CSCI 136
Data Structures &
Advanced Programming

Lecture 18

Fall 2019

Instructor: B&S

Administrative Details

- Lab 7 Today: PostScript
  - No partners this week
  - Review before lab; come to lab with design doc
  - Check out the javadoc pages for the 3 provided classes
    - Token – A wrapper for semantic PS elements,
    - Reader – An iterator to produce a stream of Tokens from standard input or a List of Tokens,
    - SymbolTable – A dictionary with String keys and Token values: For user-defined names

Last Time:

- Ordered Structures
- Trees
  - Structure, Terminology, Examples

Today

- Trees
  - Implementation
  - Recursion/Induction on Trees
  - Applications
  - Traversals

Type Safety & Generic Types

- Question: Since String extends Object, does List<String> extend List<Object>?
  - I.e., can I say List<Object> = new List<String>()?
- No. It would compromise the type system:

```
List<String> slist = new List<String>();
List<Object> olist = slist;    // If this were possible
olist.add(new Object());       // This would be bad!
```

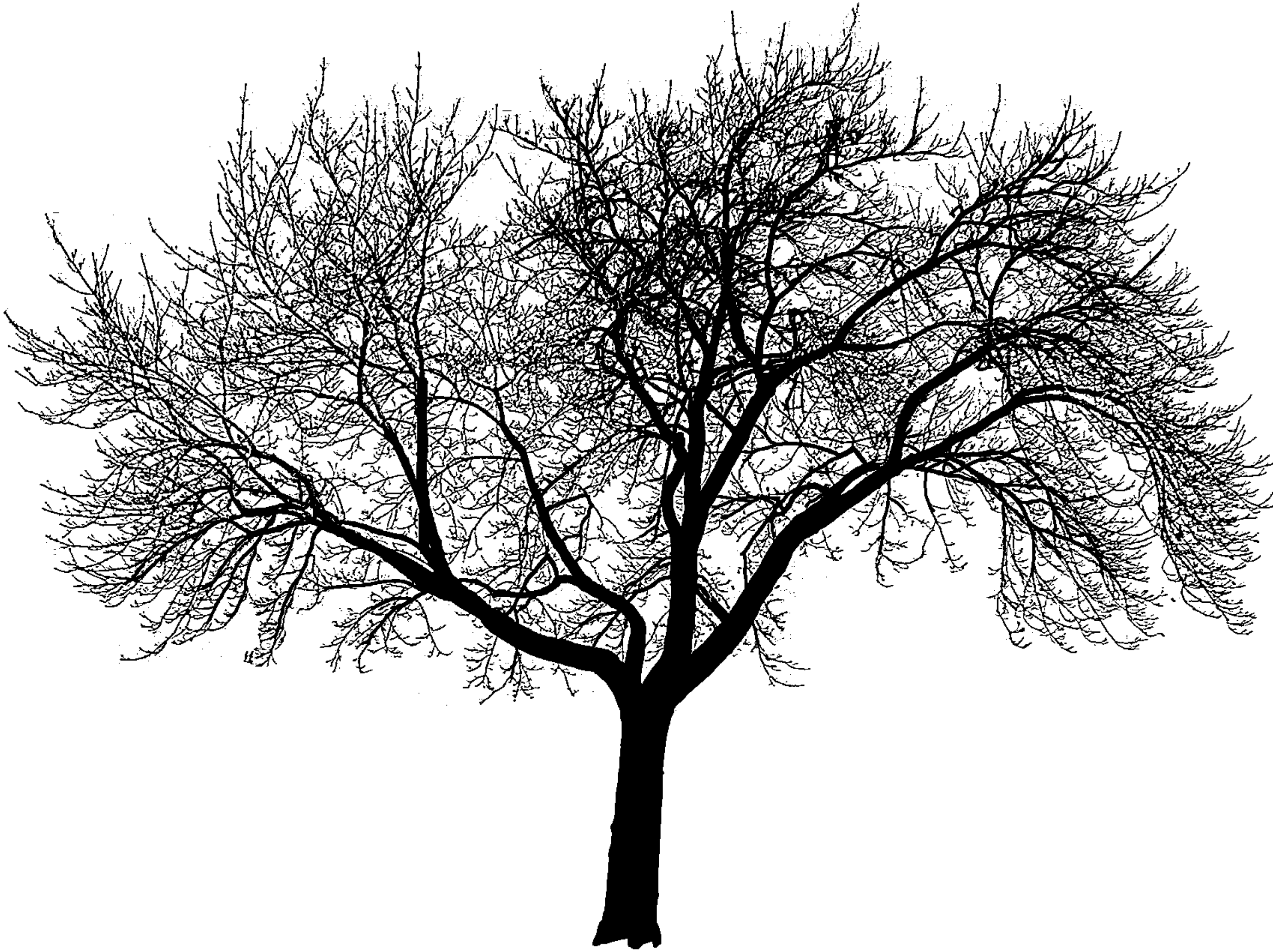
- It generates a compiler error.
- On the other hand…

```
String[] sa = {"I", "love", "java", "!"};
Object[] oa = sa;
oa[1] = new Object()); // This would be bad!
```

- …actually compiles
  - But causes a run-time error!

Introducing Trees

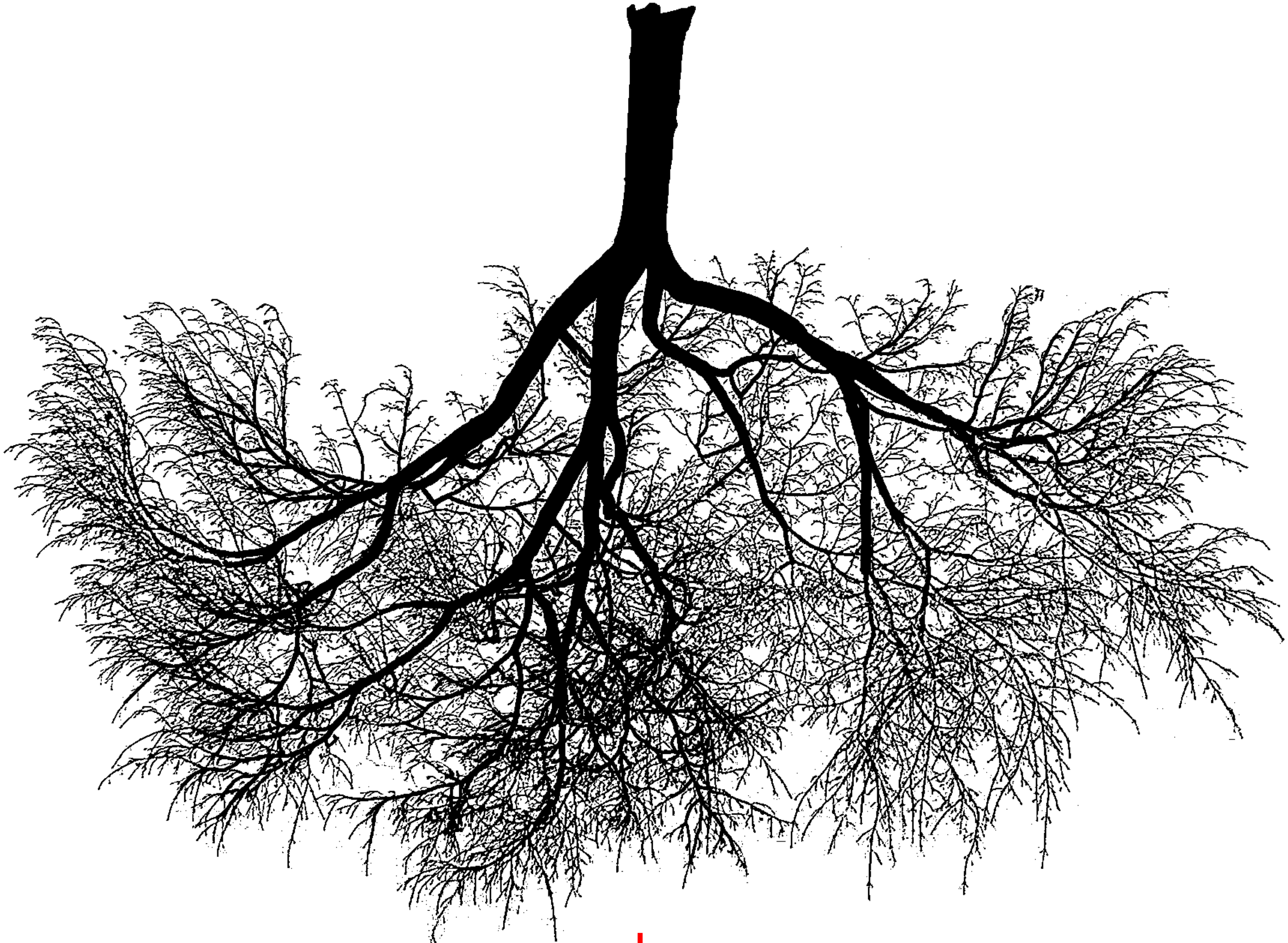
- Our structures have had a linear organization
  - Stacks, queues
  - Even ordered vectors, ordered lists, arrays, vectors, lists are visualized linearly
- By linear we essentially mean that each element has at most one successor and at most one predecessor…



Branching Out: Trees

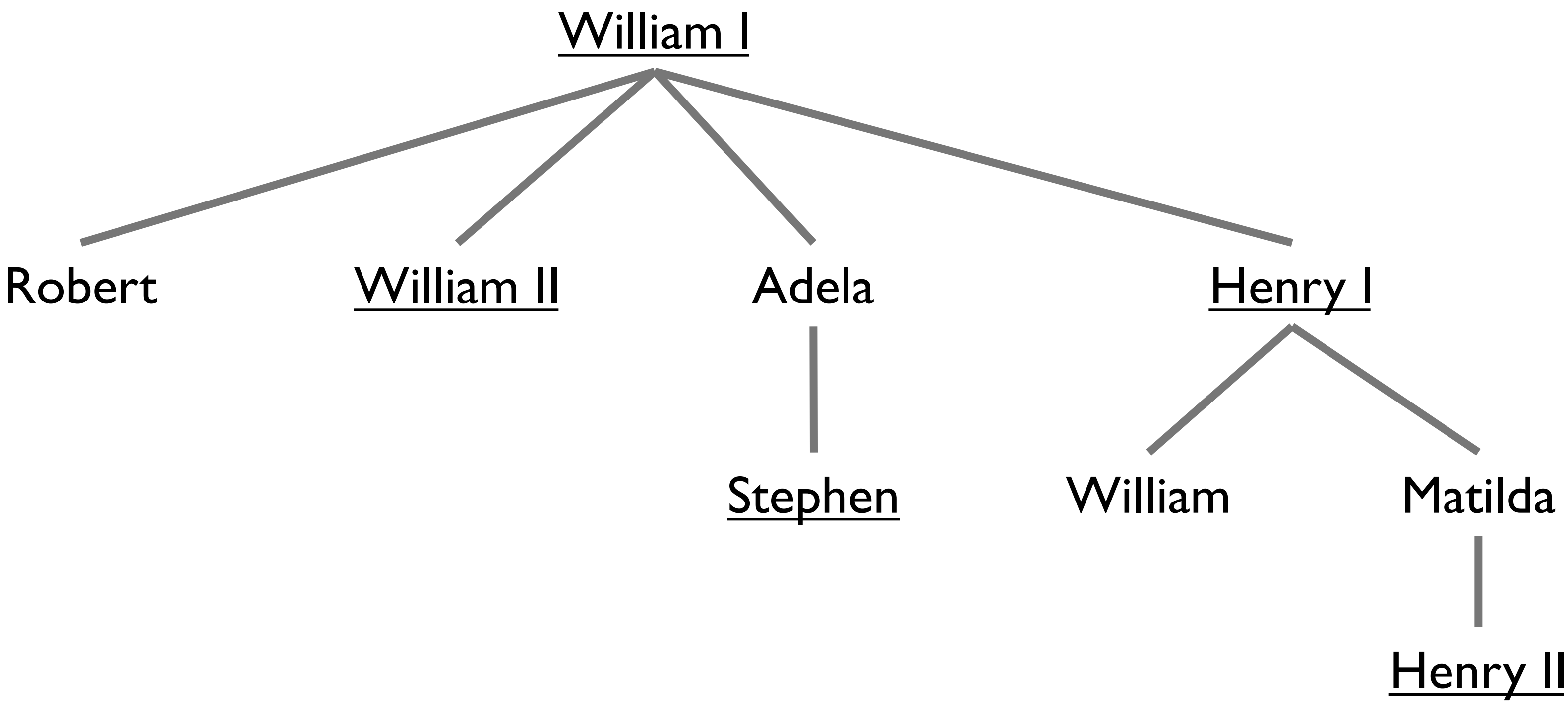
- A tree is a data structure where elements can have multiple successors (called children)
- But still only one predecessor (called parent)

Root



Leaves

House of Normandy, Battle of Hastings, 1066

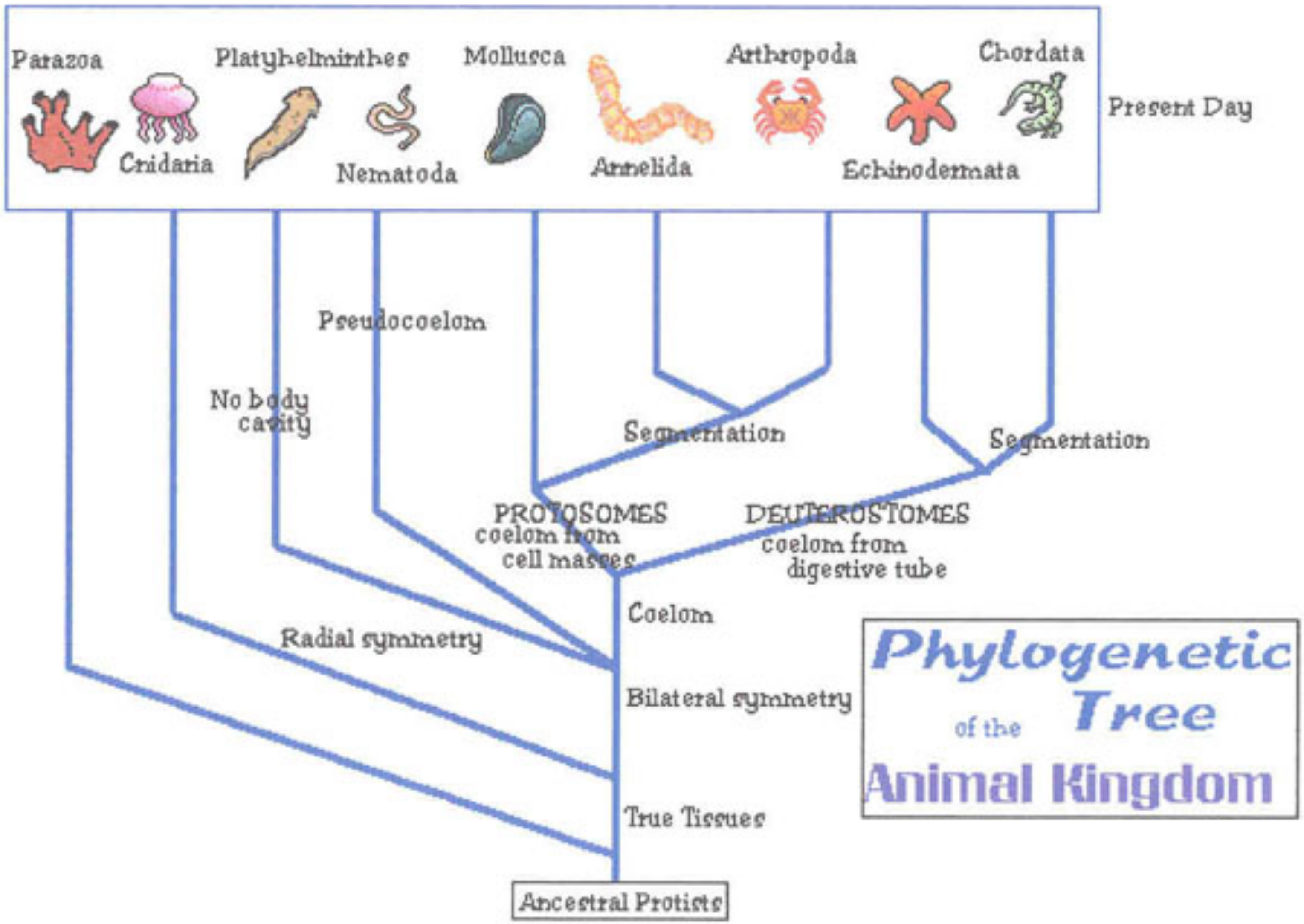


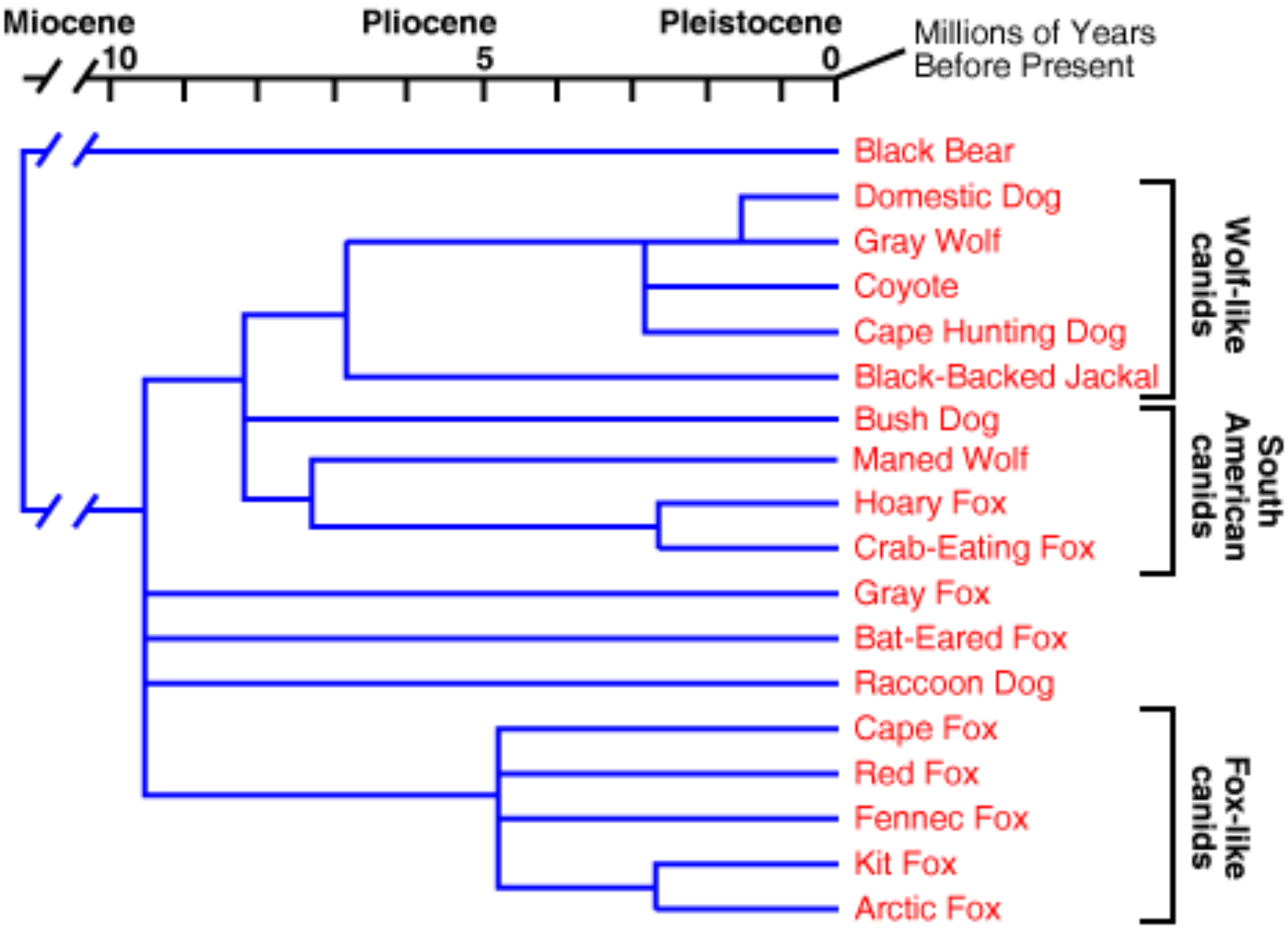
Tree Features

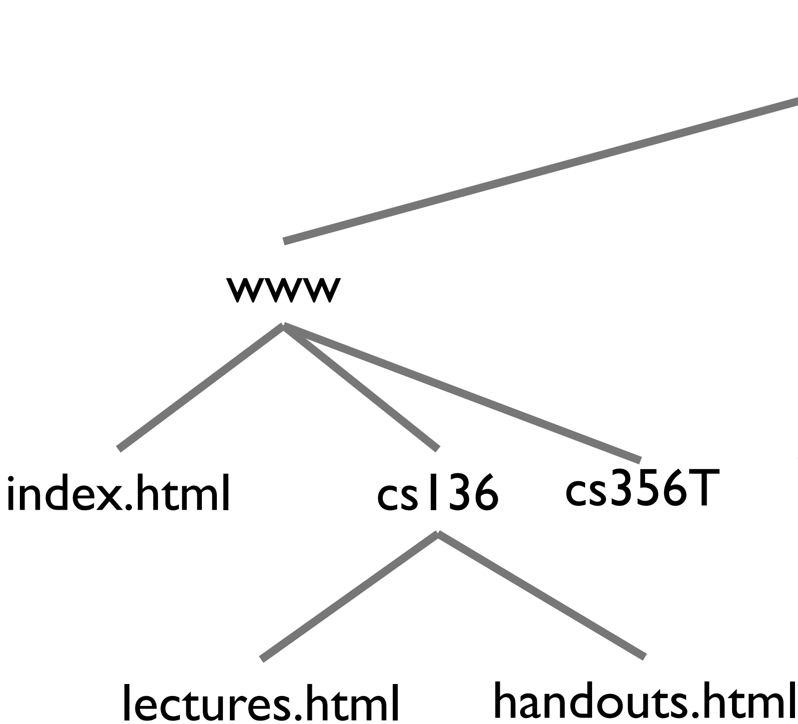
- Hierarchical relationship
- Root at the top
- Leaf at the bottom
- Interior nodes in middle
- Parents, children, ancestors, descendants, siblings
- Degree (of node): number of children of node
- Degree (of tree): maximum degree (across all nodes)
- Depth of node: number of *edges* from root to node
- Height of tree: maximum depth (across all nodes)

Other Trees

- Phylogenetic tree
- Directories of files
- Game trees
  - Build a tree
  - Search it for moves with high likelihood of winning
- Expression trees


Parazoa
Cnidaria
Platyhelminthes
Nematoda
Mollusca
Annelida
Arthropoda
Echinodermata
Chordata
Present Day
Pseudocoelom
No body cavity
Segmentation
Segmentation
PROTOSOMES
coelom from cell masses
DEUTEROSTOMES
coelom from digestive tube
Coelom
Radial symmetry
Bilateral symmetry
True Tissues
Ancestral Protists
Phylogenetic Tree of the Animal Kingdom


Miocene
Pliocene
Pleistocene
10
5
0
Millions of Years Before Present
Black Bear
Domestic Dog
Gray Wolf
Coyote
Cape Hunting Dog
Black-Backed Jackal
Bush Dog
Maned Wolf
Hoary Fox
Crab-Eating Fox
Gray Fox
Bat-Eared Fox
Raccoon Dog
Cape Fox
Red Fox
Fennec Fox
Kit Fox
Arctic Fox
Wolf-like canids
South American canids
Fox-like canids


~lenhart
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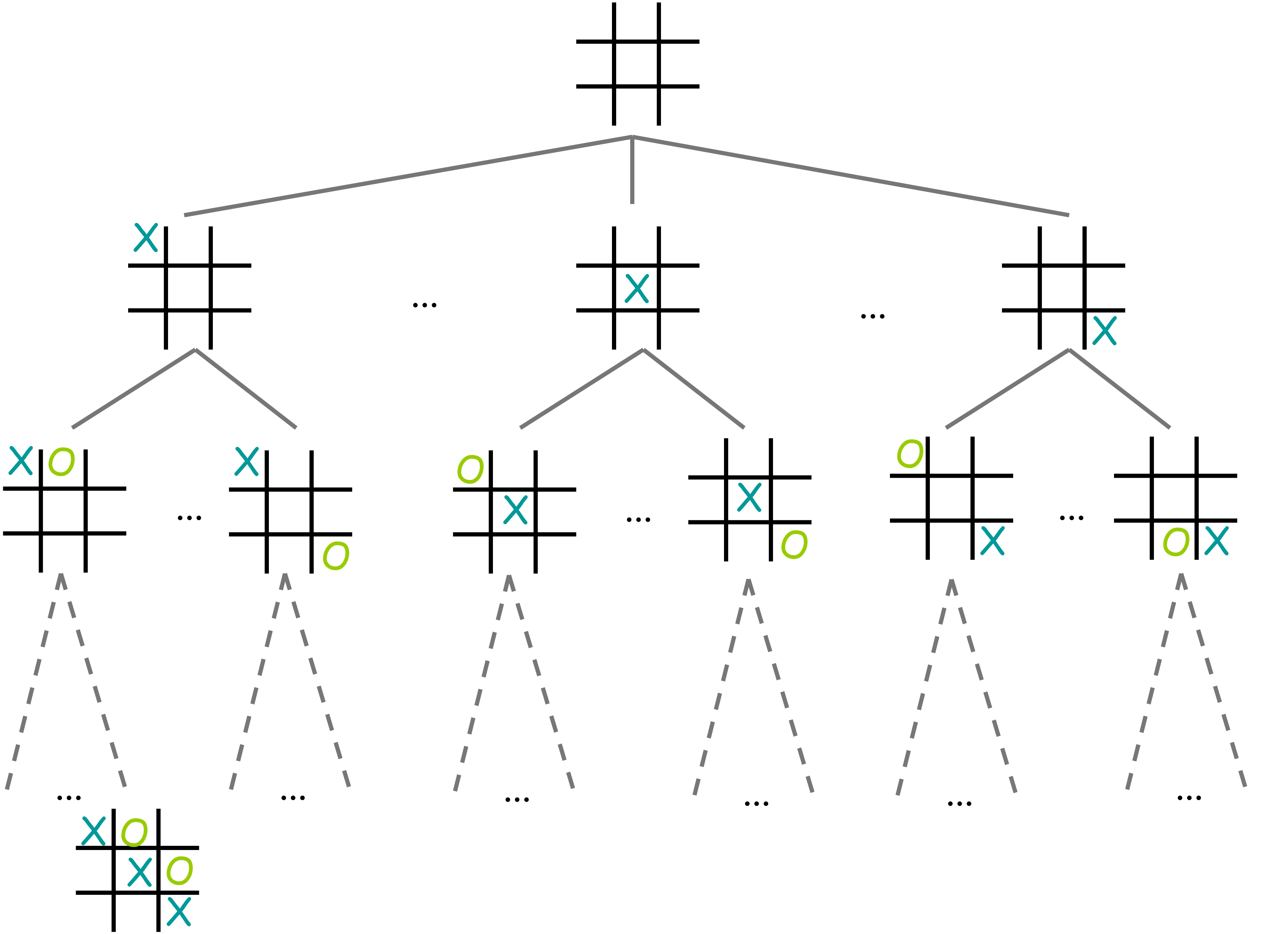

Tree Features

- Hierarchical relationship
- Root at the top
- Leaf at the bottom
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- Parents, children, ancestors, descendants, siblings
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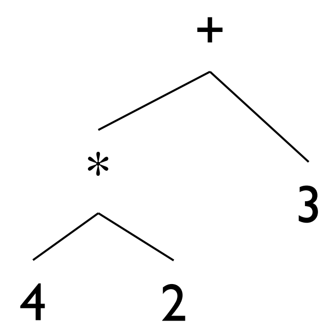
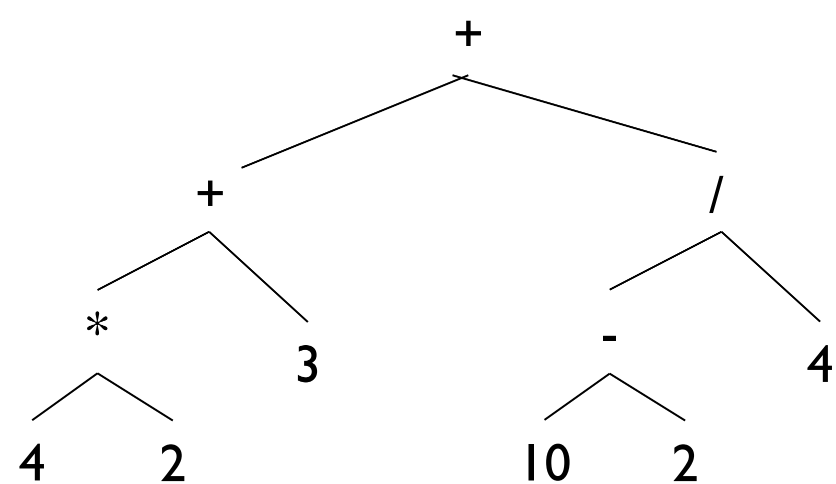
Expression Trees

4 * 2 + 3



(4 * 2 + 3) + ((10 – 2)/ 4)

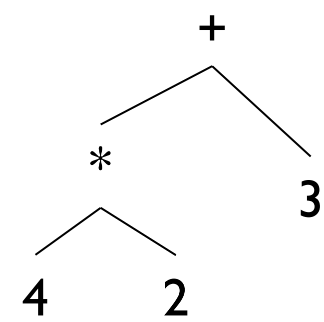


Introducing Binary Trees

- Degree of each node at most 2
- Every tree is either:
  - Empty, or
  - A root with left and right subtrees
- SLL: Recursive nature was captured by hidden node (Node<E>) class
- Binary Tree: No "inner" node class
  - Single BinaryTree class does it all
  - Is it a tree or a node?
    - It's a node that's a root of a tree!
  - And it's not part of Structure hierarchy!

Expression Trees

4 * 2 + 3



Build using constructor

```
new BinaryTree<E>(value, leftSubTree, rightSubTree)
```

```
BinaryTree<String> fourTimesTwo =  new BinaryTree<String>
        ("*",new BinaryTree<String>("4"),new BinaryTree<String>("2"));

BinaryTree<String> fourTimesTwoPlusThree = new BinaryTree<String>
        ("+", fourTimesTwo,  new BinaryTree<String>("3"));
```

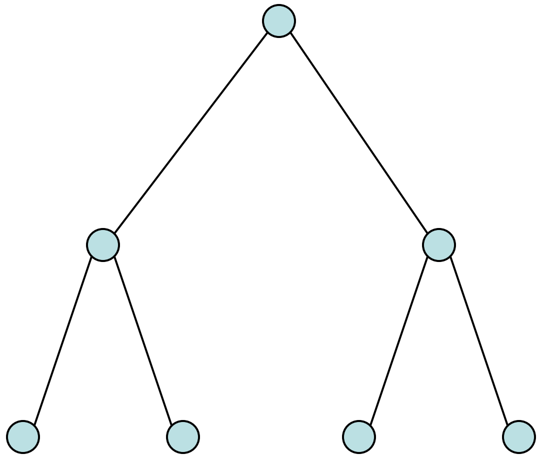
Expression Trees

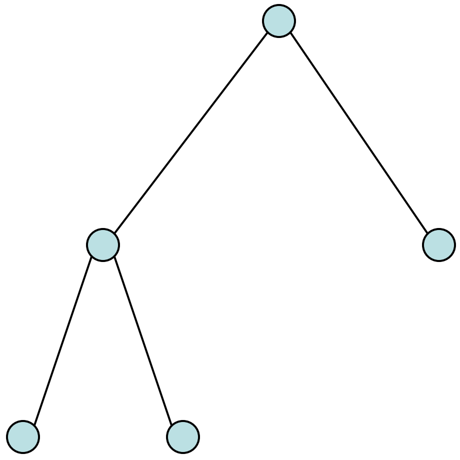
- General strategy
  - Make a binary tree (BT) for each leaf node
  - Move from bottom to top, creating BTs
  - Eventually reach the root
  - Call "evaluate" on final BT
- Example
  - How do we make a binary expression tree for (((4+3)*(10-5))/2)
    - Postfix notation: 4 3 + 10 5 - * 2 /

```java
int evaluate(BinaryTree<String> expr) {

    if (expr.height() == 0)
      return Integer.parseInt(expr.value());

    else {
      int left = evaluate(expr.left());
      int right = evaluate(expr.right());
      String op = expr.value();
      switch (op) {
      case "+" : return left + right;
      case "-" : return left - right;
      case "*" : return left * right;
      case "/" : return left / right;
      }
      Assert.fail("Bad op");
      return -1;
   }
}
```

Full vs. Complete (non-standard!)

- Full tree – A full binary tree of height h has *leaves only* on level h, and each internal node has exactly 2 children.
- Complete tree – A *complete* binary tree of height h is *full* to height h-1 and has all leaves at level h in leftmost locations.

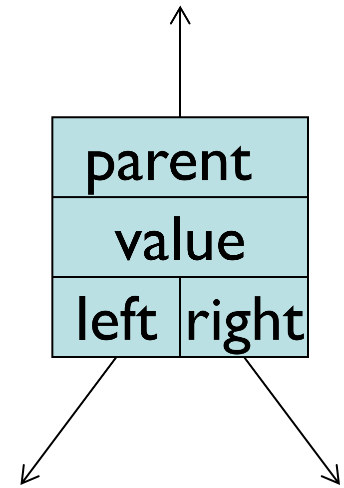


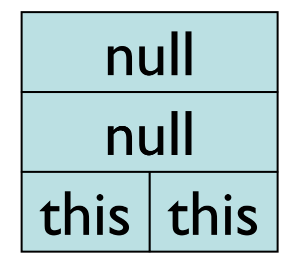


All full trees are complete, but not all complete trees are full!

Implementing BinaryTree

- BinaryTree<E> class
  - Instance variables
    - BinaryTree: parent, left, right
    - E: value
- left and right are never null
  - If no child, they point to an "empty" tree
    - Empty tree T has value null, parent null, left = right = T
  - Only empty tree nodes have null value

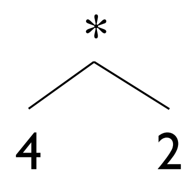


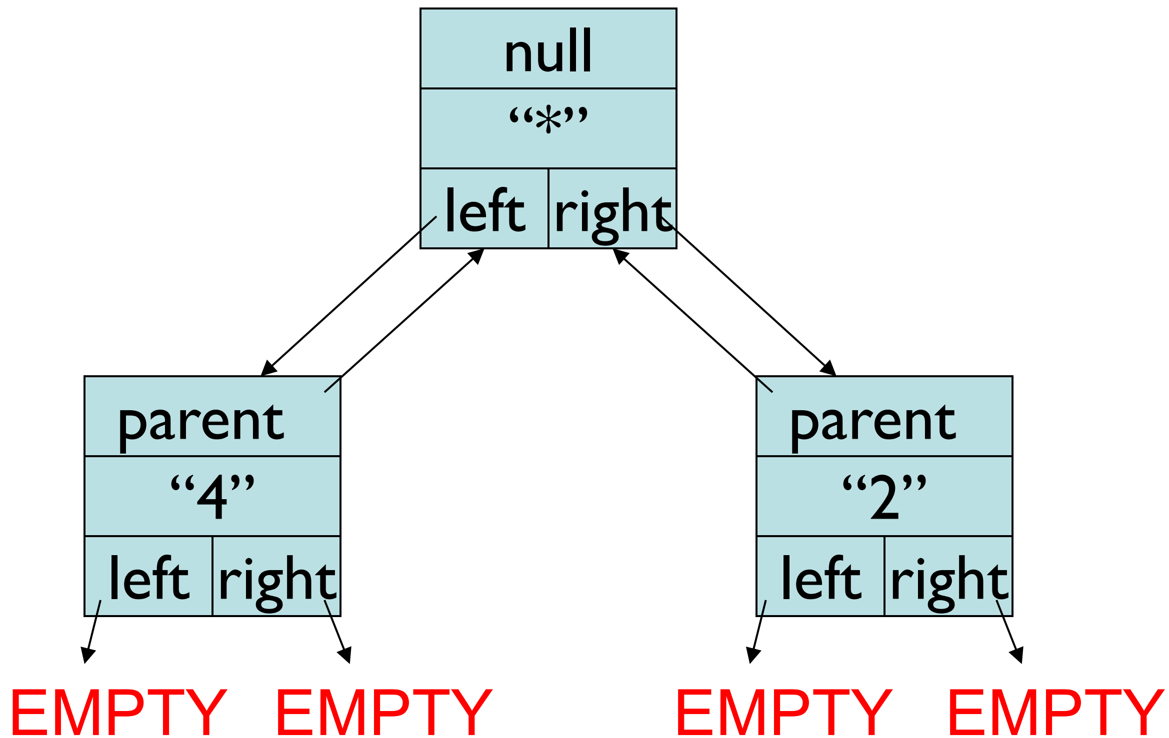


EMPTY BT

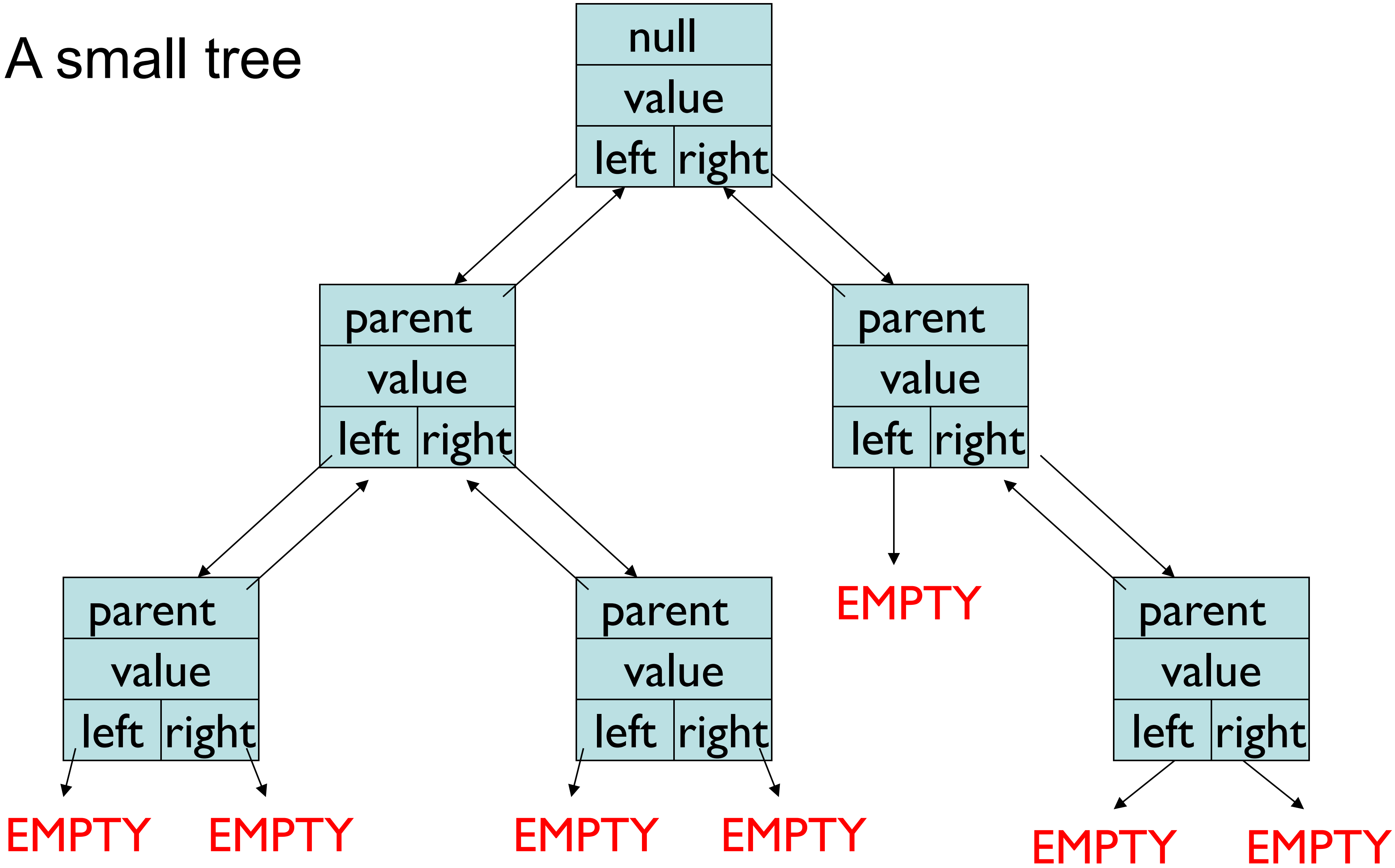
Implementing BinaryTree

- BinaryTree class
  - Instance variables
    - BT parent, BT left, BT right, E value





A small tree



EMPTY != null!

Implementing BinaryTree

- Many (!) methods: See BinaryTree javadoc page
- All "left" methods have equivalent "right" methods
  - public BinaryTree()
    - // generates an empty node (EMPTY)
    - // parent and value are null, left=right=this
  - public BinaryTree(E value)
    - // generates a tree with a non-null value and two empty (EMPTY) subtrees
  - public BinaryTree(E value, BinaryTree<E> left, BinaryTree<E> right)
    - // returns a tree with a non-null value and two subtrees
  - public void setLeft(BinaryTree<E> newLeft)
    - // sets left subtree to newLeft
    - // re-parents newLeft by calling newLeft.setParent(this)
  - protected void setParent(BinaryTree<E> newParent)
    - // sets parent subtree to newParent
    - // called from setLeft and setRight to keep all "links" consistent

Implementing BinaryTree

- Methods:
  - public BinaryTree<E> left()
    - // returns left subtree
  - public BinaryTree<E> parent()
    - // post: returns reference to parent node, or null
  - public boolean isLeftChild()
    - // returns true if this is a left child of parent
  - public E value()
    - // returns value associated with this node
  - public void setValue(E value)
    - // sets the value associated with this node
  - public int size()
    - // returns number of (non-empty) nodes in tree
  - public int height()
    - // returns height of tree rooted at this node
  - But where's "remove" or "add"?!?!

BT Questions/Proofs

- Prove
  - The number of nodes at depth n is at most 2^n
  - The number of nodes in tree of height n is at most $2^{n+1} - 1$
  - A tree with n nodes has exactly $n - 1$ edges
  - The size() method works correctly
  - The height() method works correctly
  - The isFull() method works correctly

BT Questions/Proofs

Prove: Number of nodes at depth $d \geq 0$ is at most 2^d

Idea: Induction on depth d of nodes of tree

Base case: $d = 0$: 1 node; $1 = 2^0$ ✓

Induction Hyp.: For some $d \geq 0$, there are at most 2^d nodes at depth d

Induction Step: Consider depth $d + 1$. There are at most 2 nodes at depth. $d + 1$ for every node at depth d.

Therefore it has at most $2 * 2^d = 2^{d+1}$ nodes ✓

BT Questions/Proofs

Prove that any tree on $n \geq 1$ nodes has $n-1$ edges

Idea: Induction on number of nodes

Base case: $n = 1$. There are no edges ✓

Induction Hyp: Assume that, for some $n \geq 1$, every tree on n nodes has exactly $n-1$ edges.

Induction Step: Let T have $n+1$ nodes. Show it has exactly n edges.

- Remove a leaf v (and its single edge) from T
- Now T has n nodes, so it has $n-1$ edges
- Now add v (and its single edge) back, giving $n+1$ nodes and n edges.

BT Questions/Proofs

Prove that BinaryTree method size() is correct.

- Let n be the number of nodes in the tree T

Base case: $n = 0$. T is empty---size() returns 0✓

Induction Hyp: Assume size() is correct for *all trees* having *at most* n nodes.

Induction Step: Assume T has $n+1$ nodes

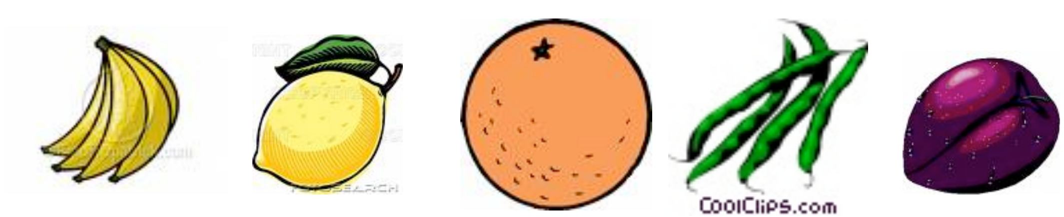
- Then left/right subtrees each have *at most* n nodes
- So size() returns correct value for each subtree
- And the size of T is 1 + size of left subtree + size of right subtree✓

Representing Knowledge

- Trees can be used to represent knowledge
  - Example: InfiniteQuestions game
- We often call these trees decision trees
  - Leaf: object
  - Internal node: question to distinguish objects
- Move down decision tree until we reach a leaf node
- Check to see if the leaf is correct
  - If not, add another question, make new and old objects children
- Let's look at the code…

Building Decision Trees

- Gather/obtain data
- Analyze data
  - Make greedy choices: Find good questions that divide data into halves (or as close as possible)
- Construct tree with shortest height
- In general this is a *hard* problem!
- Example



Representing Arbitrary Trees

- What if nodes can have many children?
  - Example: Game trees
- Replace left/right node references with a list of children (Vector, SLL, etc)
  - Allows getting "i^{th}" child
- Should provide method for getting degree of a node
- Degree 0 = Empty list = No children = Leaf