CSCI 136
Data Structures & Advanced Programming

Lecture 20

Fall 2019

Instructor: Bill & Sam

Administrative Details

- Lab 7 is available online
  - No partners this week
  - Review before lab; come to lab with design doc
  - We'll give an overview (possibly on Wednesday)

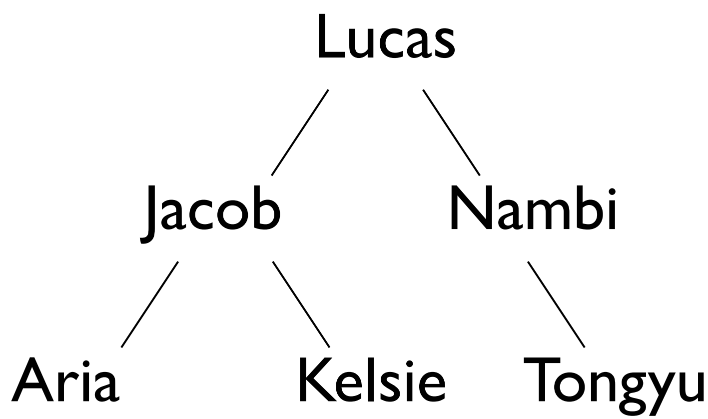
Last Time

- Recursion/Induction on Trees
- Applications: Decision Trees
- Trees with more than 2 children
  - Representations
- Traversing Binary Trees

Today

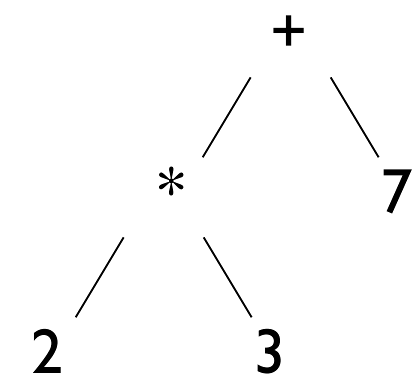
- Traversing Binary Trees
- Big Trees
- Lab 7 Discussion
- Storing Trees in Arrays

Tree Traversals



In-order: Aria, Jacob, Kelsie, Lucas, Nambi, Tongyu
Pre-order: Lucas, Jacob, Aria, Kelsie, Nambi, Tongyu
Post-order: Aria, Kelsie, Jacob, Tongyu, Nambi, Lucas,
Level-order: Lucas, Jacob, Nambi, Aria, Kelsie, Tongyu

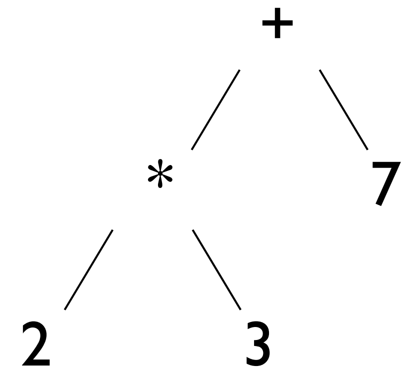
Tree Traversals



- Pre-order
  - Each node is visited before any children. Visit node, then each node in left subtree, then each node in right subtree. (node, left, right)
    - +*237
- In-order
  - Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree. (left, node, right)
    - 2*3+7

("pseudocode")

Tree Traversals

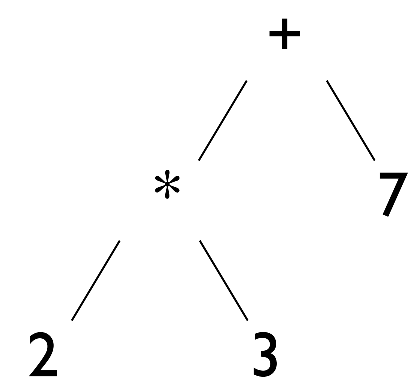


- Post-order
  - Each node is visited after its children are visited. Visit all nodes in left subtree, then all nodes in right subtree, then node itself. (left, right, node)
    - 23*7+
- Level-order (not obviously recursive!)
  - All nodes of level i are visited before nodes of level i+1. (visit nodes left to right on each level)
    - +*723

("pseudocode")

Tree Traversals

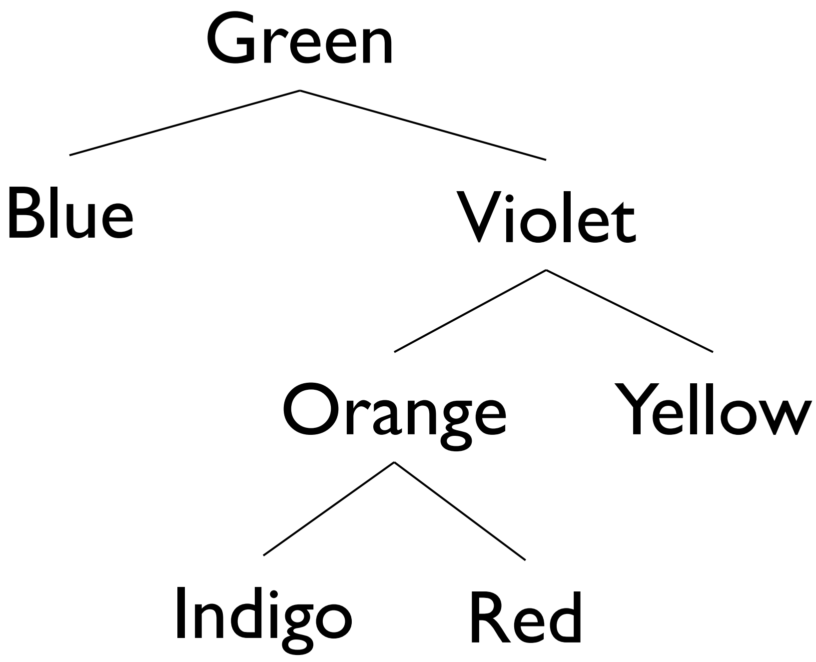
```
public void pre-order(BinaryTree t) {
      if(t.isEmpty()) return;
      touch(t); // some method
      preOrder(t.left());
      preOrder(t.right());
}
```



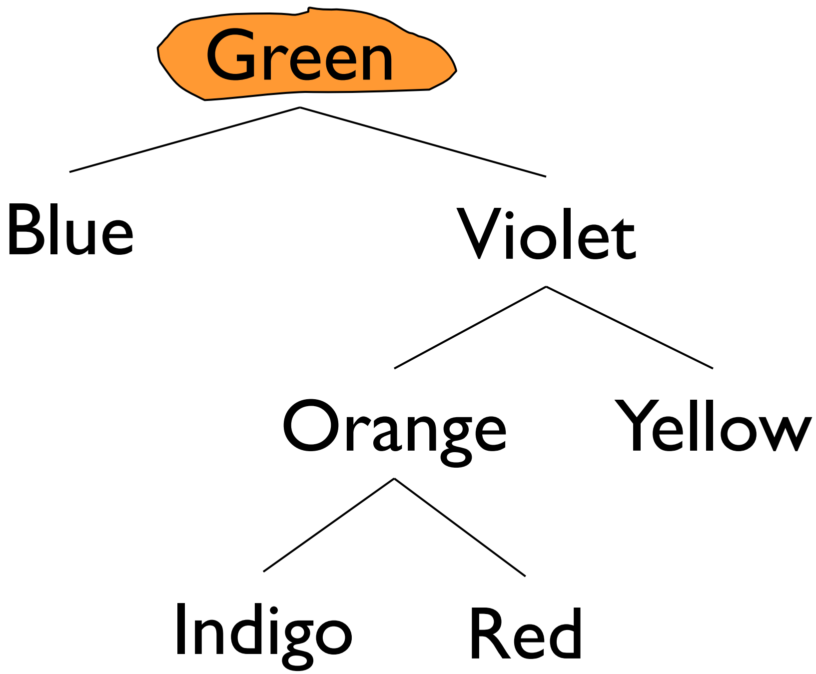
For in-order and post-order: just move touch(t)!

But what about level-order???

Level-Order Traversal

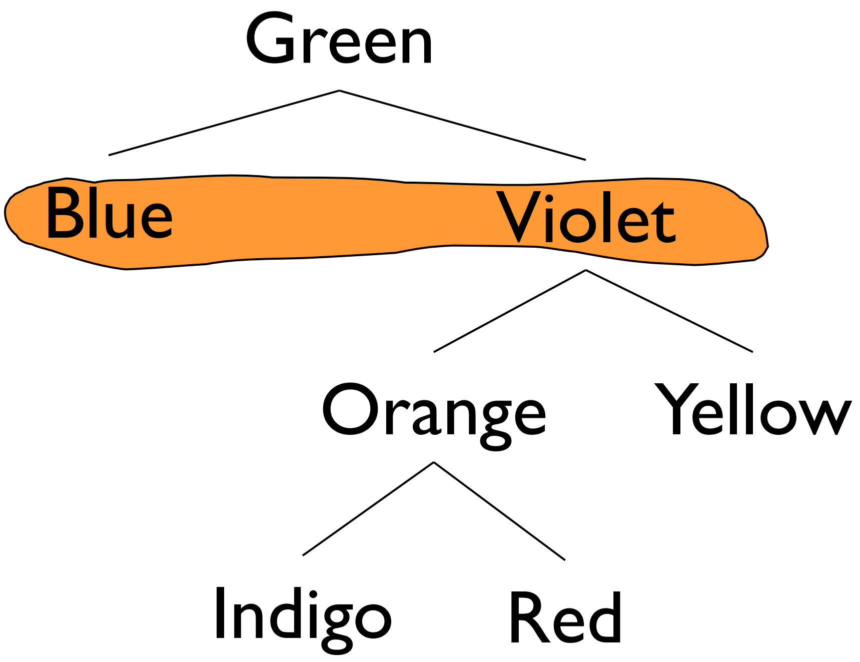


Level-Order Traversal



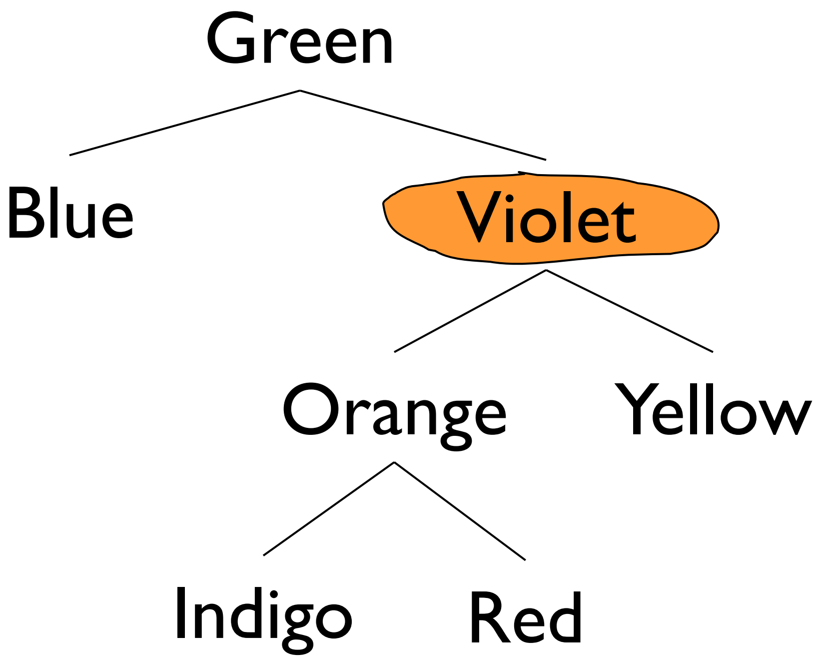
G

Level-Order Traversal



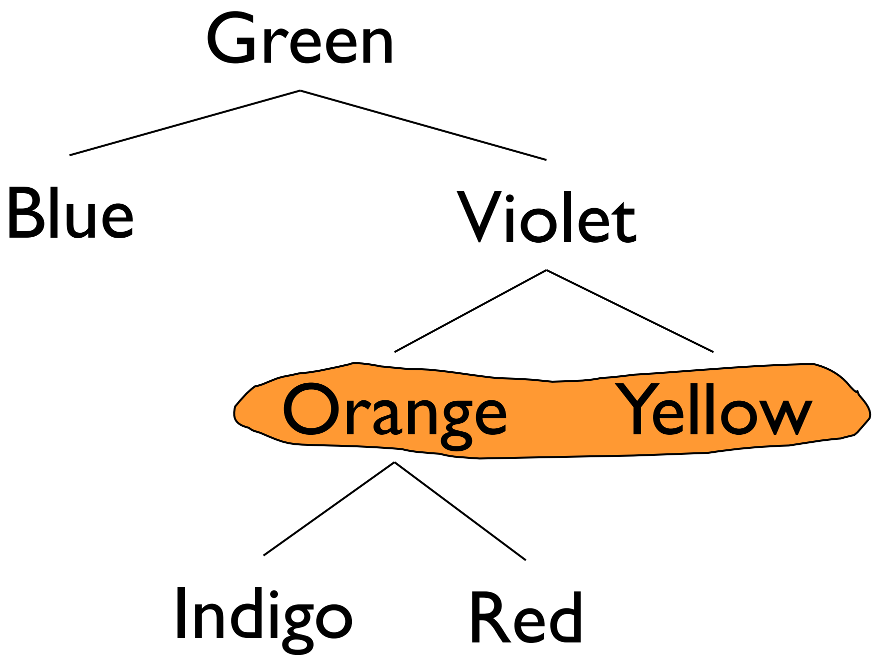
G

Level-Order Traversal



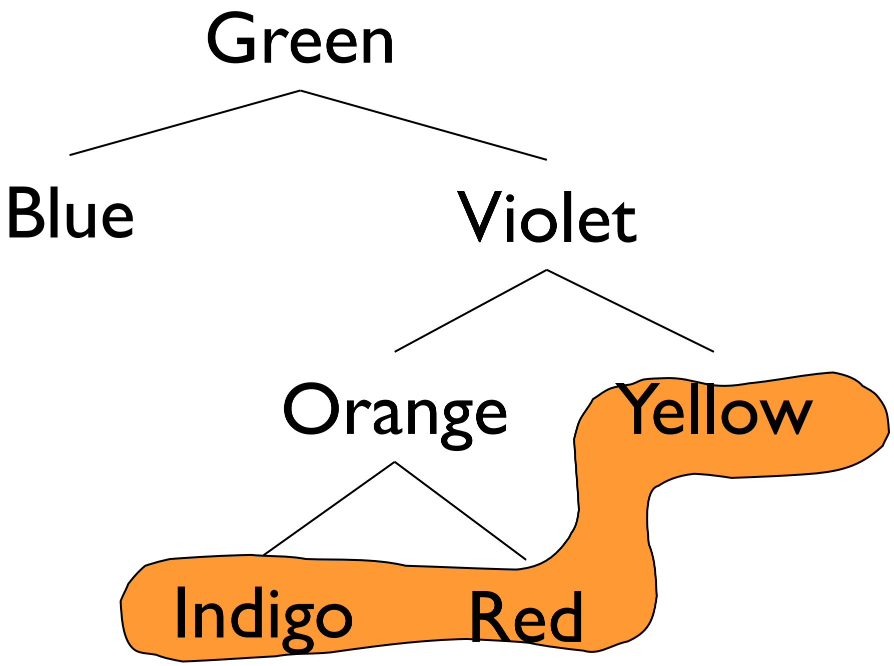
G B

Level-Order Traversal



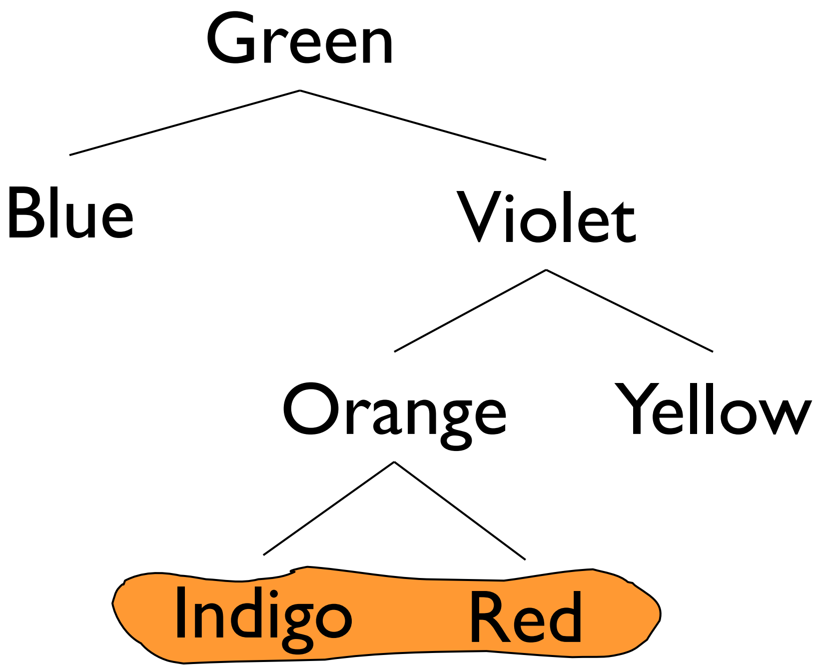
G B V

Level-Order Traversal



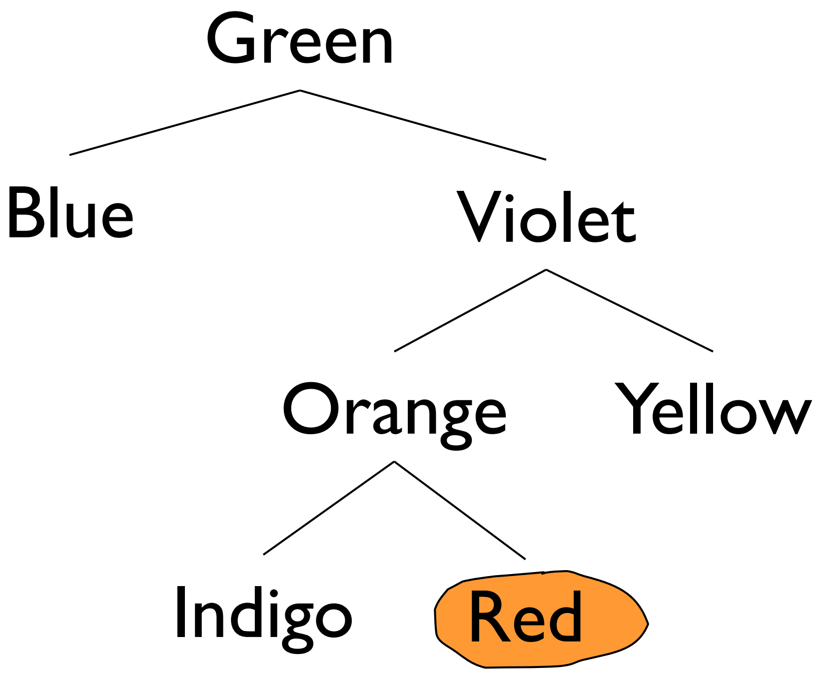
G B V O

Level-Order Traversal



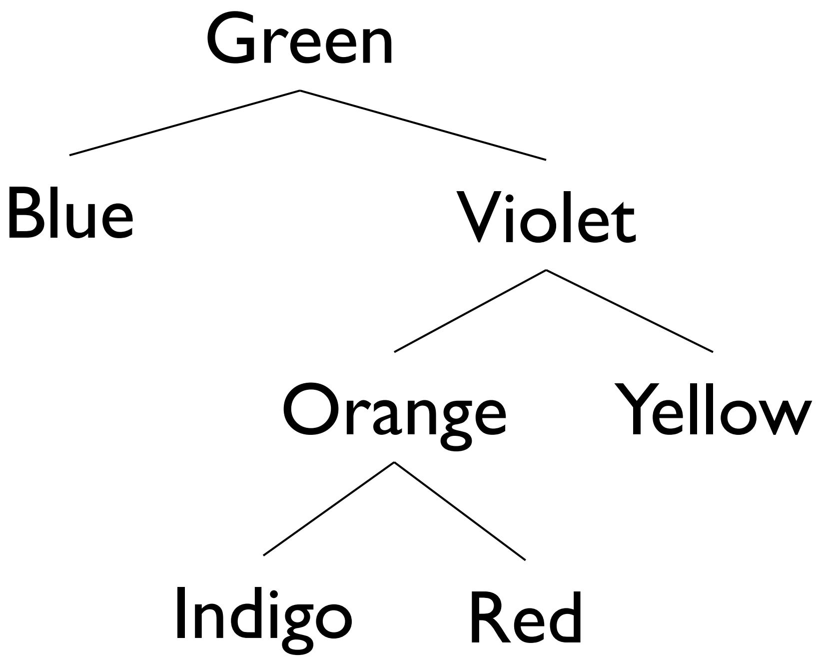
G B V O Y

Level-Order Traversal



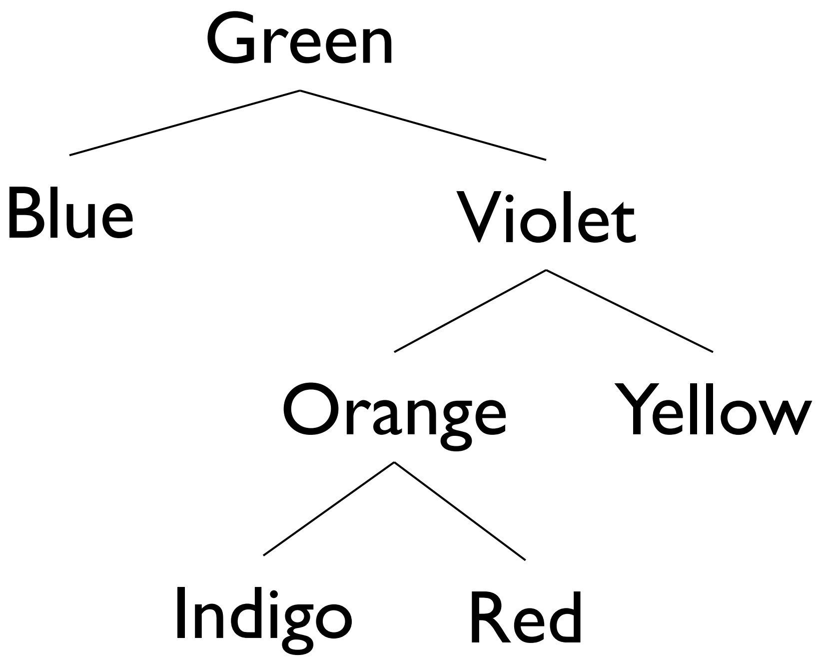
G B V O Y I

Level-Order Traversal

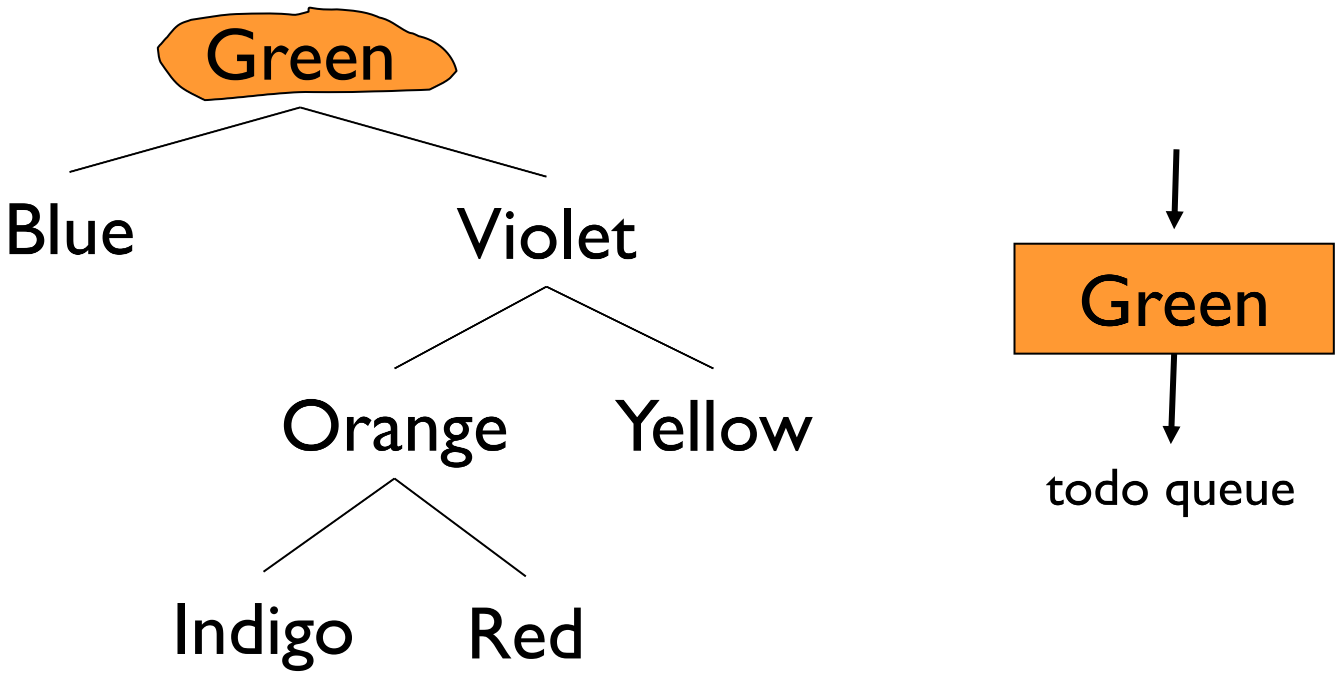


G B V O Y I R

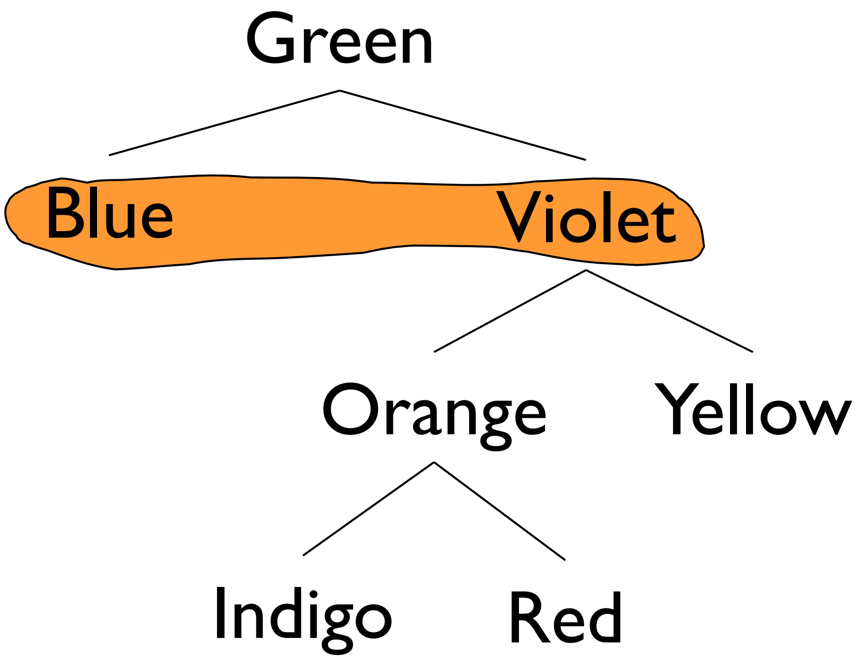
Level-Order Traversal

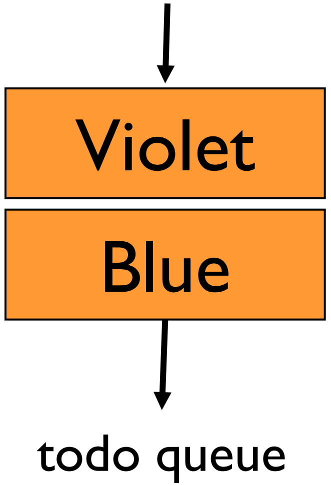


Level-Order Traversal



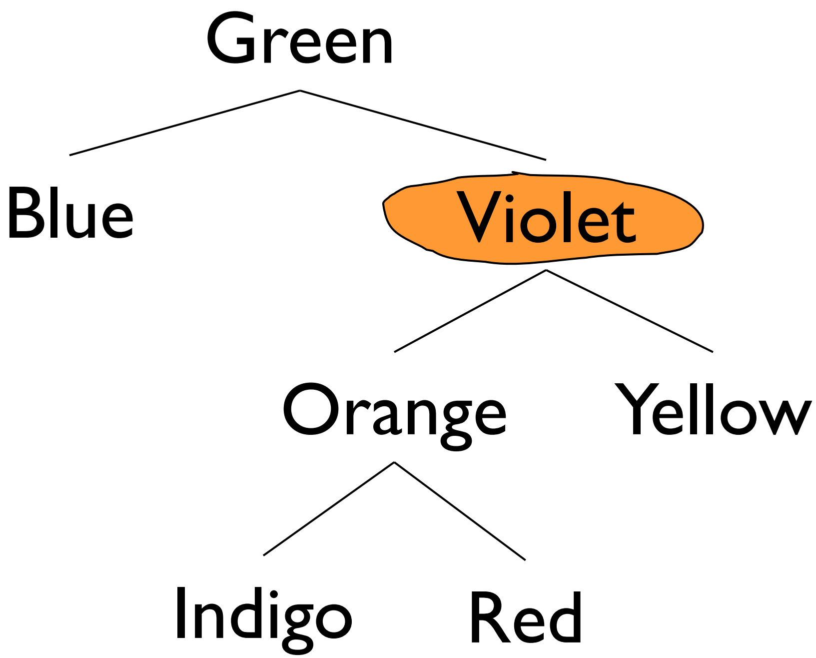
Level-Order Traversal

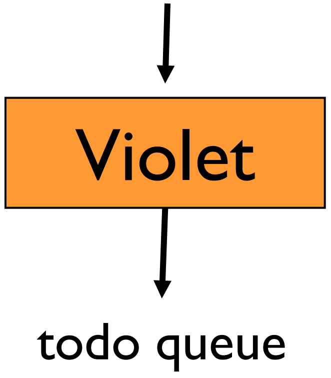




G

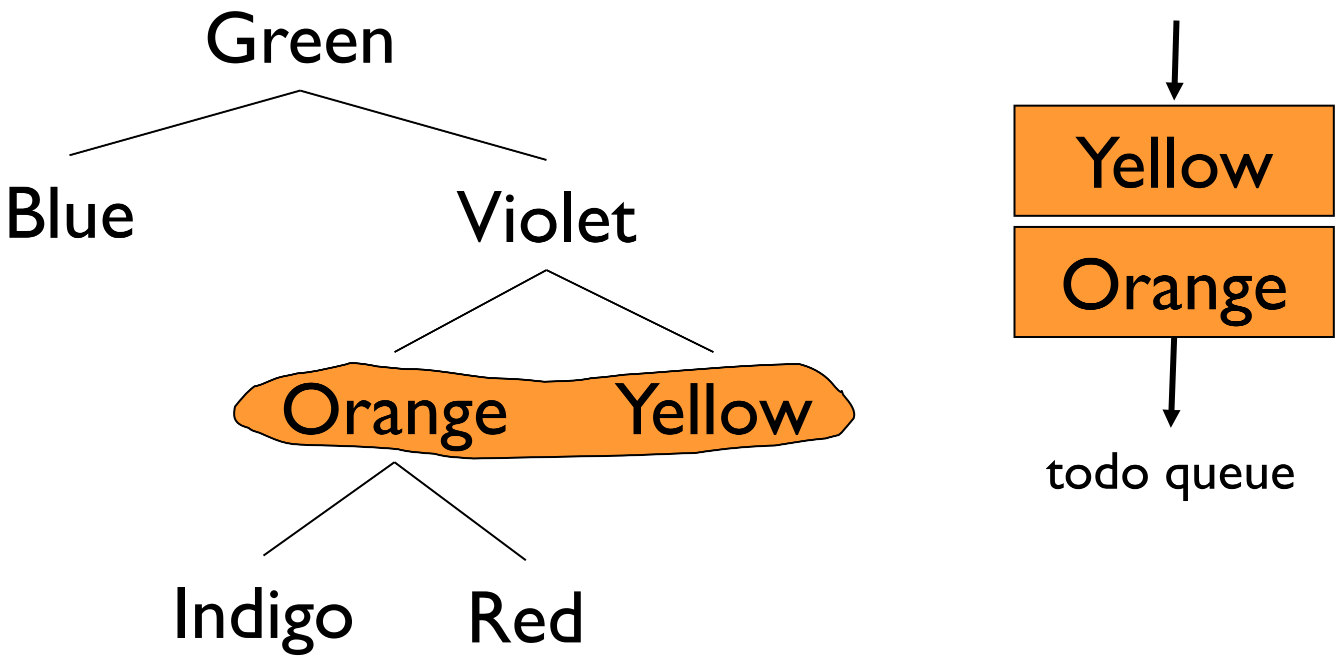
Level-Order Traversal





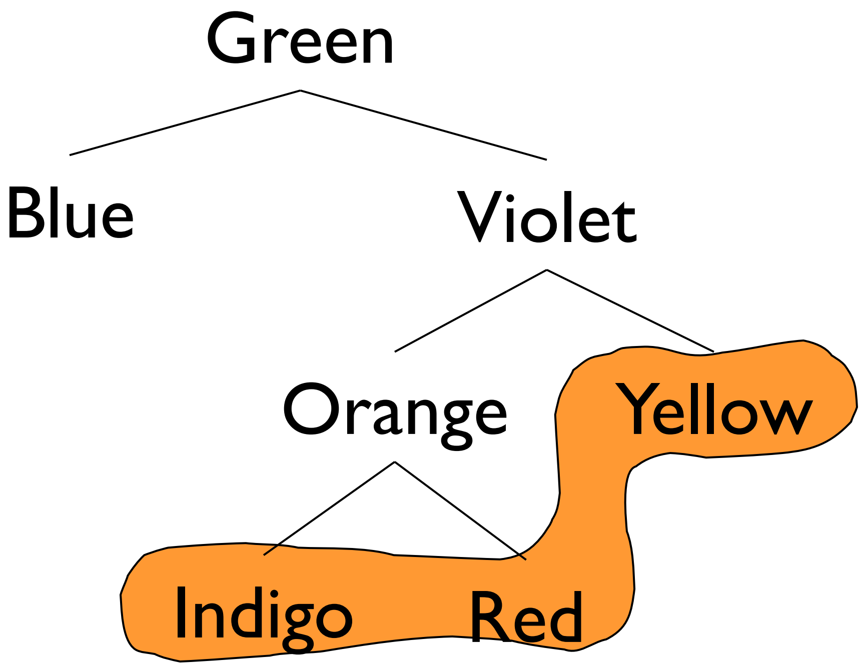
G B

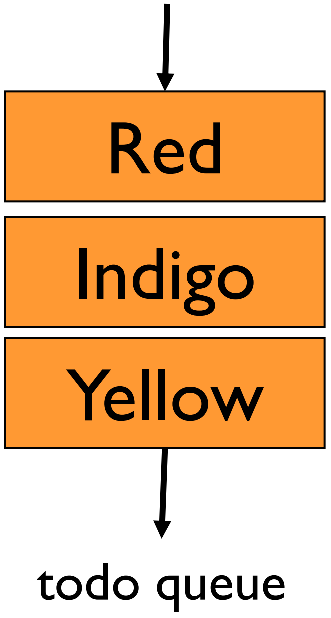
Level-Order Traversal



G B V

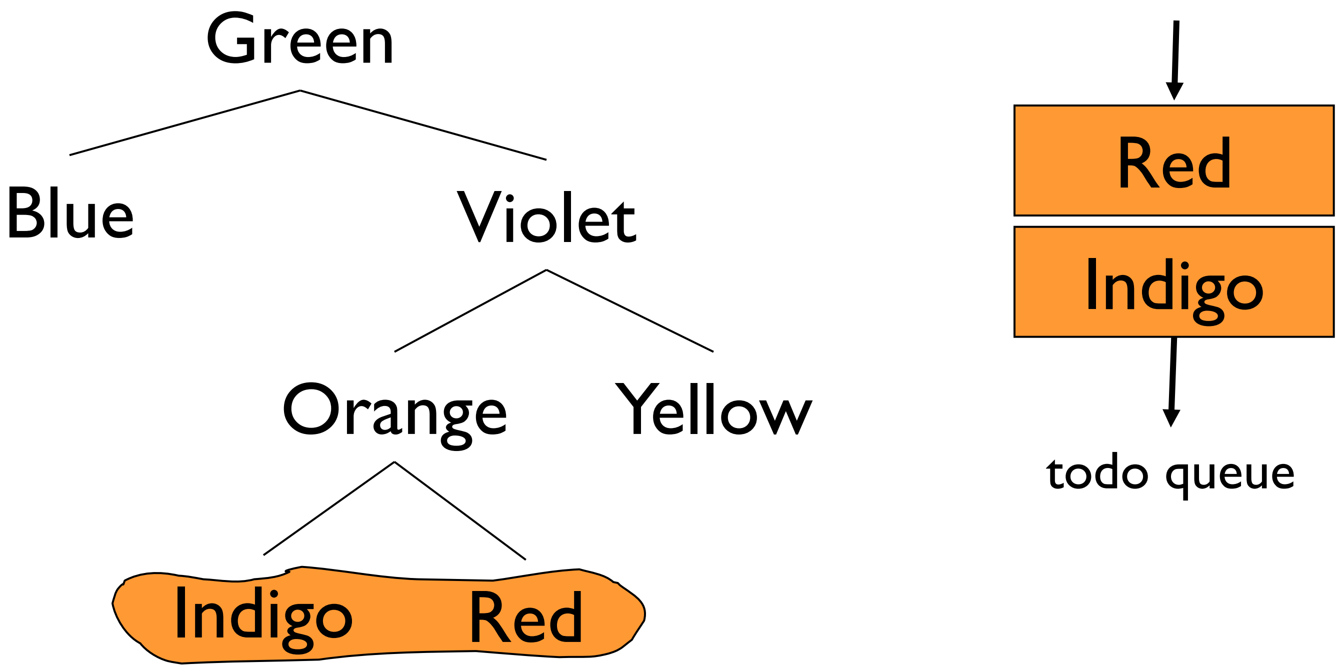
Level-Order Traversal





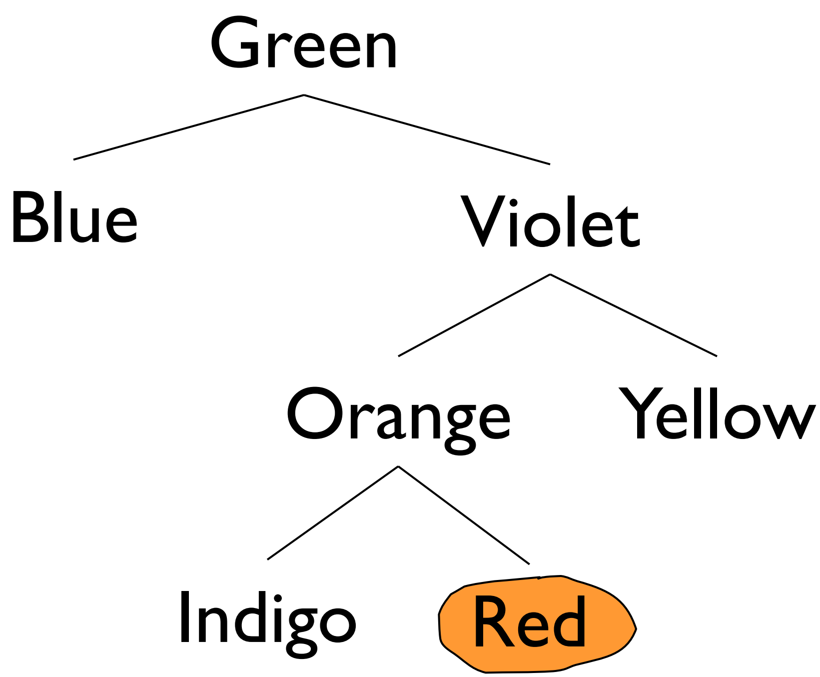
G B V O

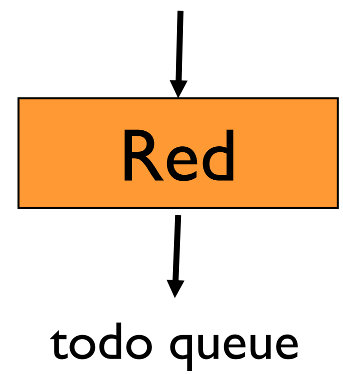
Level-Order Traversal



G B V O Y

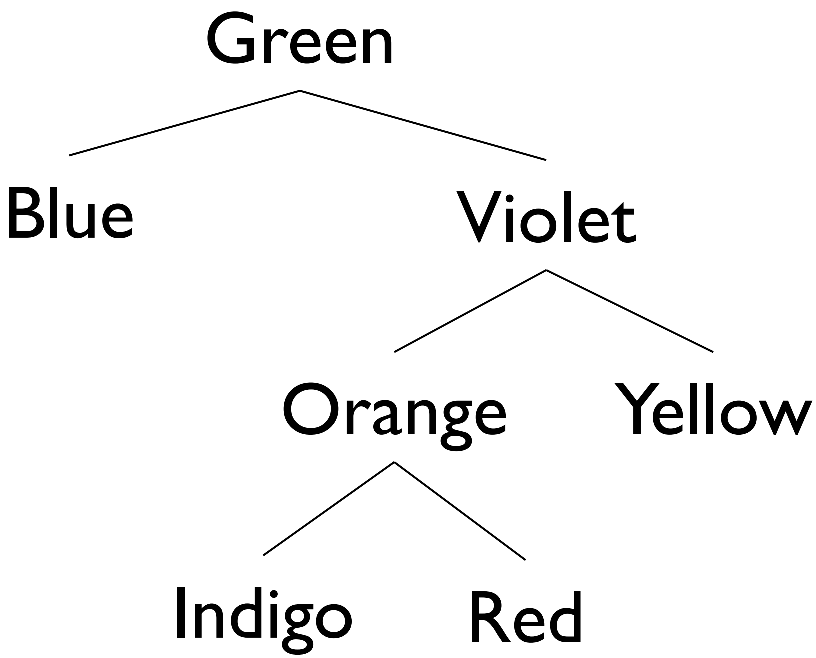
Level-Order Traversal

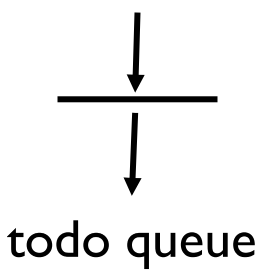




G B V O Y I

Level-Order Traversal





G B V O Y I R

Level-Order Tree Traversal

```
public static <E> void levelOrder(BinaryTree<E> t) {
   if (t.isEmpty()) return;

   // The queue holds nodes for in-order processing
   Queue<BinaryTree<E>> q = new QueueList<BinaryTree<E>>();
   q.enqueue(t); // put root of tree in queue

   while(!q.isEmpty()) {
      BinaryTree<E> next = q.dequeue();
      touch(next);
      if(!next.left().isEmpty()  ) q.enqueue( next.left()  );
      if(!next.right().isEmpty() ) q.enqueue( next.right() );
   }
}
```

Iterators

- Provide iterators that implement the different tree traversal algorithms

- Methods provided by BinaryTree class:
  - preorderIterator()
  - inorderIterator()
  - postorderIterator()
  - levelorderIterator()

Implementing the Iterators

- Basic idea
  - Should return elements in same order as corresponding traversal method shown
  - Recursive methods don't convert as easily: must phrase in terms of next() and hasNext()
  - So, let's start with levelOrder!

Level-Order Iterator

```
public BTLevelorderIterator(BinaryTree<E> root)
   {
       todo = new QueueList<BinaryTree<E>>();
       this.root = root; // needed for reset
       reset();
   }

public void reset()
   {
       todo.clear();
       // empty queue, add root
       if (!root.isEmpty()) todo.enqueue(root);
   }
```

Level-Order Iterator

```
public boolean hasNext() {
       return !todo.isEmpty();
}

public E next() {
       BinaryTree<E> current = todo.dequeue();
       E result = current.value();
       if (!current.left().isEmpty())
           todo.enqueue(current.left());
       if (!current.right().isEmpty())
           todo.enqueue(current.right());
       return result;
}
```

Pre-Order Iterator

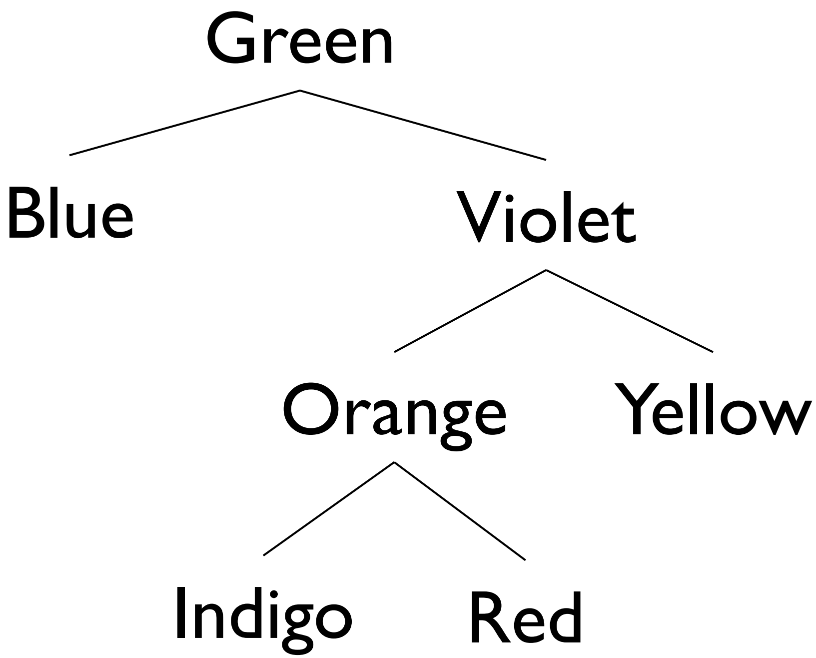
- Basic idea
  - Should return elements in same order as processed by pre-order traversal method
  - Must phrase in terms of next() and hasNext()
  - We “simulate recursion” with stack
    - The stack holds “partially processed” nodes

Pre-Order Iterator

- Outline: node - left tree – right tree
  1. Constructor: Push root onto todo stack
  2. On call to next():
     - Pop node from stack
     - Push right and then left nodes of popped node onto stack
     - Return node's value
  3. On call to hasNext():
     - return !stack.isEmpty()

Pre-Order Iterator

Visit node, then each node in left subtree, then each node in right subtree.



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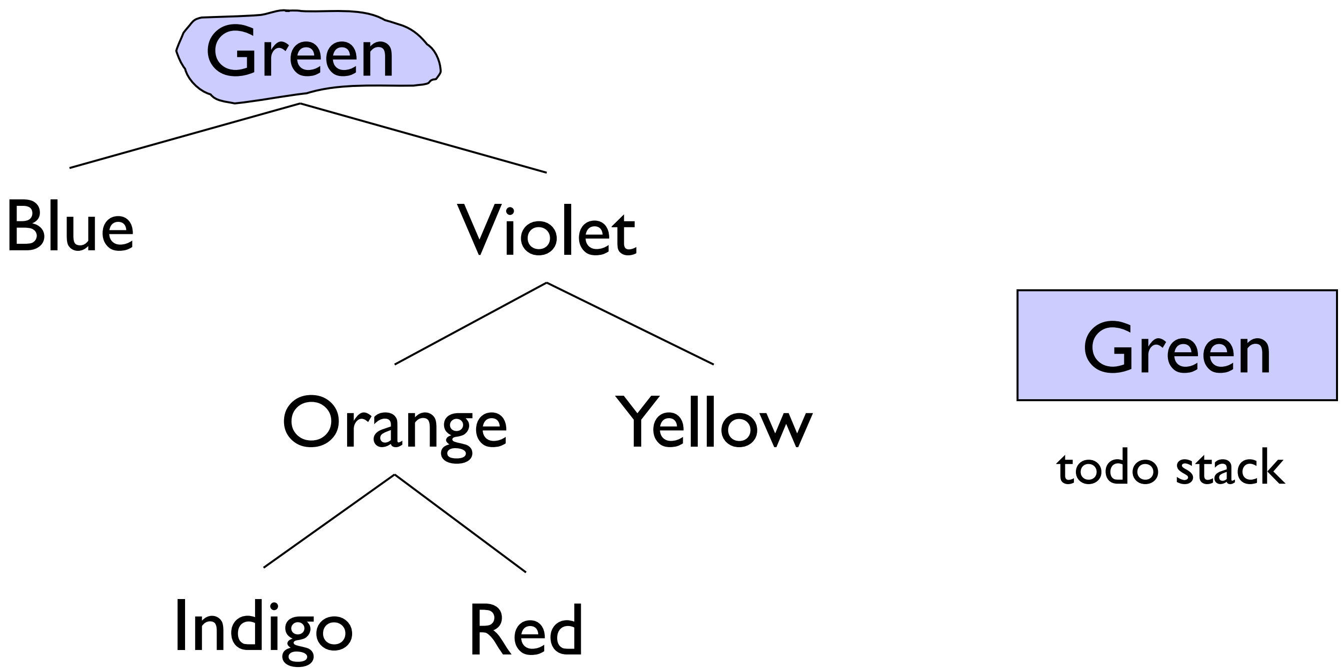
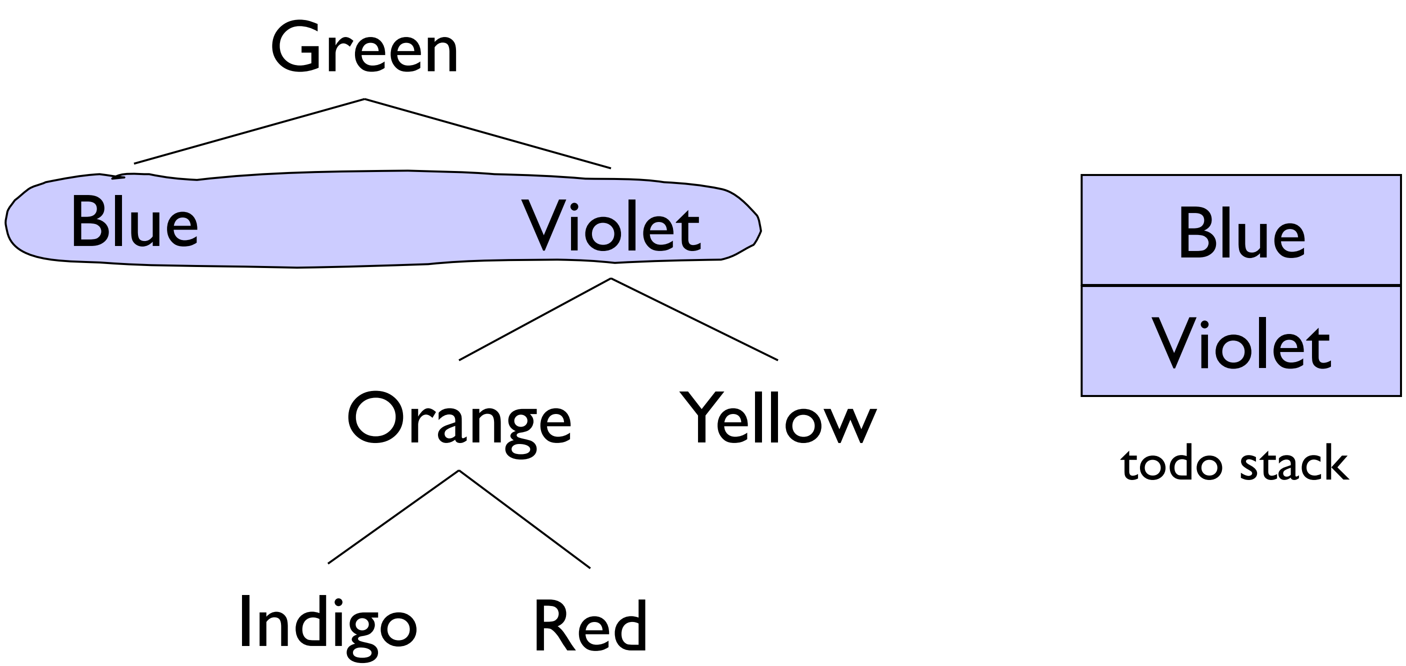


Pre-Order Iterator

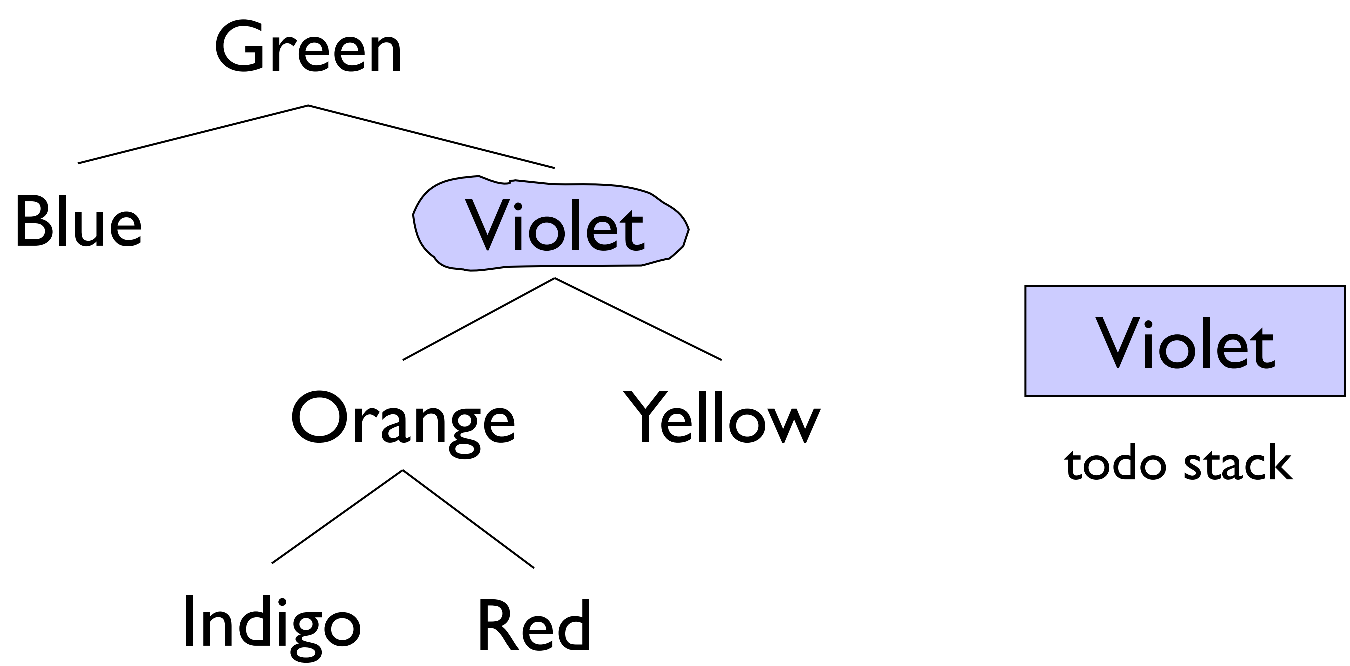
Visit node, then each node in left subtree, then each node in right subtree.



G

Pre-Order Iterator

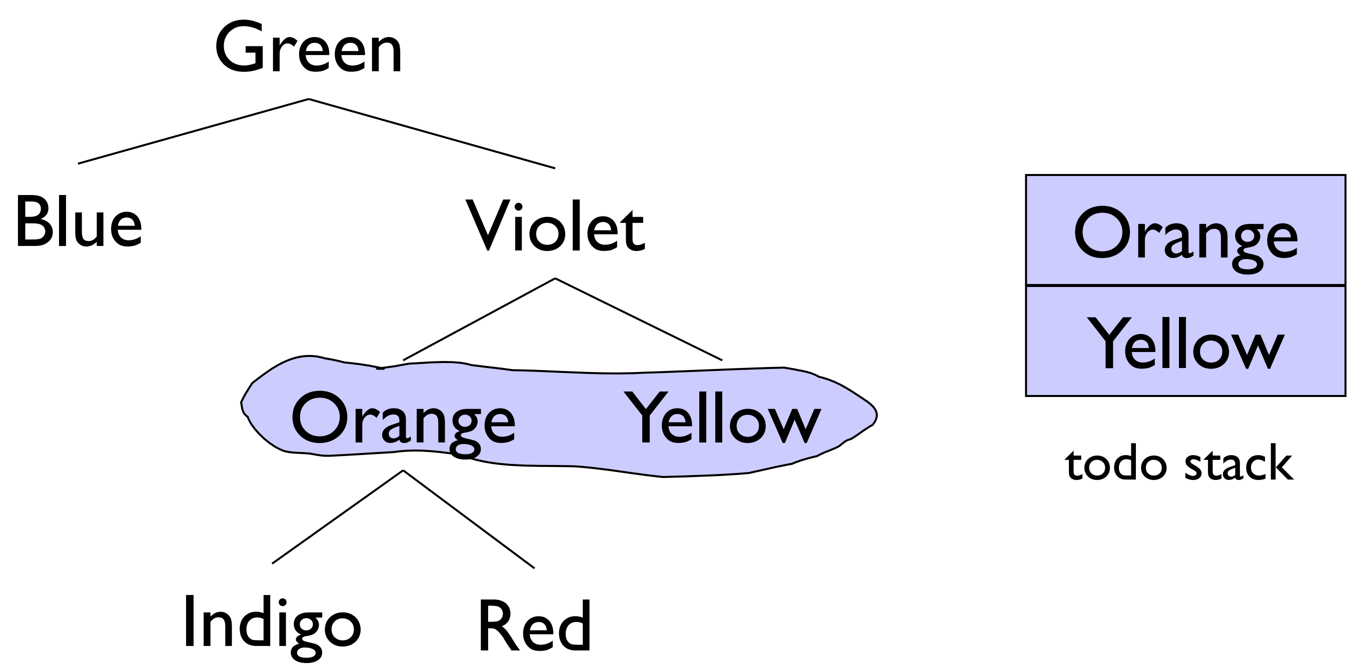
Visit node, then each node in left subtree, then each node in right subtree.



G B

Pre-Order Iterator

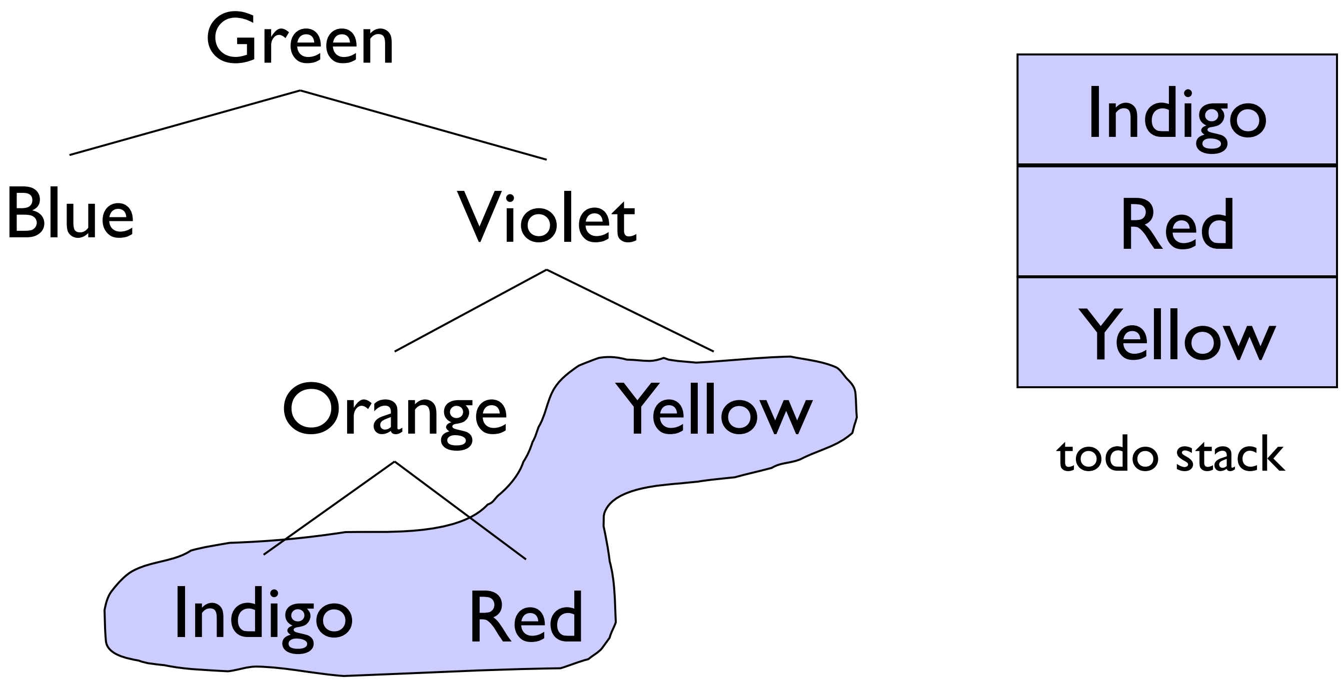
Visit node, then each node in left subtree, then each node in right subtree.



G B V

Pre-Order Iterator

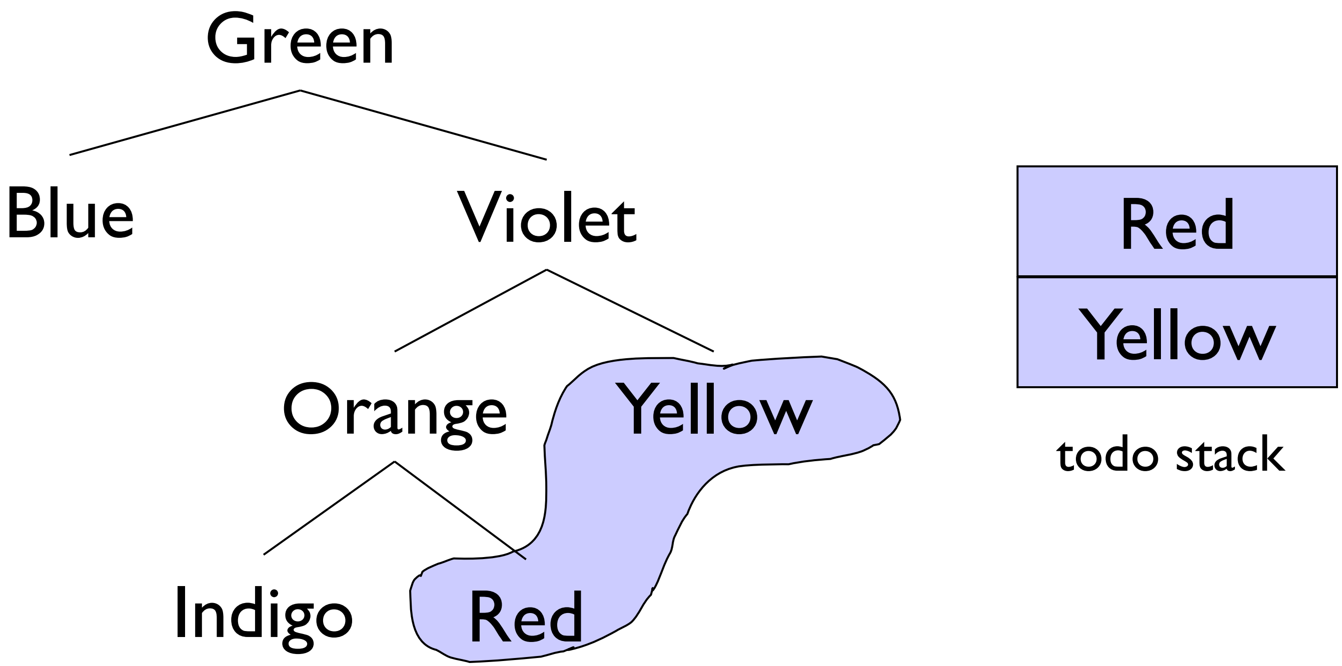
Visit node, then each node in left subtree, then each node in right subtree.



G B V O

Pre-Order Iterator

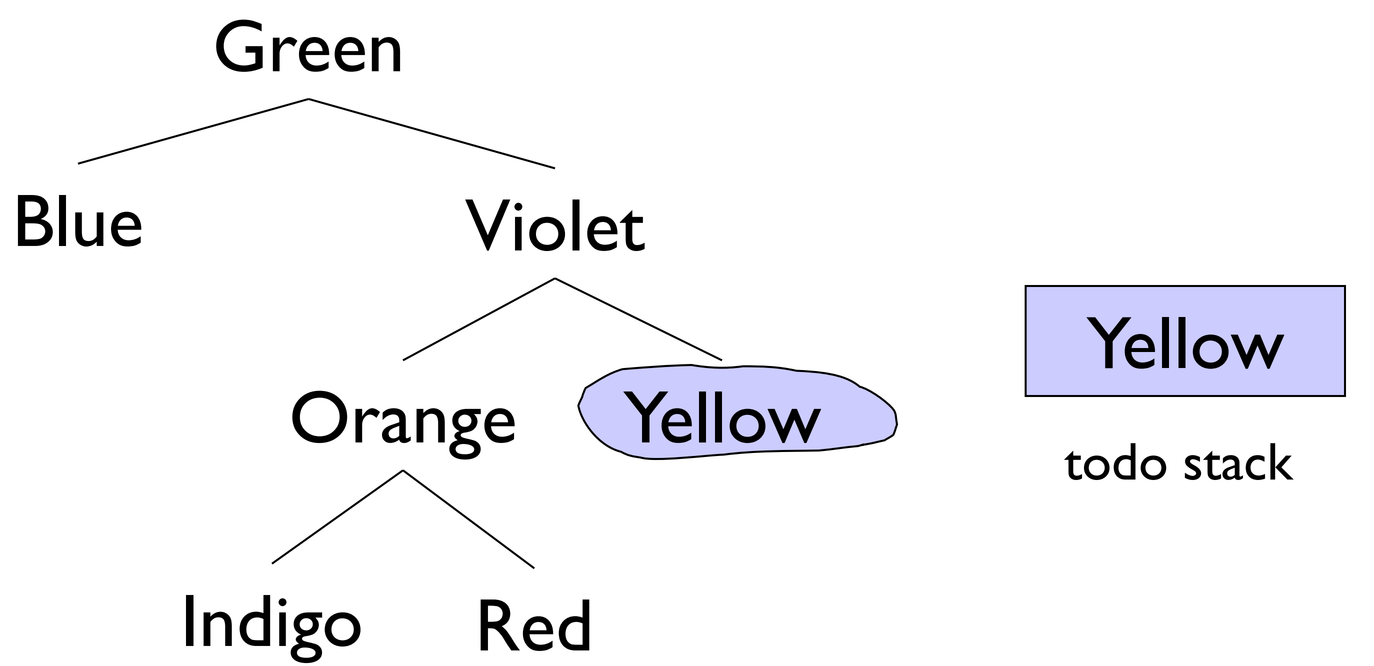
Visit node, then each node in left subtree, then each node in right subtree.



G B V O I

Pre-Order Iterator

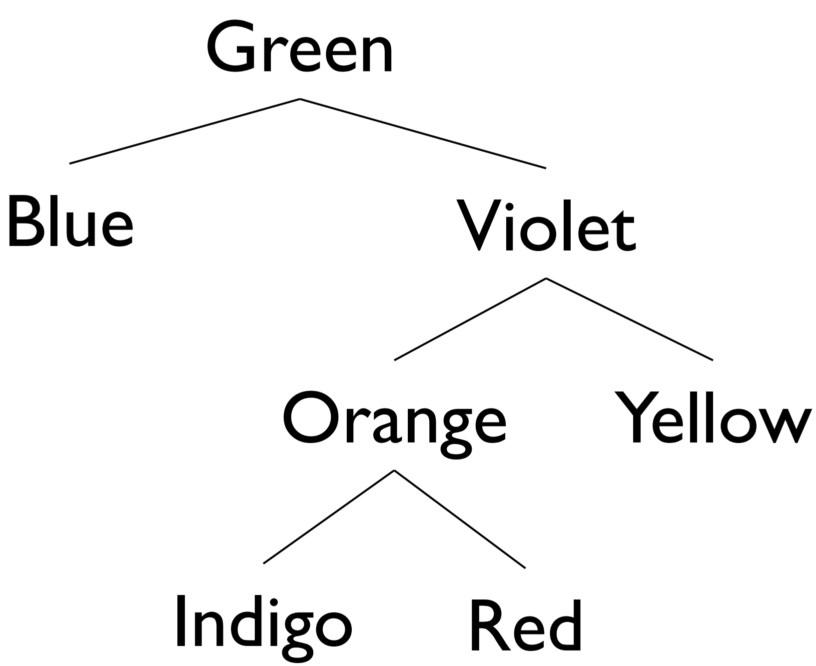
Visit node, then each node in left subtree, then each node in right subtree.



G B V O I R

Pre-Order Iterator

Visit node, then each node in left subtree, then each node in right subtree.



todo stack

G B V O I R Y

Pre-Order Iterator

```
public BTPreorderIterator(BinaryTree<E> root)
   {
      todo = new StackList<BinaryTree<E>>();
      this.root = root;
      reset();
   }

public void reset()
   {
      todo.clear(); // stack is empty; push on root
      if ((!root.isEmpty()) todo.push(root);
   }
```

Pre-Order Iterator

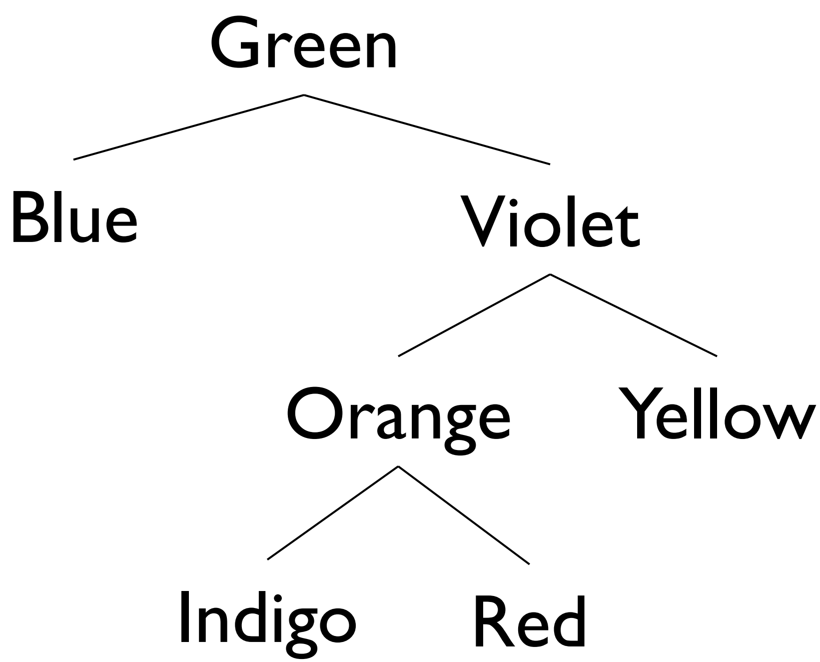
```
public boolean hasNext() {
       return !todo.isEmpty();
}

public E next() {
     BinaryTree<E> old = todo.pop();
     E result = old.value();

     if (!old.right().isEmpty())
           todo.push(old.right());
     if (!old.left().isEmpty())
           todo.push(old.left());
       return result;
   }
```

Tree Traversal Practice Problems

- Prove that levelOrder() is correct: that is, that it touches the nodes of the tree in the correct order (Hint: induction by level)
- Prove that levelOrder() takes O(n) time, where n is the size of the tree
- Prove that the PreOrder (LevelOrder) Iterator visits the nodes in the same order as the PreOrder (LevelOrder) traversal method

In-Order Iterator

Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree.



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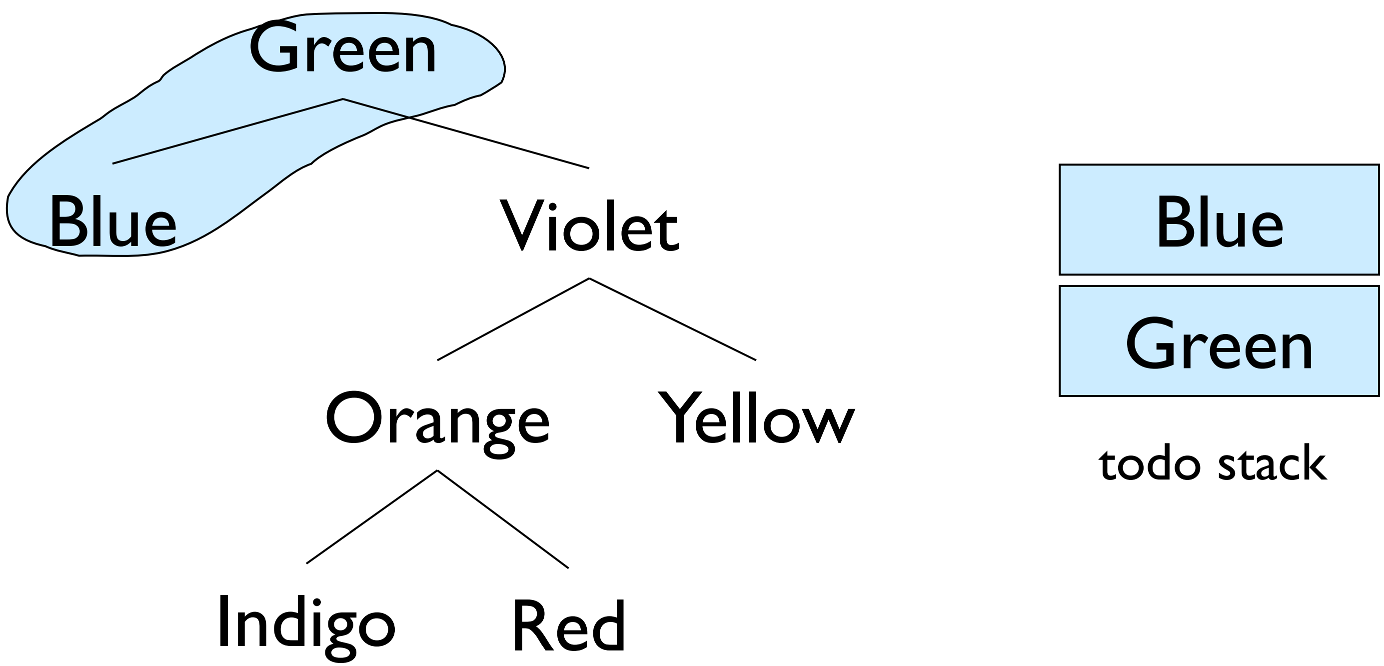
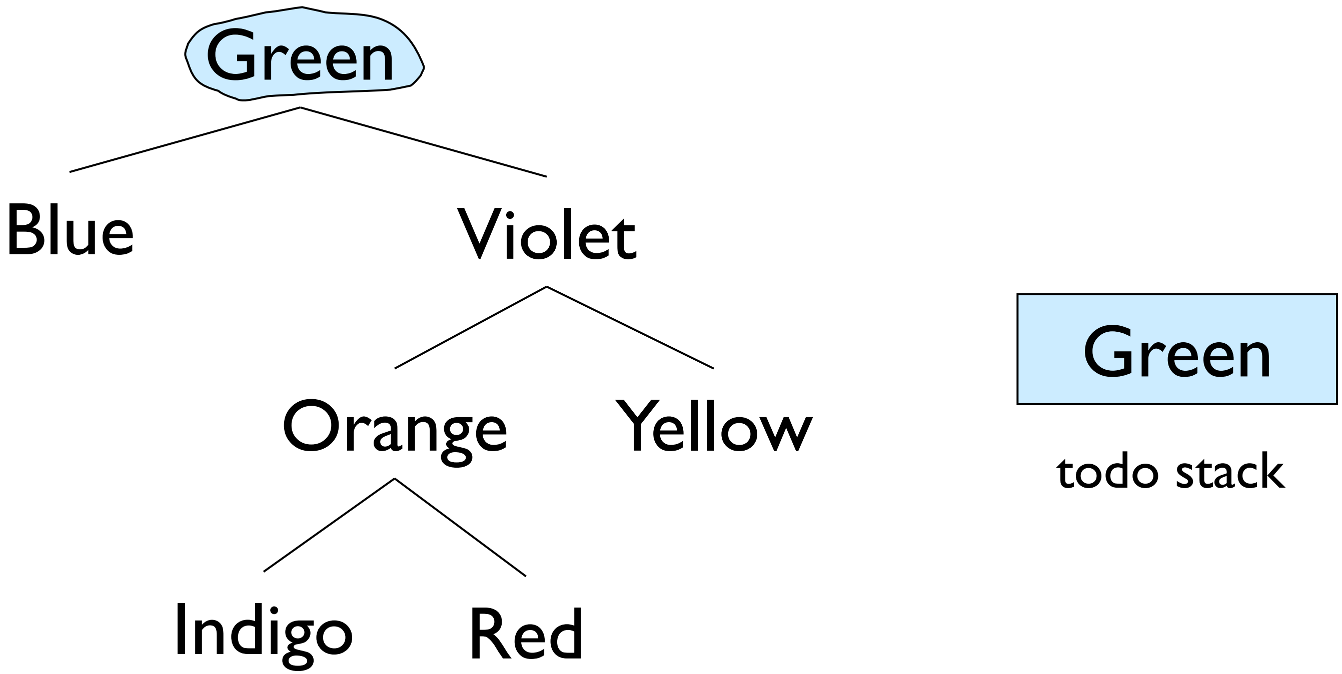


In-Order Iterator

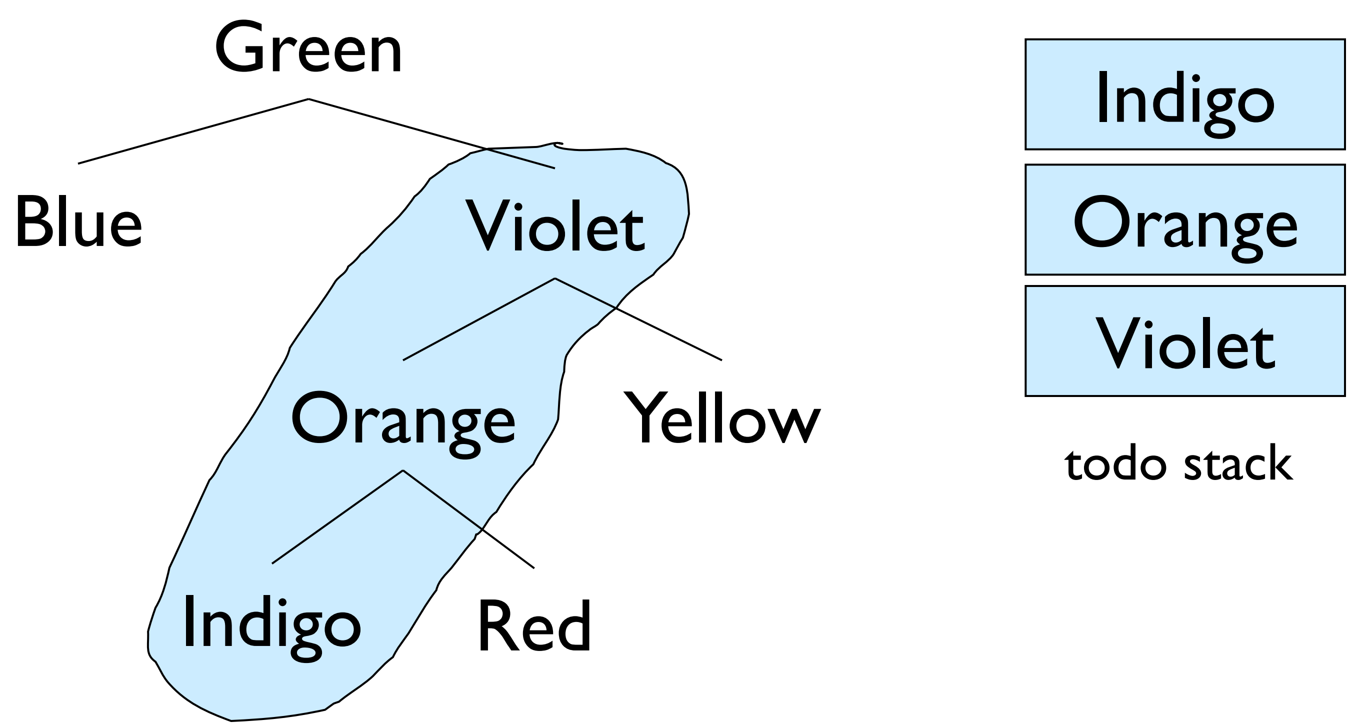
Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree.



B

In-Order Iterator

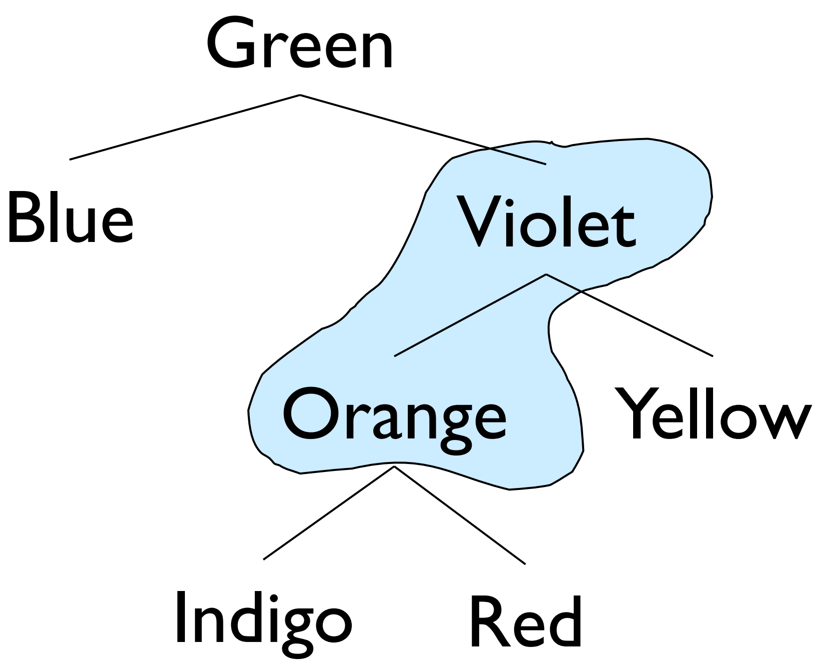
Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree.



B G

In-Order Iterator

Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree.

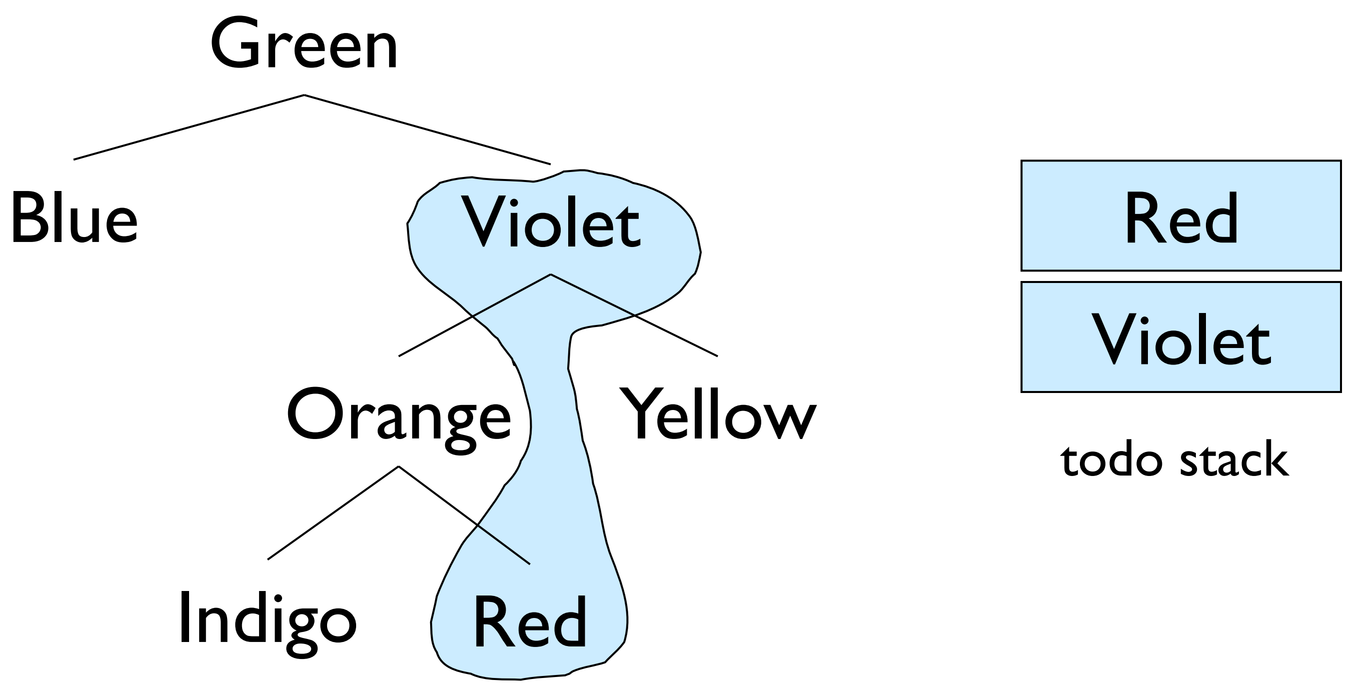


todo stack

B G I

In-Order Iterator

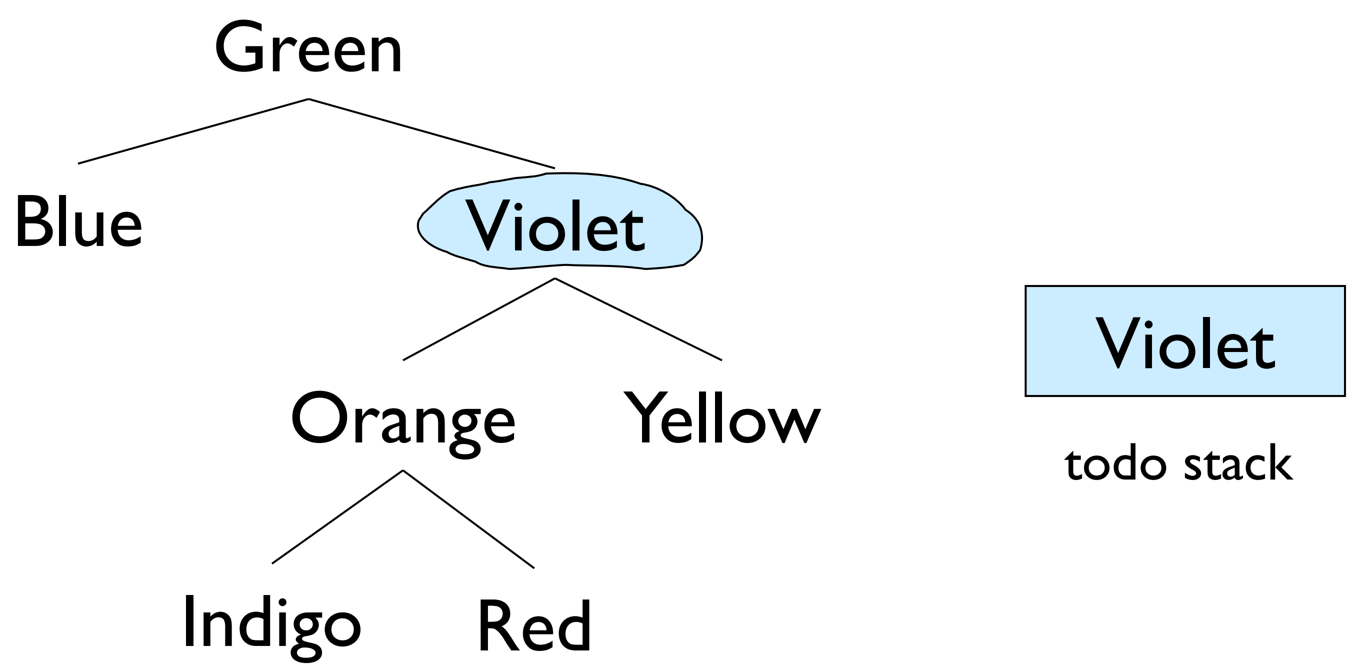
Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree.



B G I O

In-Order Iterator

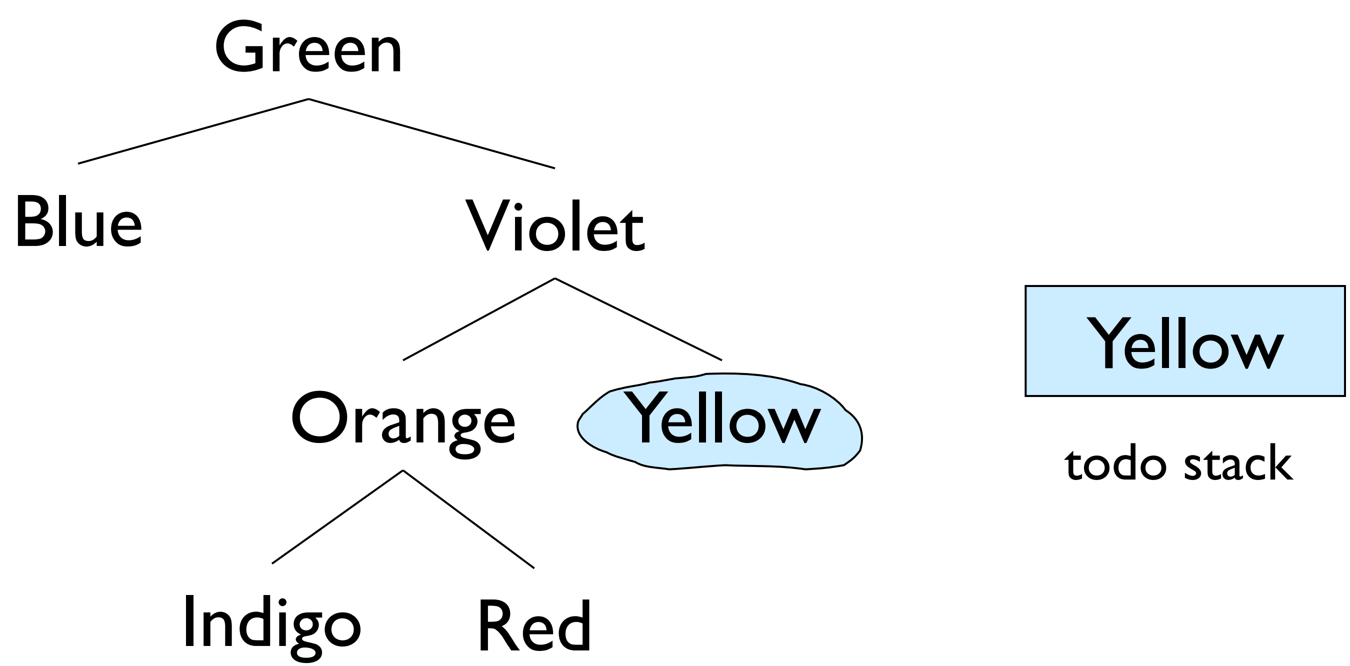
Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree.



B G I O R

In-Order Iterator

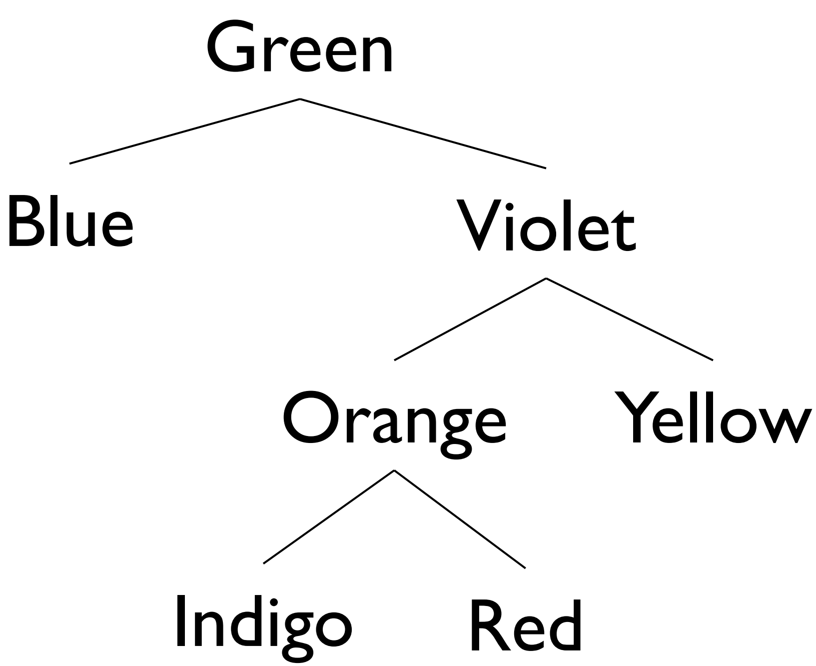
Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree.



B G I O R V

In-Order Iterator

Each node is visited after all nodes in left subtree are visited and before any nodes in right subtree.



todo stack

B G I O R V Y

In-Order Iterator

- Outline: left - node - right
  1. Push left children (as far as possible) onto stack
  2. On call to next():
     - Pop node from stack
     - Push right child and follow left children as far as possible
     - Return node's value
  3. On call to hasNext():
     - return !stack.isEmpty()

Tree Traversals

In summary:

- In-order: "left, node, right"
- Pre-order: "node, left, right"
- Post-order: "left, right, node"

Stack

- Level-order: visit all nodes at depth i before depth i+1

Queue

Traversals & Searching

- We can use traversals for searching trees
- How might we search a tree for a value?
  - Breadth-First: Explore nodes near the root before nodes far away (level order traversal)
    - Nearest gas station
  - Depth-First: Explore nodes deep in the tree first (post-order traversal)
    - Solution to a maze

Loose Ends – Really Big Trees!

- In some situations, the tree we need might be too big or expensive to build completely
  - Or parts of it might not be needed
- Example: Game Trees
  - Chess: you wouldn’t build the entire tree, you would grow portions of it as needed (with some combination of depth/breadth first searching)

Lab 7: Representing Numbers

- Humans usually think of numbers in base 10
- But even though we write `int x = 23;` the computer stores `x` as a sequence of `1`s and `0`s
- Recall Lab 3:

```
public static String printInBinary(int n) {
       if (n <= 1)
           return "" + n%2;

       return printInBinary(n/2)+n%2;
}
```

- 00000000 00000000 00000000 00010111

Bitwise Operations

- We can use *bitwise* operations to manipulate the `1`s and `0`s in the binary representation
  - Bitwise ‘and’: `&`
  - Bitwise ‘or’: `|`
- Also useful: bit shifts
  - Bit shift left: `<<`
  - Bit shift right: `>>`

& and |

- Given two integers `a` and `b`, the bitwise *or* expression `a | b` returns an integer s.t.
  - At each bit position, the result has a `1` if that bit position had a `1` in EITHER `a` OR `b` (or both)
  - `3 | 6 = ?`
- Given two integers `a` and `b`, the bitwise *and* expression `a & b` returns an integer s.t.
  - At each bit position, the result has a `1` if that bit position had a `1` in BOTH `a` AND `b`
  - `3 & 6 = ?`

>> and <<

- Given two (small) integers `a` and `i`,
  `(a << i)` returns `(a * 2`^{i}`)`
  - Why? It shifts all bits left by `i` positions
  - `1 << 4 = ?`
- Given two positive integers `a` and `i`,
  `(a >> i)` returns `(a / 2`^{i}`)`
  - Why? It shifts all bits right by `i` positions
  - `1 >> 4 = ?`
  - `97 >> 3 = ?` `(97 = 1100001)`
- Be careful about shifting left and “overflow”!!!

Revisiting printInBinary(int n)

- How would we rewrite a recursive `printInBinary` using bit shifts and bitwise operations?

```
public static String printInBinary(int n) {
    if (n <= 1) {
        return "" + n;
    return printInBinary(n >> 1) + (n & 1);
}
```

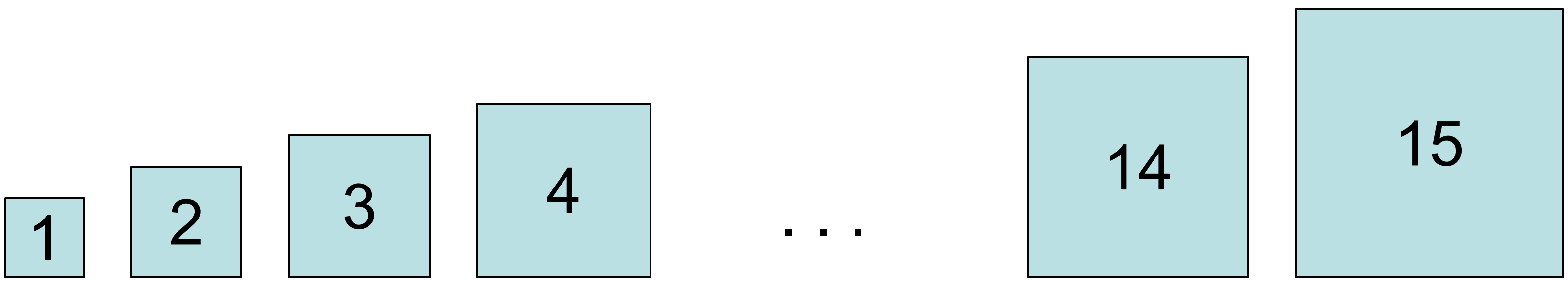
Revisiting printInBinary(int n)

- How would we write an iterative `printInBinary` using bit shifts and bitwise operations?

```
public static String printInBinary(int n,
                                   int width) {
    String result = "";
    for(int i = 0; i < width; i++)
        if ((n & (1<<i)) == 0)
            result = 0 + result;
        else
            result = 1 + result;
    return result;
}
```

Lab 7: Two Towers

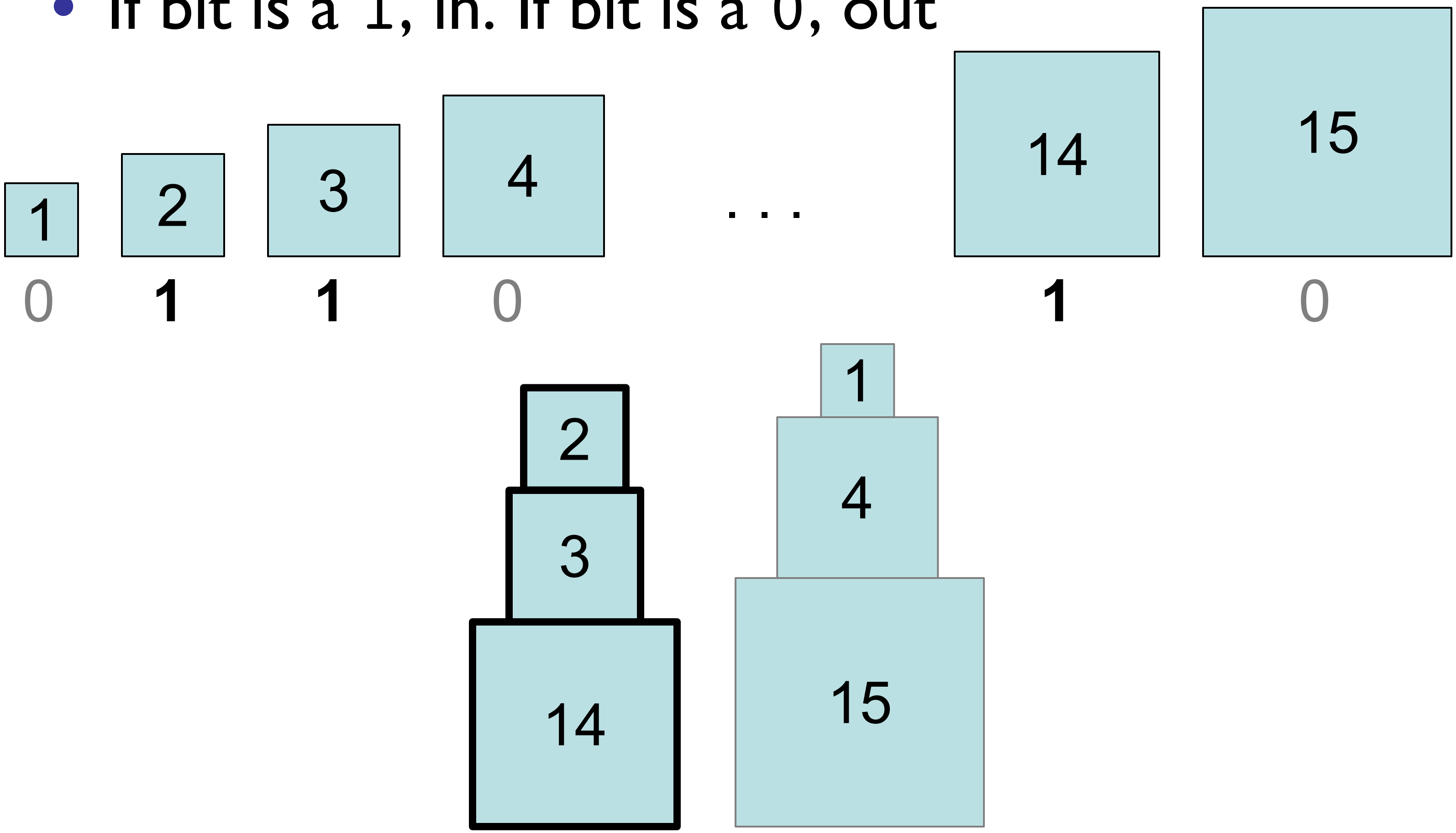
- Goal: given a set of blocks, iterate through all possible subsets to find the *best* set



- "Best" set produces the most balanced towers
- Strategy: create an iterator that uses the bits in a binary number to represent subsets

Lab 7: Two Towers

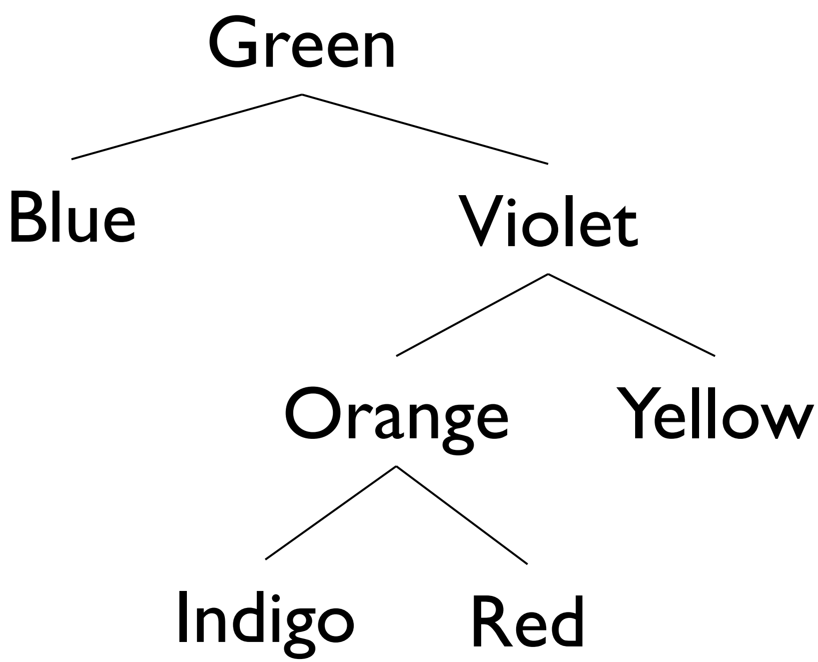
- A block can either be in the set or out
  - If bit is a 1, in. If bit is a 0, out



Questions?

- We will write a “SubsetIterator” to enumerate all possible subsets of a Vector<E>
- We will use SubsetIterator to solve this problem
- Can also be used to solve other problems
    - Identify all Subsequences of a String that are words
        - You just need a dictionary of legal words
        - Coming soon!

Alternative Tree Representations



- Total # “slots” = 4n
  - Since each BinaryTree maintains a reference to left, right, parent, value
- 2-4x more overhead than vector, SLL, array, …
- But trees capture successor and predecessor relationships that other data structures don’t…

Array-Based Binary Trees

- Encode structure of tree in array indexes
  - Put root at index 0
- Where are children of node i?
  - Children of node i are at 2i+1 and 2i+2
  - Look at example
- Where is parent of node j?
  - Parent of node j is at (j-1)/2

ArrayTree Tradeoffs

- Why are ArrayTrees good?
  - Save space for links
  - No need for additional memory allocated/garbage collected
  - Works well for full or complete trees
    - Complete: All levels except last are full and all gaps are at right
    - "A *complete* binary tree of height h is a full binary tree with 0 or more of the rightmost leaves of level h removed"
- Why bad?
  - Could waste a lot of space
  - Tree of height of n requires $2^{n+1}-1$ array slots even if only O(n) elements

Application: Huffman Codes (a CS 256 Preview)

- Computers encode a text as a sequence of bits

ASCII TABLE

Decimal	Hex	Char	Decimal	Hex	Char	Decimal	Hex	Char	Decimal	Hex	Char
0	0	[NULL]	32	20	[SPACE]	64	40	@	96	60	`
1	1	[START OF HEADING]	33	21	!	65	41	A	97	61	a
2	2	[START OF TEXT]	34	22	"	66	42	B	98	62	b
3	3	[END OF TEXT]	35	23	#	67	43	C	99	63	c
4	4	[END OF TRANSMISSION]	36	24	$	68	44	D	100	64	d
5	5	[ENQUIRY]	37	25	%	69	45	E	101	65	e
6	6	[ACKNOWLEDGE]	38	26	&	70	46	F	102	66	f
7	7	[BELL]	39	27	'	71	47	G	103	67	g
8	8	[BACKSPACE]	40	28	(	72	48	H	104	68	h
9	9	[HORIZONTAL TAB]	41	29	)	73	49	I	105	69	i
10	A	[LINE FEED]	42	2A	*	74	4A	J	106	6A	j
11	B	[VERTICAL TAB]	43	2B	+	75	4B	K	107	6B	k
12	C	[FORM FEED]	44	2C	,	76	4C	L	108	6C	l
13	D	[CARRIAGE RETURN]	45	2D	-	77	4D	M	109	6D	m
14	E	[SHIFT OUT]	46	2E	.	78	4E	N	110	6E	n
15	F	[SHIFT IN]	47	2F	/	79	4F	O	111	6F	o
16	10	[DATA LINK ESCAPE]	48	30	0	80	50	P	112	70	p
17	11	[DEVICE CONTROL 1]	49	31	1	81	51	Q	113	71	q
18	12	[DEVICE CONTROL 2]	50	32	2	82	52	R	114	72	r
19	13	[DEVICE CONTROL 3]	51	33	3	83	53	S	115	73	s
20	14	[DEVICE CONTROL 4]	52	34	4	84	54	T	116	74	t
21	15	[NEGATIVE ACKNOWLEDGE]	53	35	5	85	55	U	117	75	u
22	16	[SYNCHRONOUS IDLE]	54	36	6	86	56	V	118	76	v
23	17	[ENG OF TRANS. BLOCK]	55	37	7	87	57	W	119	77	w
24	18	[CANCEL]	56	38	8	88	58	X	120	78	x
25	19	[END OF MEDIUM]	57	39	9	89	59	Y	121	79	y
26	1A	[SUBSTITUTE]	58	3A	:	90	5A	Z	122	7A	z
27	1B	[ESCAPE]	59	3B	;	91	5B	[	123	7B	{
28	1C	[FILE SEPARATOR]	60	3C	<	92	5C	\	124	7C	|
29	1D	[GROUP SEPARATOR]	61	3D	=	93	5D	]	125	7D	}
30	1E	[RECORD SEPARATOR]	62	3E	>	94	5E	^	126	7E	~
31	1F	[UNIT SEPARATOR]	63	3F	?	95	5F	_	127	7F	[DEL]

Huffman Codes

- Goal: Encode a text as a sequence of bits
- Normally, use ASCII: 1 character = 8 bits (1 byte)
  - Allows for $2^8 = 256$ different characters
- 'A' = 01000001, 'B' = 01000010
- Space to store "AN_ANTARCTIC_PENGUIN"
  - 20 characters -> 20*8 bits = 160 bits
- Is there a better way?
  - Only 11 symbols are used (ANTRCIPEGU_)
  - Only need 4 bits per symbol (since $2^4>11$)!
    - 20*4 = 80 bits instead of 160!
  - Can we still do better??

Huffman Codes

- Example
  - AN_ANTARCTIC_PENGUIN
  - Compute letter frequencies

A	C	E	G	I	N	P	R	T	U	_
3	2	1	1	2	4	1	1	2	1	2

- Key Idea: Use fewer bits for most common letters

A	C	E	G	I	N	P	R	T	U	_
3	2	1	1	2	4	1	1	2	1	2
110	111	1011	1000	000	001	1001	1010	0101	0100	011

- Uses 67 bits to encode entire string

Huffman Codes

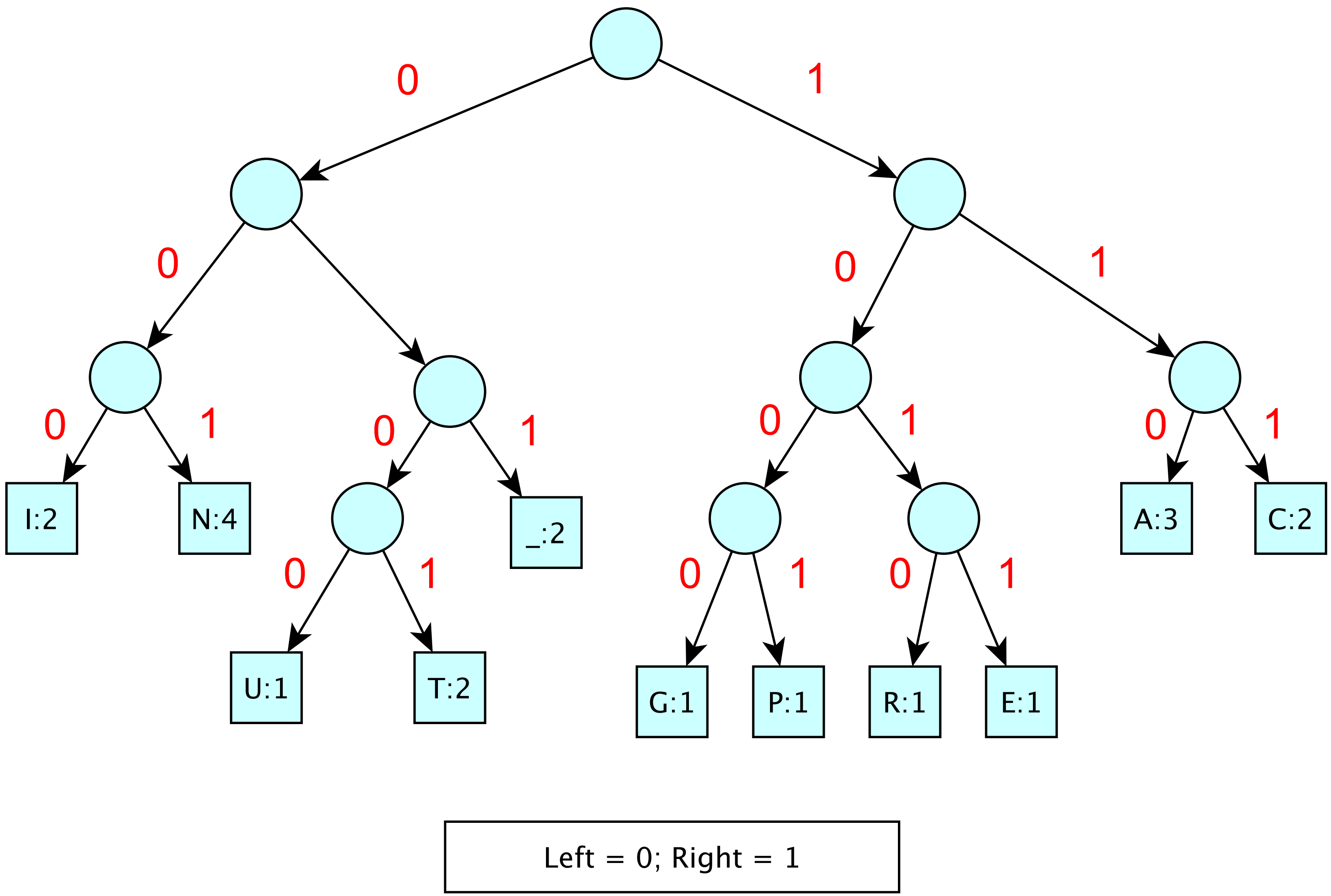
A	C	E	G	I	N	P	R	T	U	_
3	2	1	1	2	4	1	1	2	1	2
110	111	1011	1000	000	001	1001	1010	0101	0100	011

- Uses 67 bits to encode entire string
- Can we do better?

A	C	E	G	I	N	P	R	T	U	_
3	2	1	1	2	4	1	1	2	1	2
100	010	1100	1101	011	101	0001	0000	001	1110	1111

- Uses 67 bits to encode entire string

The Encoding Tree



Features of Good Encoding

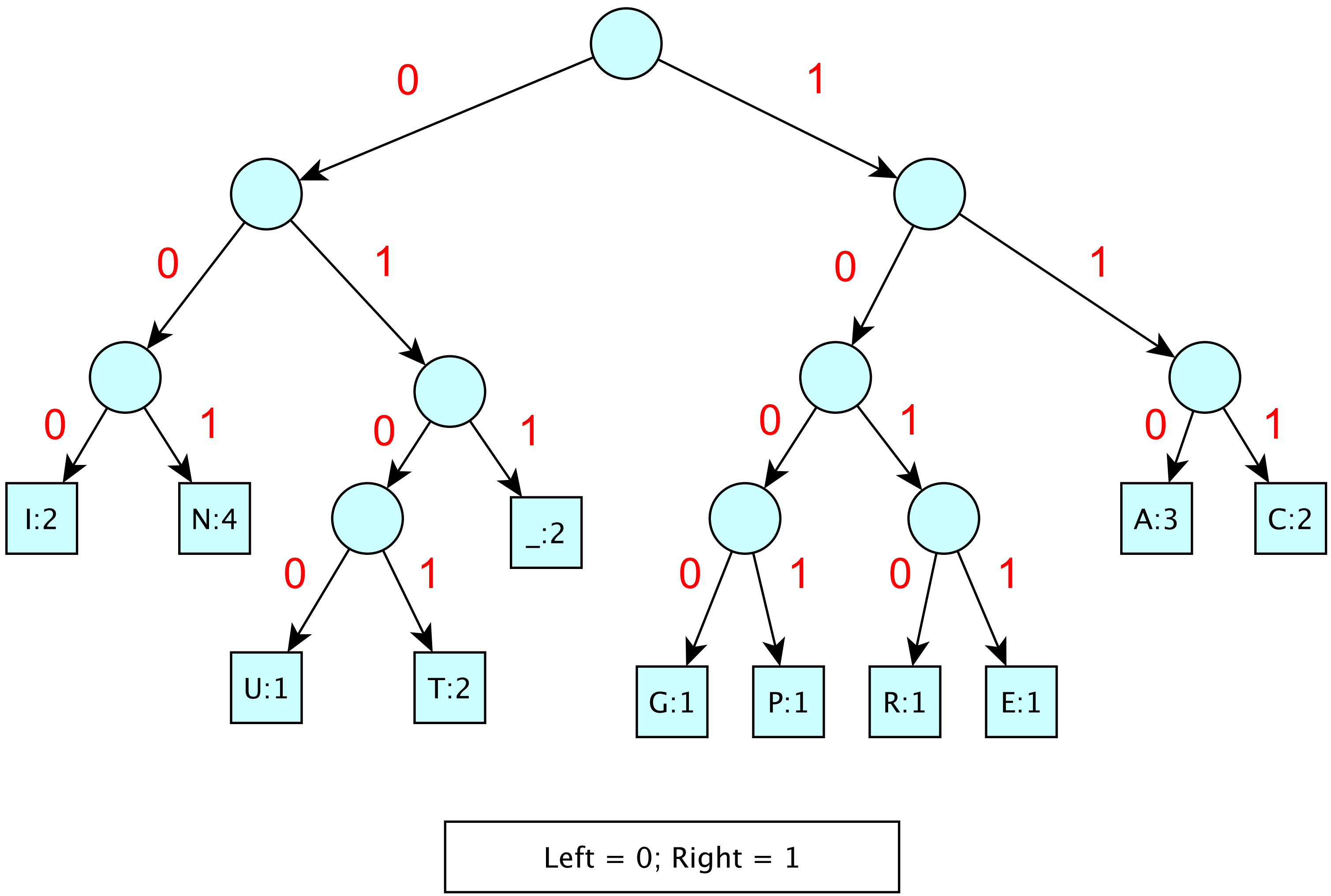
- Prefix property: No encoding is a prefix of another encoding (letters appear at leaves)
- No internal node has a single child
- Nodes with lower frequency have greater depth

- All optimal length unambiguous encodings have these features

Huffman Encoding

- Input: symbols of alphabet with frequencies
- Huffman encode as follows
  - Create a single-node tree for each symbol: key is frequency; value is letter
  - while there is more than one tree
    - Find two trees T1 and T2 with lowest keys
    - Merge them into new tree T with dummy value and key= T1.key+ T2.key
- Theorem: The tree computed by Huffman is an optimal encoding for given frequencies

The Encoding Tree



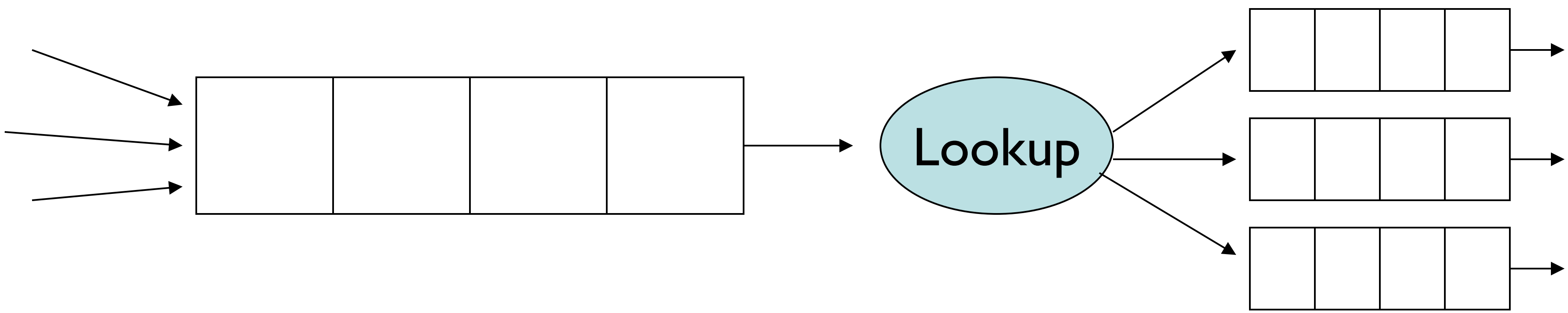
How To Implement Huffman

- Keep a Vector of Binary Trees
- Sort them by decreasing frequency
  - Removing two smallest frequency trees is fast
- Insert merged tree into correct sorted location in Vector
- Running Time:
  - O(n log n) for initial sorting
  - $O(n^2)$ for rest: O(n) re-insertions of merged trees
- Can we do better...?

What Huffman Encoder Needs

- A structure S to hold items with *priorities*
- S should support operations
  - add(E item); // add an item
  - E removeMin(); // remove min priority item
- S should be designed to make these two operations fast
- If, say, they both ran in O(log n) time, the Huffman algorithm would take O(n log n) time instead of $O(n^2)$!
- We've seen this situation before….

Priority Queues



Packet Sources May Be Ordered by Sender

sysnet.cs.williams.edu	priority = 1 (best)
bull.cs.williams.edu	2
yahoo.com	10
spammer.com	100 (worst)

Priority Queues

- Priority queues are also used for:
  - Scheduling processes in an operating system
    - Priority is function of time lost + process priority
  - Order services on server
    - Backup is low priority, so don't do when high priority tasks need to happen
  - Scheduling future events in a simulation
  - Medical waiting room
  - Huffman codes - order by tree size/weight
  - A variety of graph/network algorithms
  - To roughly order choices that are generated out of order

Priority Queues

- Name is misleading: They are **not FIFO**
- Always dequeue object with **highest priority** (smallest rank) regardless of when it was enqueued
- Data can be received/inserted in any order, but it is always returned/removed according to priority
- Like ordered structures (i.e., OrderedVectors and OrderedLists), PQs require comparisons of values

An Apology

- On behalf of computer scientists everywhere, I'd like to apologize for the confusion that inevitably results from the fact that

Higher Priority   Lower Rank

- The PQ removes the *lowest ranked* value in an ordering: that is, the *highest priority* value!

We're sorry!

PQ Interface

```
public interface PriorityQueue<E extends Comparable<E>> {
  public E getFirst(); // peeks at minimum element
  public E remove();   // removes minimum element
  public void add(E value); // adds an element
  public boolean isEmpty();
  public int size();
  public void clear();
}
```

Notes on PQ Interface

- Unlike previous structures, we do not extend any other interfaces
  - Many reasons: For example, it's not clear that there's an obvious iteration order
- PriorityQueue uses Comparables: methods *consume* Comparable parameters and *return* Comparable values
  - Could be made to use Comparators instead…

Implementing PQs

- Queue?
  - Wouldn’t work so well because we can’t insert and remove in the “right” way (i.e., keeping things ordered)
- OrderedVector?
  - Keep ordered vector of objects
  - O(n) to add/remove from vector
  - Details in book…
  - Can we do better than O(n)?
- Heap!
  - Partially ordered binary tree

Heap

- A heap is a special type of tree
- A heap is a tree where:
  - Root holds smallest (highest priority) value
  - Subtrees are also heaps (this is important!)
- So values increase in priority (decrease in rank) from leaves to root (from descendant to ancestor)
- *Invariant for nodes*
  - node.value() >= node.parent.value()
    - Tree need not be binary….
- Several valid heaps for same data set (no unique representation)

Inserting into a PQ

- Add new value as a leaf
- "Percolate" it up the tree
  - while (value < parent's value) swap with parent
- This operation preserves the heap property since new value was the only one violating heap property
- Efficiency depends upon speed of
  - Finding a place to add new node
  - Finding parent
  - Tree height

Removing From a PQ

- Find a leaf, delete it, put its *data* in the root
- “Push” *data* down through the tree
  - while (*data.value* > value of (at least) one child)
    - Swap *data* with data of **smaller** child
- This operation preserves the heap property
- Efficiency depends upon speed of
  - Finding a leaf
  - Finding locations of children
  - Height of tree