CSCI 136: Fall 2019
Handout PS 2
Due: Friday, Oct. 4 *at start of your lecture section*

Problem Set 2

Instructions: I encourage you to do all of these problems, but please hand in only the ones labeled "Hand In"! If Friday ends up being Mountain Day, drop the problems off in your instructor's mail cubby on the third floor of TCL (right across from TCL 303).

You might find the *Second Principle of Mathematical Induction* handout useful to read before attempting these problems!

Second Principle of Induction

(*3 points*) .... Hand In

Prove that every integer $n \geq 8$ can be written as a sum of 3s and 5s.

(*4 points*) .... Hand In

Define a sequence of numbers $t_1, \ldots, t_n, \ldots$ as follows:

1. $t_1 = 1$
2. For all $n \geq 2$, if n is even, then $t_n = 2t_{n/2}$; if n is odd, $t_n = 2t_{(n-1)/2}$.

So $t_2 = 2t_1 = 2, t_3 = 2t_1 = 2, t_4 = 2t_2 = 4, t_5 = 2t_2 = 4, t_6 = 2t_3 = 4, \ldots$.

Prove that for all $n \geq 1$, $t_n \leq n$. Hint: In the induction step, consider 2 cases: $n+1$ is even and $n+1$ is odd.

(*4 points*) .... Hand In

A string s of left and right "square" and/or "round" parentheses is *balanced* if,

1. `s` is the empty string (`s = ""`),
2. `s = "(" + t + ")"`, where `t` is a balanced string of parentheses,
3. `s = "[" + t + "]"`, where `t` is a balanced string of parentheses, or
4. `s = t + u`, where `t` and `u` are balanced strings of parentheses each shorter than `s`.

Note: In the definition above, + is the string concatenation operator. So, for example, "()", "[]", "[()[]]", "([])[[]()]", are all balanced strings, but "[()[(])]" is not.

Prove by induction if a non-empty string is balanced it must contain as a substring either "()" or "[]".* You can assume that all strings are even length. Hint: Here your induction step must consider 3 cases!

(*0 points*) .... Practice

Prove that every integer $n > 1$ can be written as a product of prime numbers. Hint: Use the fact proven in the handout that every $n > 1$ has a prime factor as the basis for an induction proof. Could you have solved this problem as easily using the First Principle of Induction?

(*0 points*) .... Practice—Challenge!

A *triomino* (pronounced like "try" followed by "dominoes" without the "d") is a shape made of three unit squares in an "L"-shape. Think of it as the result of removing a single square from a 2×2-checkerboard. If I remove *any* single square from a 4×4 checkerboard, I can cover the remaining 15 squares with 5 non-overlapping tri-ominoes. Try it and see!
Prove by induction that if *any* square is removed from a $2^n \times 2^n$-checkerboard, then the remaining squares can be covered by non-overlapping tri-ominoes. I love this problem: The proof is very short but requires just the right induction step!

*Note: This recursive definition of balanced is different from that used in Lab 3!