CSCI 136
Data Structures &
Advanced Programming

Lecture 10

Fall 2019

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Administrative Details

- Problem Set I due at beginning of class today!
  - Problem Set 2 is now online; it's due next Friday
    - If Mountain Day, drop in instructor's mailbox by 6pm
- Lab 4 Wednesday: Sorting!
  - The lab will soon be posted on the Labs page
  - You may again work with a partner
    - Needn't be same partner as Lab 3
    - Fill out the Google Form!
  - Produce a design before lab
    - Each member of pair should produce their own and then discuss/decide on final design

Last Time

- Strong Induction
- Basic Sorting
  - Insertion, Selection Sorts
  - Including time and space analysis

This Time

- Comparable Interface
- Better Sorting Methods
  - MergeSort
  - QuickSort
- More Flexible Comparing: Comparator Interface

Making Sorting Generic

- We need *comparable* items
- Unlike with equality testing, the Object class doesn't define a "compareTo()" method 😟
- We want a uniform way of saying objects can be compared, so we can write generic versions of methods like binary search
- Use an interface!
- Two approaches
  - Comparable interface
  - Comparator interface

Java Interfaces : Motivating Example

- Idea: Implement a class that describes a single playing card (e.g., "Queen of Diamonds")
- Start simple: a single class – BasicCard
- Think about alternative implementations
- Use an *interface* to allow implementation independent coding
- Let's look at BasicCard

Aside : Enum Types are Class Types

```
enum Rank { TWO, THREE, FOUR, FIVE, SIX, SEVEN,
      EIGHT, NINE, TEN, JACK, QUEEN, KING, ACE;
}
```

Notes

- Creates an ordered sequence of named constants
- Can find position of an enum value in sequence
  - `int i = r.ordinal(); // r is of type Rank`
- Can get an array of all values in the enum
  - `Rank[] allRanks = Rank.values();`
- Can use in for loops
  - `for (Rank r : Rank.values() ) { ... }`
- Can have its own instance variables and methods

Implementing a Card Object

- Think before we code!
- Many ways to implement a card
  - An index from 0 to 51; a rank and a suit, ...
- Start general.
  - Build an *interface* that advertises all public features of a card
  - Not an implementation (define methods, but don't include code)
- Then get specific.
  - Build specific implementation of a card using our general card interface

Start General: Card: An Interface

- What data do we have to represent?
  - Properties of cards
  - How can we represent these properties?
    - There are often multiple options—name some!
- What methods do we need?
  - Capabilities of cards
  - Do we need *accessor* and/or *mutator* methods?

*

A Card Interface

```
public interface Card {

    // Methods - must be public
    public Suit getSuit();
    public Rank getRank();
}
```

Notes

- Don't allow card to change its value
  - Only need accessor methods
- Support `enums` for rank and suit

Get Specific: Card Implementations

- Now suppose we want to build a specific card object
- We want to use the properties/capabilities defined in our interface
  - That is, we want to *implement* the interface

```
public class CardRankSuit implements Card {
    . . .
}
```

The Enums for Cards

```
public enum Suit {
    CLUBS, DIAMONDS, HEARTS, SPADES; // the values

public String toString() {
        switch (this) {
        case CLUBS : return "clubs";
        case DIAMONDS : return "diamonds";
        case HEARTS : return "hearts";
        case SPADES : return "spades";
        }
        return "Bad suit!";
    }
}
```

A similar declaration is defined for Rank

A First Card Implementation

```
public class CardRankSuit implements Card {
// instance variables
      protected Suit suit;
      protected Rank rank;
// Constructors
      public CardRankSuit( Rank r, Suit s)
            {suit = r; rank = s;}
// returns suit of card
      public Suit getSuit() { return suit;}
// returns rank of card
      public Rank getRank() { return rank;}
// create String representation of card
 public String toString()
           {return getRank() + " of " + getSuit();}
}
```

A Second Card Implementation

```
public class Card52 implements Card {
// instance variables
protected int code; // 0 <= code < 52;
// rank is code/13 and suit is code%13
// Constructors
public CardRankSuit( int index )
      {code = index;}
// returns suit of card
      public Suit getSuit() {// see sample code}
// returns rank of card
      public Rank getRank() {// see sample code}
// create String representation of card
 public String toString()
           {return getRank() + " of " + getSuit();}
}
```

Interfaces: Worth Noting

- Interface methods **are always** public
  - Java does not allow non-public methods in interfaces
- Interface instance variables are always **static final**
  - static variables are shared across instances
  - final variables are constants: they can't change value
- Most classes contain constructors; interfaces do not!
- Can *declare* interface objects (just like class objects) but cannot instantiate ("new") them

Comparable Interface

- Java provides an interface for comparisons between objects
  - Provides a replacement for "<" and ">" in recBinarySearch
- Java provides the *Comparable* interface, which specifies a method *compareTo()*
  - Any class that implements Comparable must provide compareTo()

```
public interface Comparable<T> {
    //post: return < 0 if this smaller than other
            return 0 if this equal to other
            return > 0 if this greater than other
      int compareTo(T other);
}
```

Comparable Interface

- Many Java-provided classes implement Comparable
  - String (alphabetical order)
  - Wrapper classes: Integer, Character, Boolean
  - All Enum classes
- We can write methods that work on any type that implements Comparable
  - Example: RecBinSearch.java and BinSearchComparable.java

compareTo in Card Example

We could write

```
public class CardRankSuit implements
      Comparable<CardRankSuit> {

  public int compareTo(CardRankSuit other) {
      if (this.getSuit() != other.getSuit())
         return getSuit().compareTo(other.getSuit());
      else
         return getRank().compareTo(other.getRank());
  }
// rest of code for the class....
}
```

Comparable & compareTo

- The Comparable interface (Comparable<T>) is part of the java.lang (not structure5) package.
- Other Java-provided structures can take advantage of objects that implement Comparable
  - See the Arrays class in java.util
  - Example JavaArraysBinSearch
- Users of Comparable are urged to ensure that *compareTo()* and *equals()* are *consistent*. That is,
  - x.compareTo(y) == 0 exactly when x.equals(y) == true
- Note that Comparable limits user to a *single ordering*
- The syntax can get kind of dense
  - See BinSearchComparable.java : a generic binary search method
  - And even more cumbersome….

ComparableAssociation

- Suppose we want an *ordered* Dictionary, so that we can use binary search instead of linear
- Structure5 provides a ComparableAssociation class that implements Comparable.
- The class declaration for ComparableAssociation is

…wait for it...

public class ComparableAssociation<K extends Comparable<K>, V>
Extends Association<K,V> implements
Comparable<ComparableAssociation<K,V>>

(Yikes!)

- Example: Since Integer implements Comparable, we can write
  - ComparableAssociation<Integer, String> myAssoc =
    new ComparableAssociation(new Integer(567), “Bob”);
- We could then use Arrays.sort on an array of these

Comparators

- Limitations with Comparable interface
  - Only permits one order between objects
  - What if it isn't the desired ordering?
  - What if it isn't implemented?
- Solution: Comparators

Comparators (Ch 6.8)

- A comparator is an object that contains a method that is capable of comparing two objects
- Sorting methods can be written to apply a comparator to two objects when a comparison is to be performed
- Different comparators can be applied to the same data to sort in different orders or on different keys

```
public interface Comparator <E> {
     // pre: a and b are valid objects
     // post: returns a value <, =, or > than 0 determined by
     // whether a is less than, equal to, or greater than b
     public int compare(E a, E b);
}
```

Example

Note that Patient does not implement Comparable or Comparator!

```
class Patient {
    protected int age;
    protected String name;
    public Patient (String s, int a) {name = s; age = a;}
    public String getName() { return name; }
    public int getAge() {return age;}
}

class NameComparator implements Comparator <Patient>{
    public int compare(Patient a, Patient b) {
       return a.getName().compareTo(b.getName());
    }
} // Note: No constructor; a "do-nothing" constructor is added by Java
```

```
public void sort(T a[], Comparator<T> c) {
   …
   if (c.compare(a[i], a[max]) > 0) {…}
}
```

```
sort(patients, new NameComparator());
```

Comparable vs Comparator

- Comparable Interface for class X
  - Permits just one order between objects of class X
  - * Class X must implement a compareTo method *
  - Changing order requires rewriting compareTo
    - And recompiling class X
- Comparator Interface
  - Allows creation of "Compator classes" for class X
  - * Class X isn't changed or recompiled *
  - Multiple Comparators for X can be developed
    - Sort Strings by length (alphabetically for equal-length)

Selection Sort with Comparator

```
public static <E> int findPosOfMax(E[] a, int last,
              Comparator<E> c) {
       int maxPos = 0       // A wild guess
       for(int i = 1; i <= last; i++)
              if (c.compare(a[maxPos], a[i]) < 0) maxPos = i;
       return maxPos;
}
public static <E> void selectionSort(E[] a, Comparator<E> c) {
       for(int i = a.length - 1; i>0; i--) {
           int big= findPosOfMin(a,i,c);
           swap(a, i, big);
       }
}
```

- The same array can be sorted in multiple ways by passing different Comparator<E> values to the sort method;

Merge Sort

- A *divide and conquer* algorithm
- Merge sort works as follows:
  - If the list is of length 0 or 1, then it is already sorted.
  - Divide the unsorted list into two sublists of about half the size of original list.
  - Sort each sublist recursively by re-applying merge sort.
  - Merge the two sublists back into one sorted list.
- Time Complexity?
  - Spoiler Alert! We'll see that it's O(n log n)
- Space Complexity?
  - O(n)

Merge Sort

- [8 14 29 1 17 39 16 9]
- [8 14 29 1] [17 39 16 9] split
- [8 14] [29 1] [17 39] [16 9] split
- [8] [14] [29] [1] [17] [39] [16] [9] split
- [8 14] [1 29] [17 39] [9 16] merge
- [1 8 14 29] [9 16 17 39] merge
- [1 8 9 14 16 17 29 39] merge

Merge Sort

- How would we implement it?
- First pass…

// recursively mergesorts A[from .. To] “in place”

void recMergeSortHelper(A[], int from, int to)

    if (from ≤ to)
        mid = (from + to)/2
        recMergeSortHelper(A, from, mid)
        recMergeSortHelper(A, mid+1, to)
        merge(A, from, to)

But *merge* hides a number of important details….

Merge Sort

- How would we implement it?
  - Review MergeSort.java
  - Note carefully how temp array is used to reduce copying
  - Make sure the data is in the correct array!
- Time Complexity?
  - Takes at most 2k comparisons to merge two lists of size k
  - Number of splits/merges for list of size n is log n
  - Claim: At most time O(n log n)…We'll see soon...
- Space Complexity?
  - O(n)?
  - Need an extra array, so really O(2n)! But O(2n) = O(n)

Merge Sort = O(n log n)

[8	14	29	1	17	39	16	9]		
[8	14	29	1]	[17	39	16	9]	split	
[8	14]	[29	1]	[17	39]	[16	9]	split	log n
[8]	[14]	[29]	[1]	[17]	[39]	[16]	[9]	split	
[8	14]	[1	29]	[17	39]	[9	16]	merge	
[1	8	14	29]	[9	16	17	39]	merge	log n
[1	8	9	14	16	17	29	39]	merge	

merge takes at most n comparisons per line

Time Complexity Proof

- Prove for n = 2^k (true for other n but harder)
- That is, MergeSort for performs at most
  - $n * \log(n) = 2^k * k$ comparisions of elements
- Base case $k \le 1$: 0 comparisons: $0 < 1 * 2^1$ ✓
- Induction Step: Suppose true for all integers smaller than k. Let T(k) be # of comparisons for 2^k elements. Then
- $T(k) \le 2^k + 2 * T(k-1)$

$$\le 2^k + 2(k-1)2^{k-1} \le k2^k$$

Merge Sort

- Unlike Bubble, Insertion, and Selection sort, Merge sort is a divide and conquer algorithm
  - Bubble, Insertion, Selection sort complexity: $O(n^2)$
  - Merge sort complexity: $O(n \log n)$
- Are there any problems or limitations with Merge sort?
- Why would we ever use any other algorithm for sorting?

Problems with Merge Sort

- Need extra temporary array
  - If data set is large, this could be a problem
- Waste time copying values back and forth between original array and temporary array
- Can we avoid this?

Quick Sort

- Quick sort is designed to behave much like Merge sort, without requiring extra storage space

Merge Sort	Quick Sort
Divide list in half	Partition* list into 2 parts
Sort halves	Sort parts
Merge halves	Join* sorted parts

Recall Merge Sort

```
private static void mergeSortRecursive(Comparable data[],
                    Comparable temp[], int low, int high) {
   int n = high-low+1;
   int middle = low + n/2;
   int i;

   if (n < 2) return;
   // move lower half of data into temporary storage
   for (i = low; i < middle; i++) {
       temp[i] = data[i];
   }
   // sort lower half of array
   mergeSortRecursive(temp,data,low,middle-1);
   // sort upper half of array
   mergeSortRecursive(data,temp,middle,high);
   // merge halves together
   merge(data,temp,low,middle,high);
}
```

Quick Sort

```
public void quickSortRecursive(Comparable data[],
                               int low, int high) {
    // pre: low <= high
    // post: data[low..high] in ascending order
        int pivot;
        if (low >= high) return;

        /* 1 - place pivot */
        pivot = partition(data, low, high);
        /* 2 - sort small */
        quickSortRecursive(data, low, pivot-1);
        /* 3 - sort large */
        quickSortRecursive(data, pivot+1, high);
}
```

Partition

1. Put first element (pivot) into sorted position
2. All to the left of “pivot” are smaller and all to the right are larger
3. Return index of “pivot”

Partition

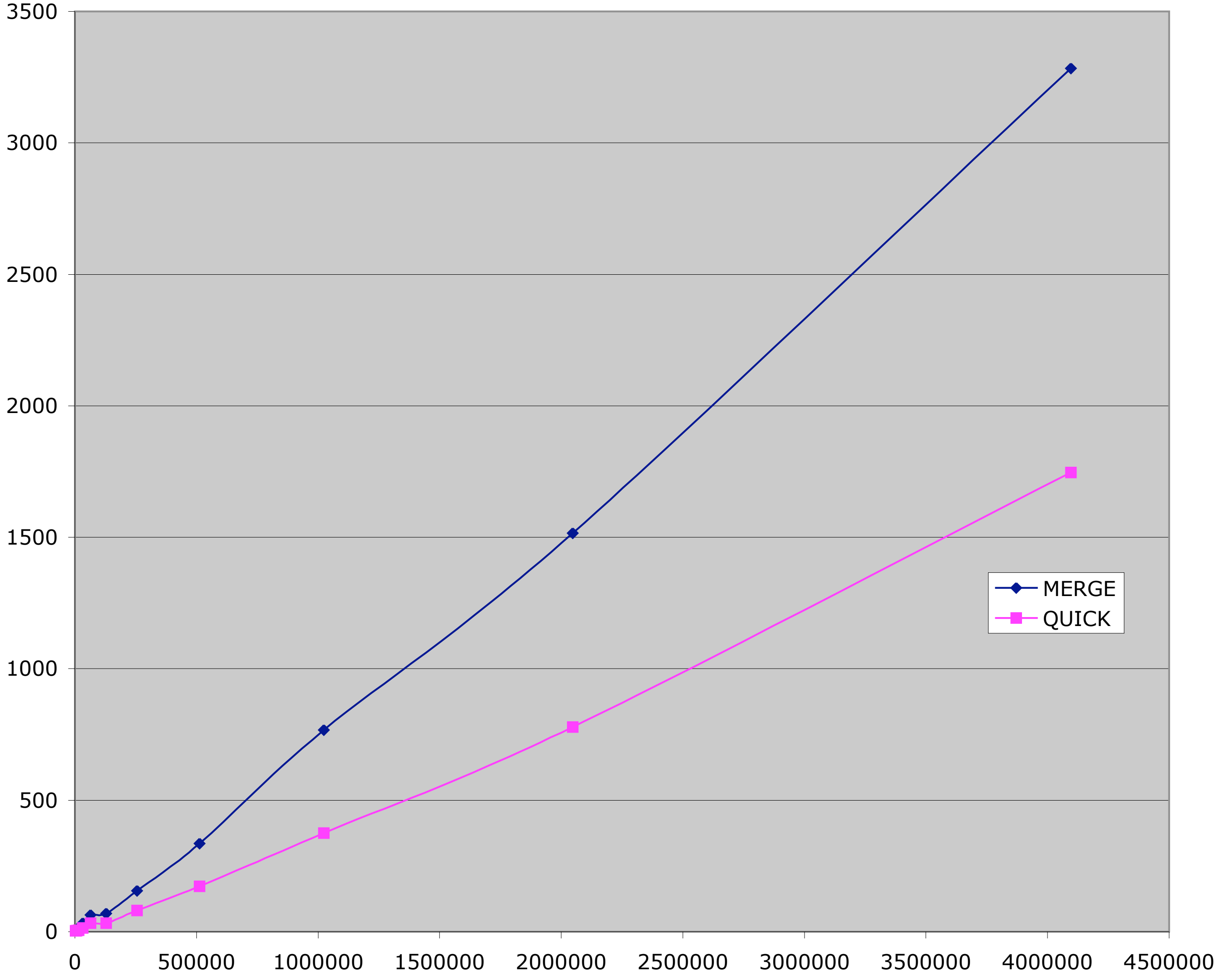
```
int partition(int data[], int left, int right) {
  while (true) {
    while (left < right && data[left] < data[right])
      right--;
    if (left < right) {
      swap(data,left++,right);
    } else {
      return left;
    }

    while (left < right && data[left] < data[right])
      left++;
    if (left < right) {
      swap(data,left,right--);
    } else {
      return right;
    }
  }
}
```

Complexity

- Time:
  - Partition is $O(n)$
  - If partition breaks list exactly in half, same as merge sort, so $O(n \log n)$
  - If data is already sorted, partition splits list into groups of 1 and n-1, so $O(n^2)$
- Space:
  - $O(n)$ (so is MergSort)
    - In fact, it's n + c compared to 2n + c for MergeSort

Merge vs. Quick



Food for Thought…

- How to avoid picking a bad pivot value?
  - Pick median of 3 elements for pivot (heuristic!)
- Combine selection sort with quick sort
  - For small n, selection sort is faster
  - Switch to selection sort when elements is <= 7
  - Switch to selection/insertion sort when the list is almost sorted (partitions are very unbalanced)
    - Heuristic!

Sorting Wrapup

	Time	Space
Bubble	Worst: $O(n^2)$ Best: O(n) - if "optimized"	O(n) : n + c
Insertion	Worst: $O(n^2)$ Best: O(n)	O(n) : n + c
Selection	Worst = Best: $O(n^2)$	O(n) : n + c
Merge	Worst = Best:: O(n log n)	O(n) : 2n + c
Quick	Average = Best: O(n log n) Worst: $O(n^2)$	O(n) : n + c

More Skill-Testing (Try these at home)

Given the following list of integers:

9 5 6 1 10 15 2 4

1) Sort the list using Insertion sort. . Show your work!
2) Sort the list using Merge sort. . Show your work!
3) Verify the best and worst case time and space complexity for each of these sorting algorithms as well as for selection sort.

Faster Sorting: Merge Sort

- A *divide and conquer* algorithm
- Typically used on arrays
- Merge sort works as follows:
  - If the array is of length 0 or 1, then it is already sorted.
  - Divide the unsorted array into two arrays of about half the size of original.
  - Sort smaller arrays recursively by re-applying merge sort.
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Merge Sort

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[1	8	14	29]	[9	16	17	39]	merge
[1	8	9	14	16	17	29	39]	merge

Merge Sort : Pseudo-code

- How would we *design* it?
- First pass…

```
// recursively mergesorts A[from .. To] "in place"
void recMergeSortHelper(A[],  int from, int to)
    if ( from ≤ to )
        mid = (from + to)/2
        recMergeSortHelper(A, from, mid)
        recMergeSortHelper(A, mid+1, to)
        merge(A, from, to)
```

But *merge* hides a number of important details….

Merge Sort : Java Implementation

- How would we *implement* it?
  - Review MergeSort.java
  - Note carefully how temp array is used to reduce copying
  - Make sure the data is in the correct array!
- Time Complexity?
  - Takes at most 2k comparisons to merge two lists of size k
  - Number of splits/merges for list of size n is log n
  - Claim: At most time O(n log n)…We'll see soon...
- Space Complexity?
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  - Need an extra array, so really O(2n)! But O(2n) = O(n)

Merge Sort = O(n log n)

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log n

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merge takes at most n comparisons per line

Time Complexity Proof

- Prove for $n = 2^k$ (true for other n but harder)
- That is, MergeSort for performs at most
  - $n * \log(n) = 2^k * k$ comparisions of elements
- Base cases $k \leq 1$: 0 comparisons: $0 < 1 * 2^1$ ✓
- Induction Step: Suppose true for all integers smaller than k. Let T(k) be # of comparisons for 2^k elements. Then
- $\underline{T(k) \leq} \; 2^k + 2 * T(k-1) \leq 2^k + 2(k-1)2^{k-1} \leq \underline{k * 2^k}$ ✓

Merge Sort

- Unlike Bubble, Insertion, and Selection sort, Merge sort is a divide and conquer algorithm
  - Bubble, Insertion, Selection sort complexity: $O(n^2)$
  - Merge sort complexity: $O(n \log n)$
- Are there any limitations with Merge sort?
- Why would we ever use any other algorithm for sorting?

Drawbacks to Merge Sort

- Need extra temporary array
  - If data set is large, this could be a problem
- Waste time copying values back and forth between original array and temporary array
- Can we avoid this?

Quick Sort

- Quick sort is designed to behave much like Merge sort, without requiring extra storage space

Merge Sort	Quick Sort
Divide list in half	Partition* list into 2 parts
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Quick Sort

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        int pivot;
        if (low >= high) return;

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        /* 2 - sort small */
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}
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Partition

1. Put first element (pivot) into sorted position
2. All to the left of “pivot” are smaller and all to the right are larger
3. Return index of “pivot”

Partition

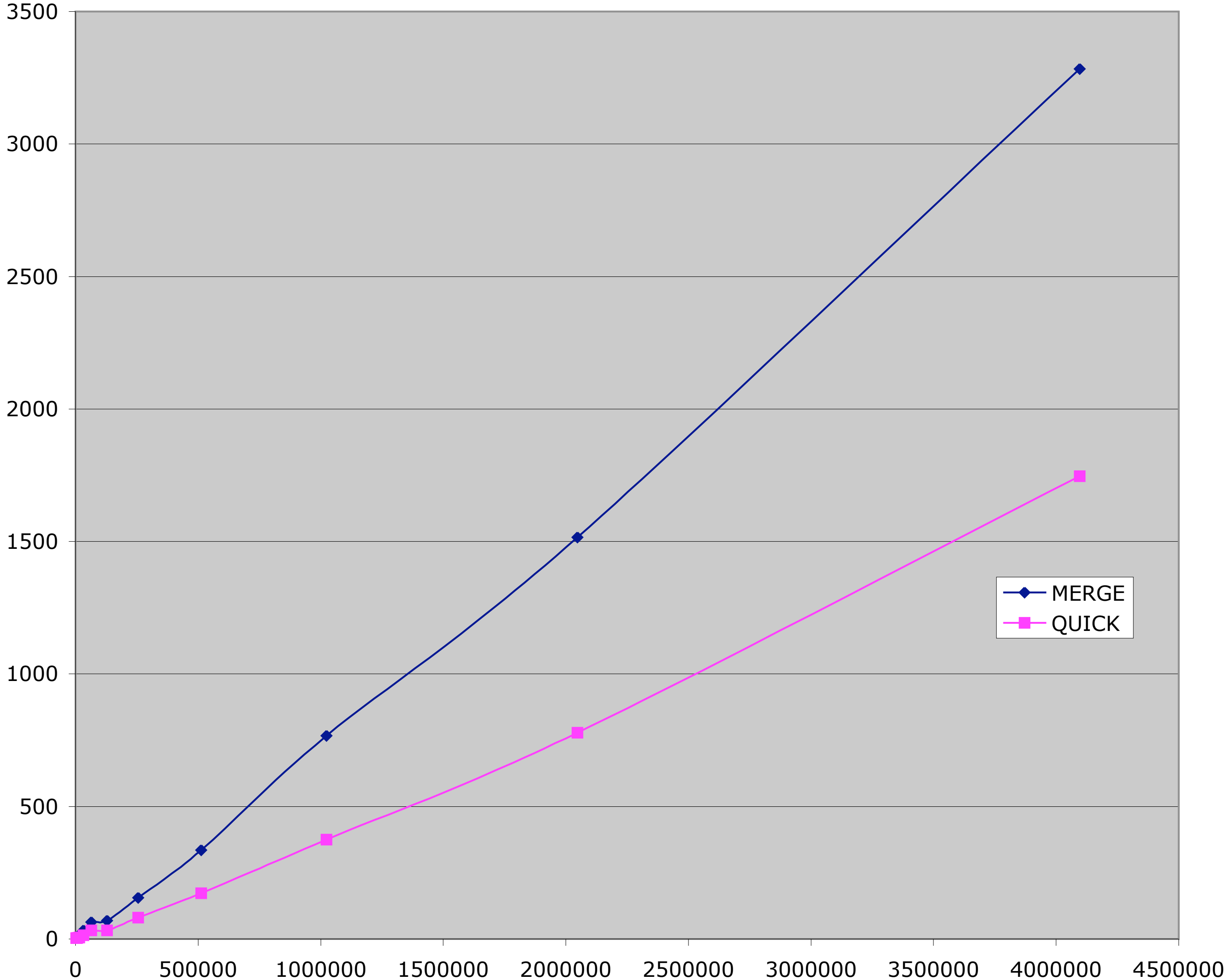
```
int partition(int data[], int left, int right) {
  while (true) {
    while (left < right && data[left] < data[right])
      right--;
    if (left < right) {
      swap(data,left++,right);
    } else {
      return left;
    }

    while (left < right && data[left] < data[right])
      left++;
    if (left < right) {
      swap(data,left,right--);
    } else {
      return right;
    }
  }
}
```

Complexity

- Time:
  - Partition is $O(n)$
  - If partition breaks list exactly in half, same as merge sort, so $O(n \log n)$
  - If data is already sorted, partition splits list into groups of 1 and n-1, so $O(n^2)$
- Space:
  - $O(n)$ (so is MergSort)
    - In fact, it's n + c compared to 2n + c for MergeSort

Merge vs. Quick (Average Time)



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Selection	Worst = Best: $O(n^2)$	$O(n)$: $n + c$
Merge	Worst = Best:: $O(n \log n)$	$O(n)$: $2n + c$
Quick	Average = Best: $O(n \log n)$ Worst: $O(n^2)$	$O(n)$: $n + c$

More Skill-Testing (Try these at home)

Given the following list of integers:

9 5 6 1 10 15 2 4

1) Sort the list using Bubble sort. Show your work!
2) Sort the list using Insertion sort. . Show your work!
3) Sort the list using Merge sort. . Show your work!
4) Verify the best and worst case time and space complexity for each of these sorting algorithms as well as for selection sort.

Sorting Material Ends Here

Class Specialization

- Classes can *extend* other classes
  - Inherit fields and **method bodies**
- By extending other classes, we can create specialized sub-classes
- Java supports class extension/specialization
- Java enforces *type-safety*: Objects behave according to their type
  - Some checks are made at compile-time
  - Some checks are made at run-time
- We'll first use this feature to factor out code

Abstract Classes

- Note: All of our Card implementations code `toString()` in identical fashion.
- It’s good to be able to “factor out” common code so that it only has to be maintained in one place
- *Abstract classes* to the rescue….
- An abstract class allows for a *partial* implementation
- We can then *extend* it to a complete implementation
- Let’s do this with our cards.
  - Examine CardAbstract.java....

Abstract Classes

Notes from `CardAbstract` class example

- `CardAbstract` *implements* `Card` (partially)
- `CardAbstract` is declared to be *abstract*
  - It contains the implementation of `toString()`

How do the full implementations (`CardRankSuit`, etc) change?

- They are declared to *extend* `CardAbstract`
- They don't need to say "`implements Card`"
- They don't contain the `toString()` method
  - They *inherit* that method from `CardAbstract`
  - But could *override* that method if desired

Extending Concrete Classes

Let's call a class *concrete* if it is not abstract

We can extend concrete classes

Example: Adding a point count to a `Card`

- Suppose we wanted to add a point value to each of the playing cards in `CardRankSuit`
- We *extend* that class

```
class CardRankSuitPoints extends CardRankSuit {… }
```

- This new class can now contain additional instance variables and methods
- Let's look at the code for CardRankSuitPoints.java….

CardRankSuitPoints Notes

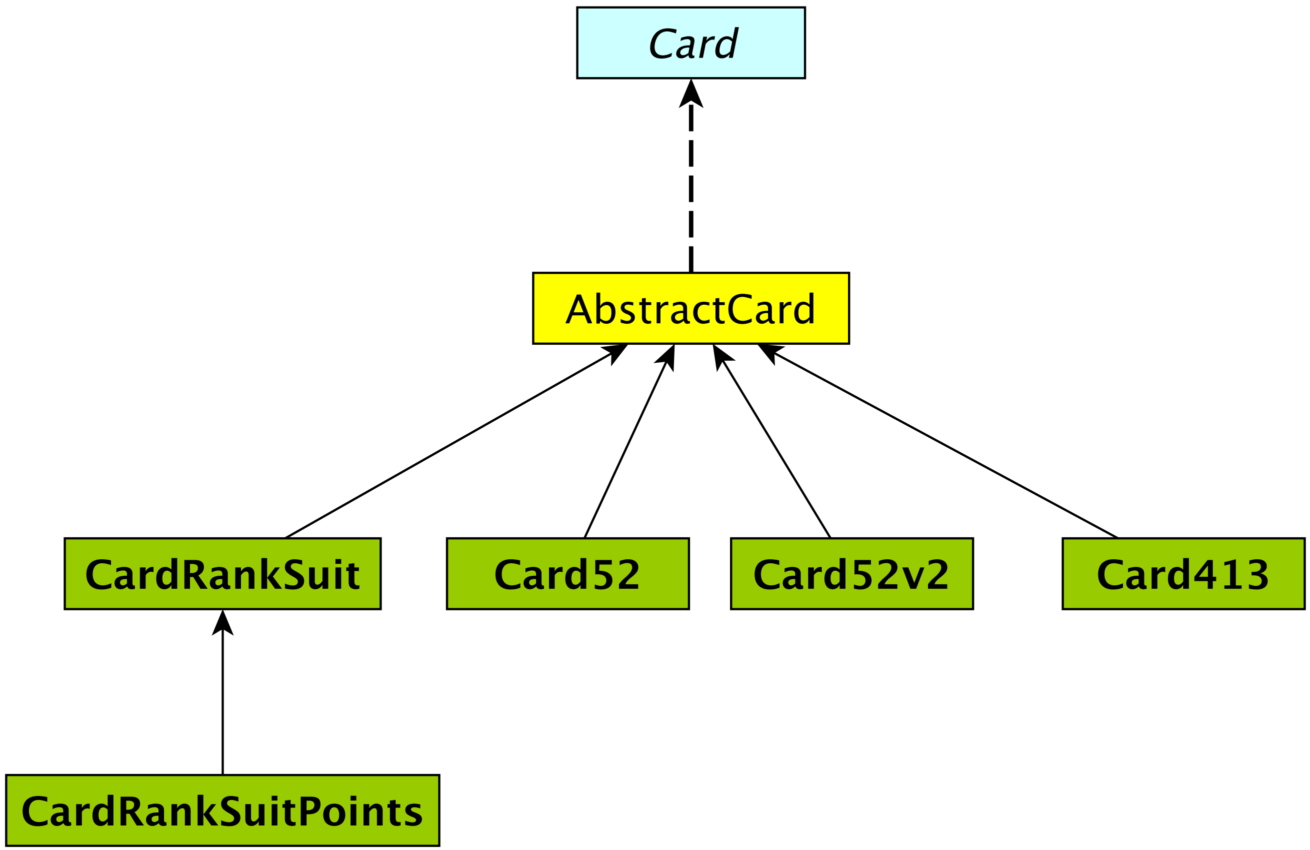
- Constructor calls `CardRankSuit` constructor using *super*
- We can override methods---e.g., `toString()`
- Can use a `CardRankSuitPoints` object wherever we use a `Card`
  - But! Can only use new features (`getPoints()`) if the object is declared to be of type `CardRankSuitPoints`

```
CardRankSuitPoints c1 = new CardRankSuitPoints(
    Rank.ACE, Suit.CLUBS, 4);
int p1 = c1.getPoints(); // Legal
Card c2 = new CardRankSuitPoints(Rank.ACE,
    Suit.CLUBS, 4);
int p2 = c2.getPoints(); // Bad! c2 is of type Card
int p3 = ((CardRankSuitPoints) c2).getPoints(); // Legal
```

- Java enforces *type-safety*: An variable of type X can only be assigned a value of type X or of a type that extends X

The Card Classes Hierarchy

Interface | Abstract Class | **Class**



compareTo in Card Example

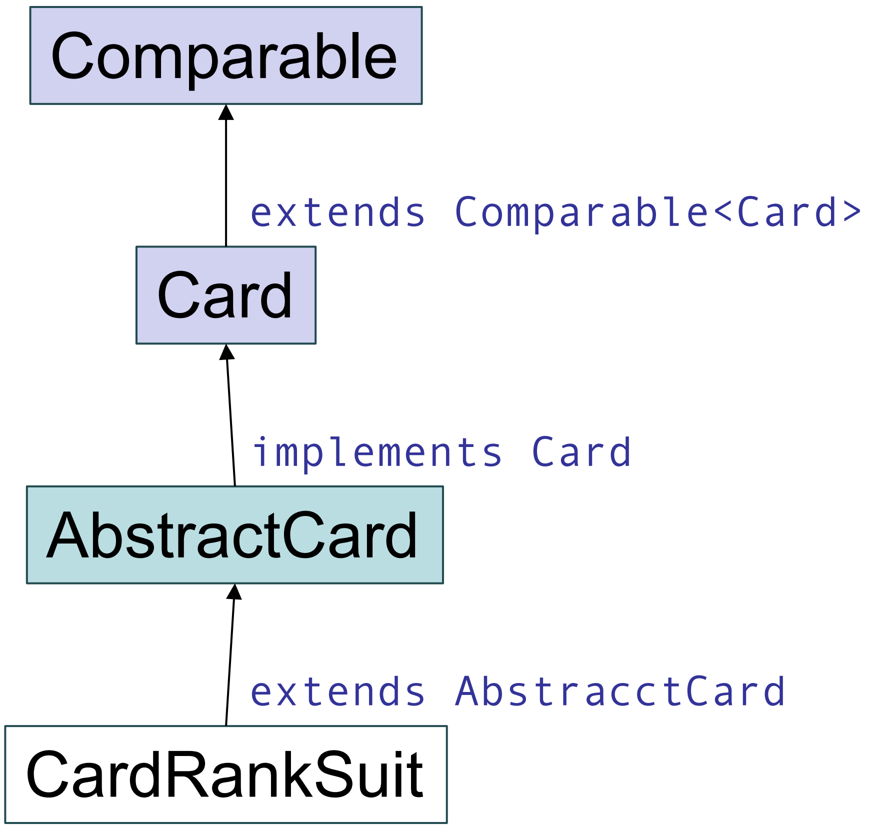
We actually wrote (in Card.java)

```
public interface Card extends Comparable<Card> {
   public int compareTo(Card other);
   // remainder of interface code
}
```

And in CardAbstract.java, we added

```
   public int compareTo(Card other) {
        if (this.getSuit() != other.getSuit())
           return getSuit().compareTo(other.Suit());
        else
           return getRank().compareTo(other.getRank());
   }
```

Class/Interface Hierarchy



- As a result, all of our implementations of the Card interface have comparable card types!