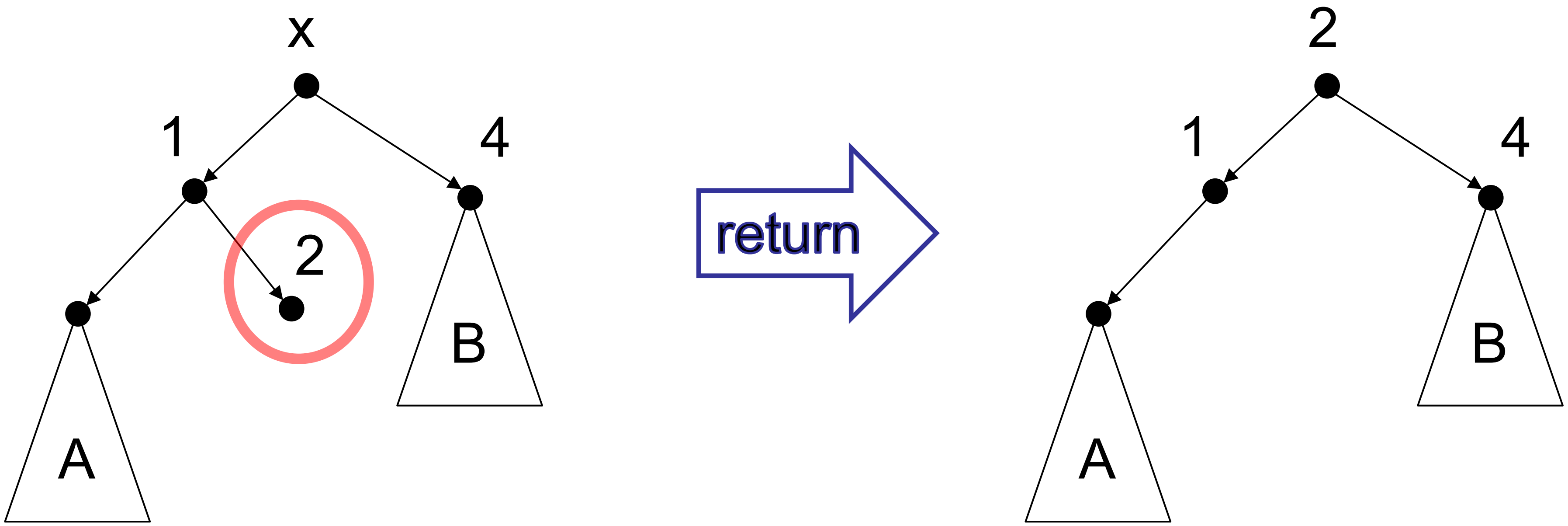
Case 3: Left has no right subtree



Case 4: General Case

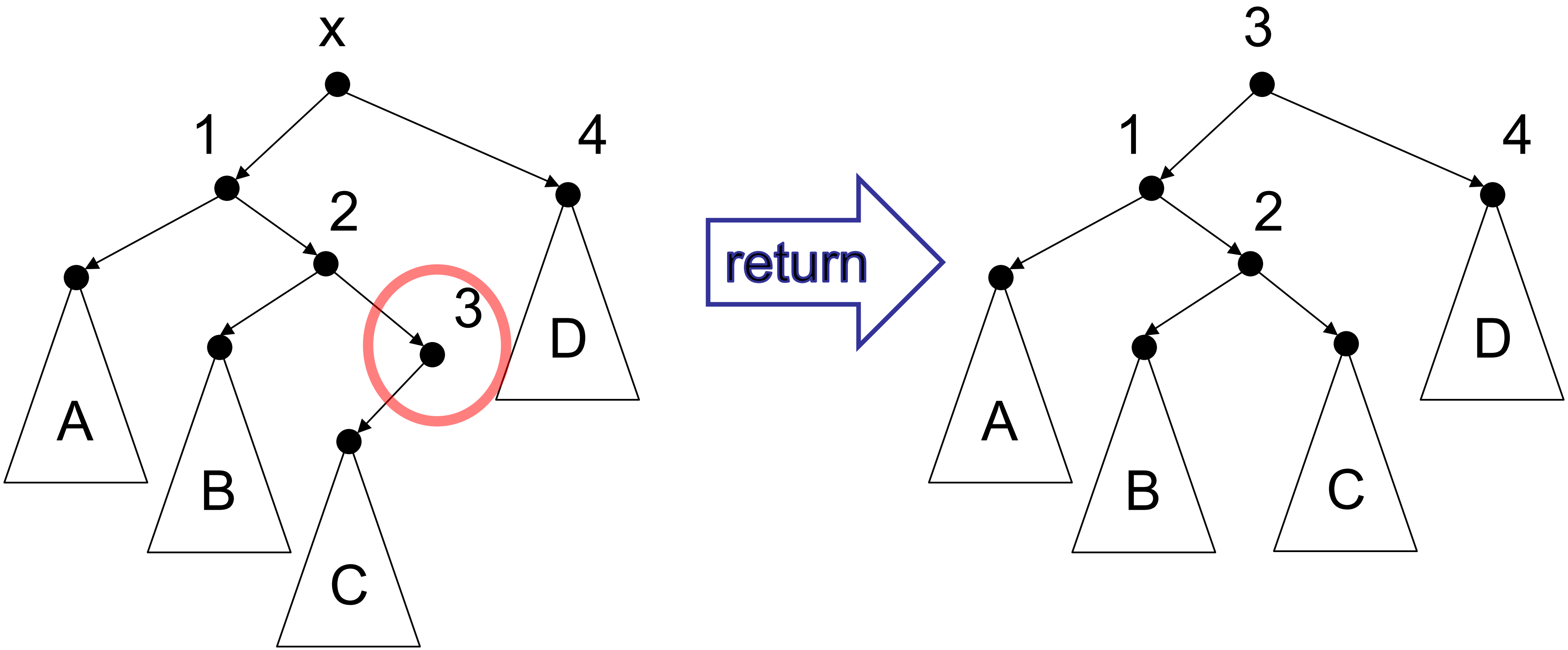
- Consider BST requirements:
  - Left subtree must be <= root
  - Right subtree must be > root
- Strategy: replace the root with the largest value that is less than or equal to it
  - predecessor(root) : rightmost left descendant
- This may require reattaching the predecessor's left subtree!

Case 4: General Case



Replace root with predecessor(root), then patch up the remaining tree

Case 4: General Case



Replace root with predecessor(root), then patch up the remaining tree

RemoveTop(topNode)

Detach left and right sub-trees from root (i.e. topNode)

If either left or right is empty, ***return*** *the other one*

If left has no right child

make right the right child of left then ***return*** *left*

Otherwise find largest node C in left

// C is the right child of its own parent P

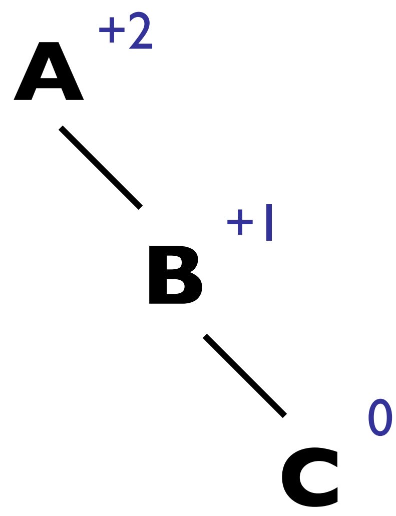
// C is the predecessor of right (ignoring topNode)

Detach C from P; make C's left child the right child of P

Make C new root with left and right as its sub-trees

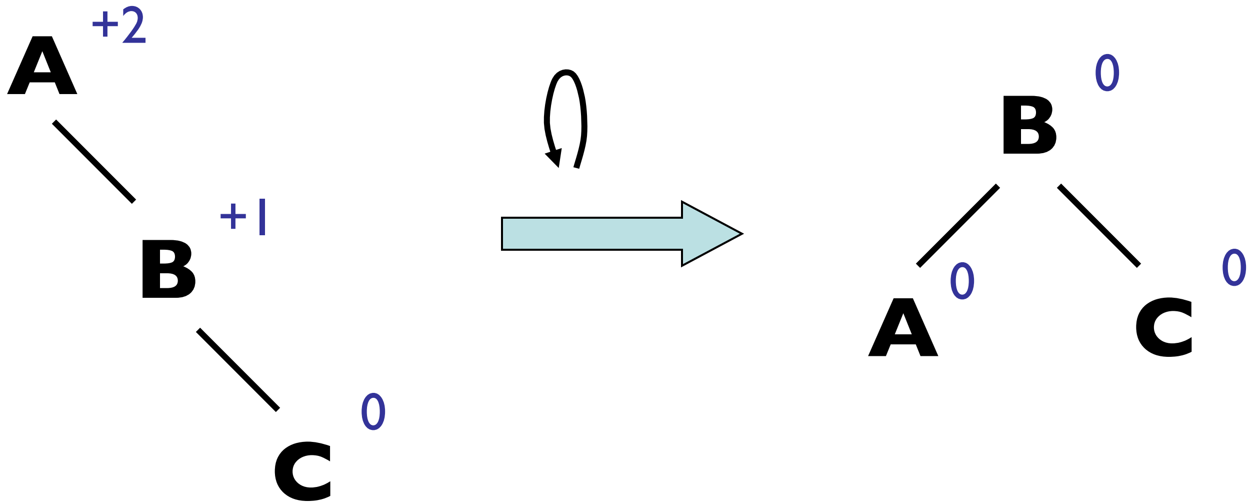
But What About Height?

- Can we design a binary search tree that is always “shallow”?
- Yes! In many ways. Here’s one
- AVL trees
  - Named after its two inventors, G.M. Adelson-Velsky and E.M. Landis, who published a paper about AVL trees in 1962 called "An algorithm for the organization of information"



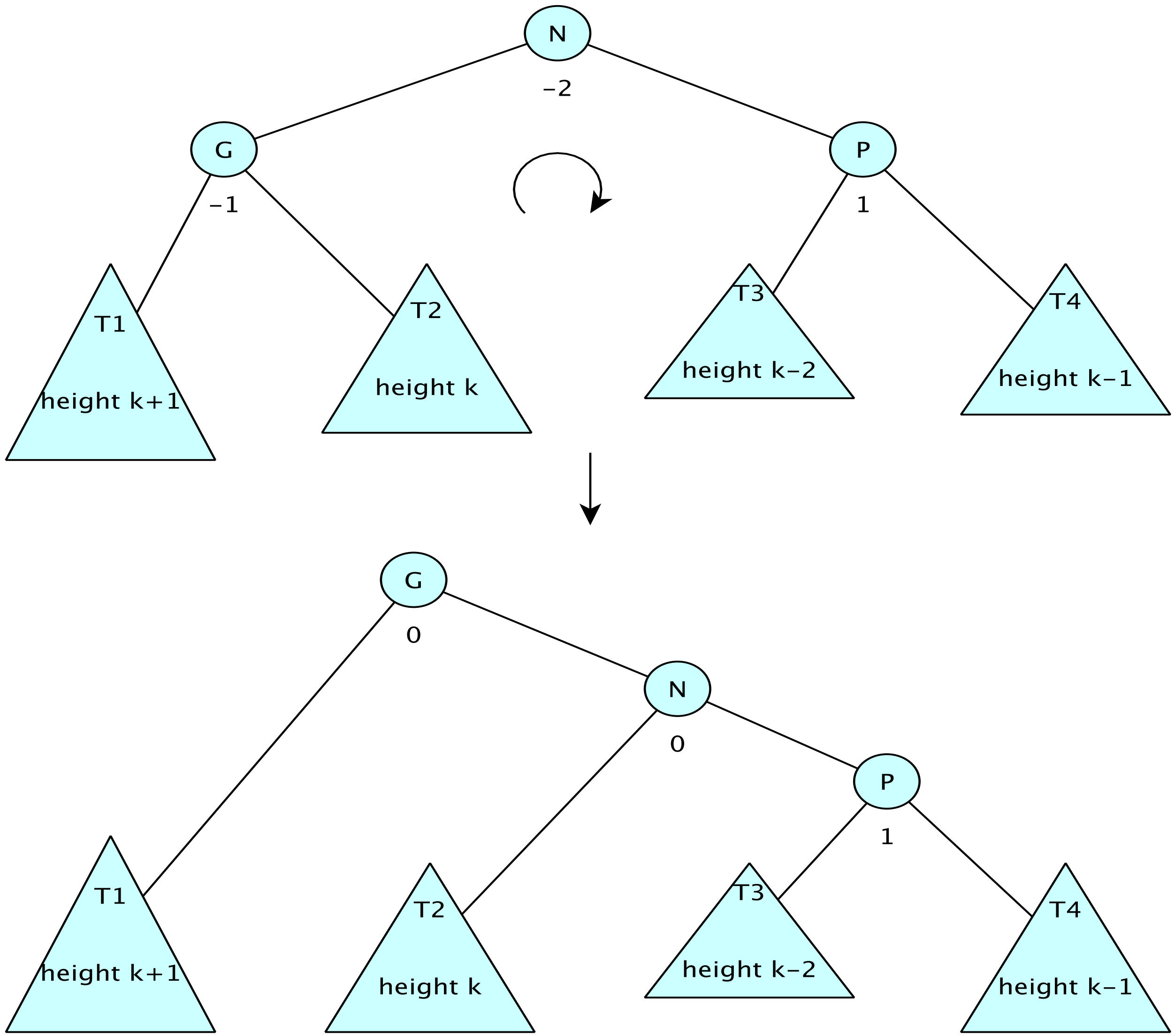
- The *balance factor* of a node is the height of its right subtree minus the height of its left subtree. A node with balance factor 1, 0, or -1 is considered *balanced*.
- A node with any other balance factor is considered *unbalanced* and requires rebalancing the tree.

Single Rotation

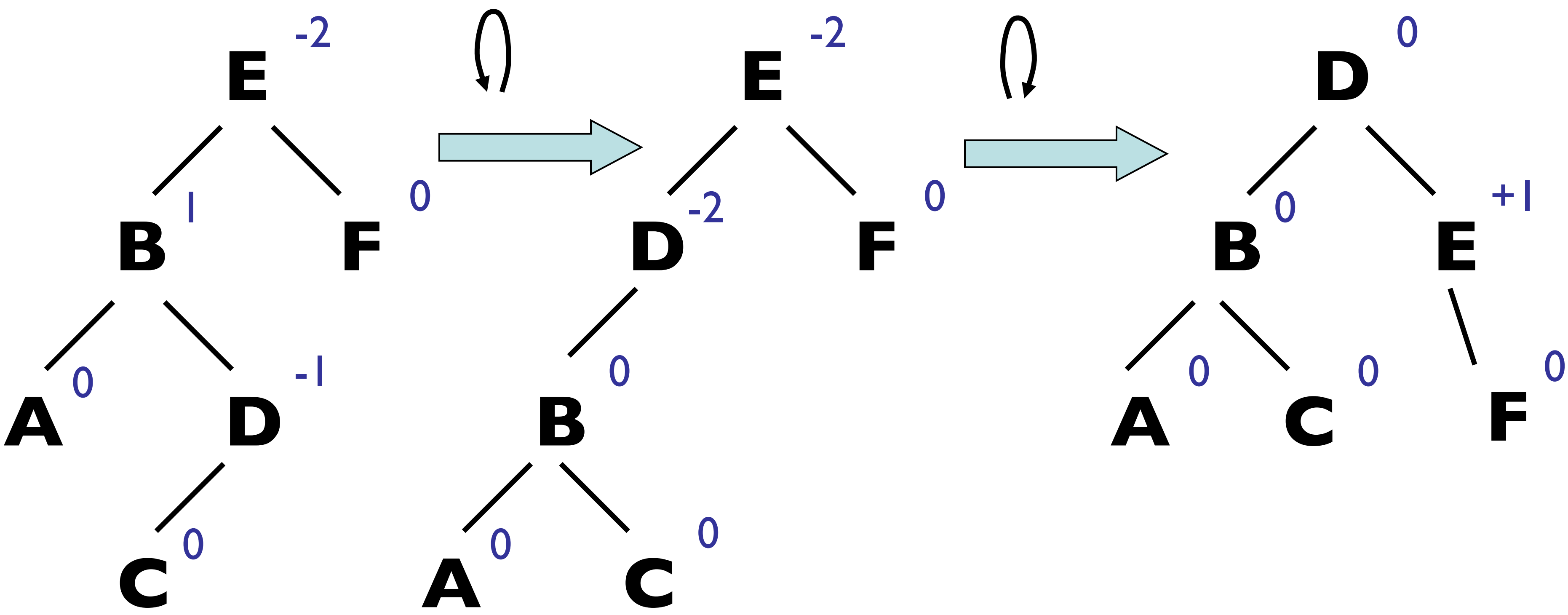


Unbalanced trees can be rotated to achieve balance.

Single Right Rotation



Double Rotation



AVL Tree Facts

- A tree that is AVL except at root, where root balance factor equals ± 2 can be rebalanced with at most 2 rotations
- add(v) requires at most O(log n) balance factor changes and one (single or double) rotation to restore AVL structure
- remove(v) requires at most O(log n) balance factor changes and (single or double) rotations to restore AVL structure

AVL Trees: One of Many

- There are many strategies for tree balancing to preserve O(log n) height, including
- AVL Trees: guaranteed O(log n) height
- Red-black trees: guaranteed O(log n) height
- B-trees (not binary): guaranteed O(log n) height
  - 2-3 trees, 2-3-4 trees, red-black 2-3-4 trees, ...
- Splay trees: *Amortized* O(log n) time operations
- Randomized trees: O(log n) expected height