Searching and Sorting

Searching in a List

- **Search Problem.** Given a list L of length n and an item e, is item e present in L?
- Function `linearSearch( L, e)`
- Examples

```
>>> linearSearch([12, 16, 23, 2, 7], 16)
True
>>> linearSearch(['hello', 'world', 'silly'], 'hi')
False
>>> linearSearch(['a', 'e', 'i', 'o', 'u'], 'u')
True
```

Searching in an Unsorted List

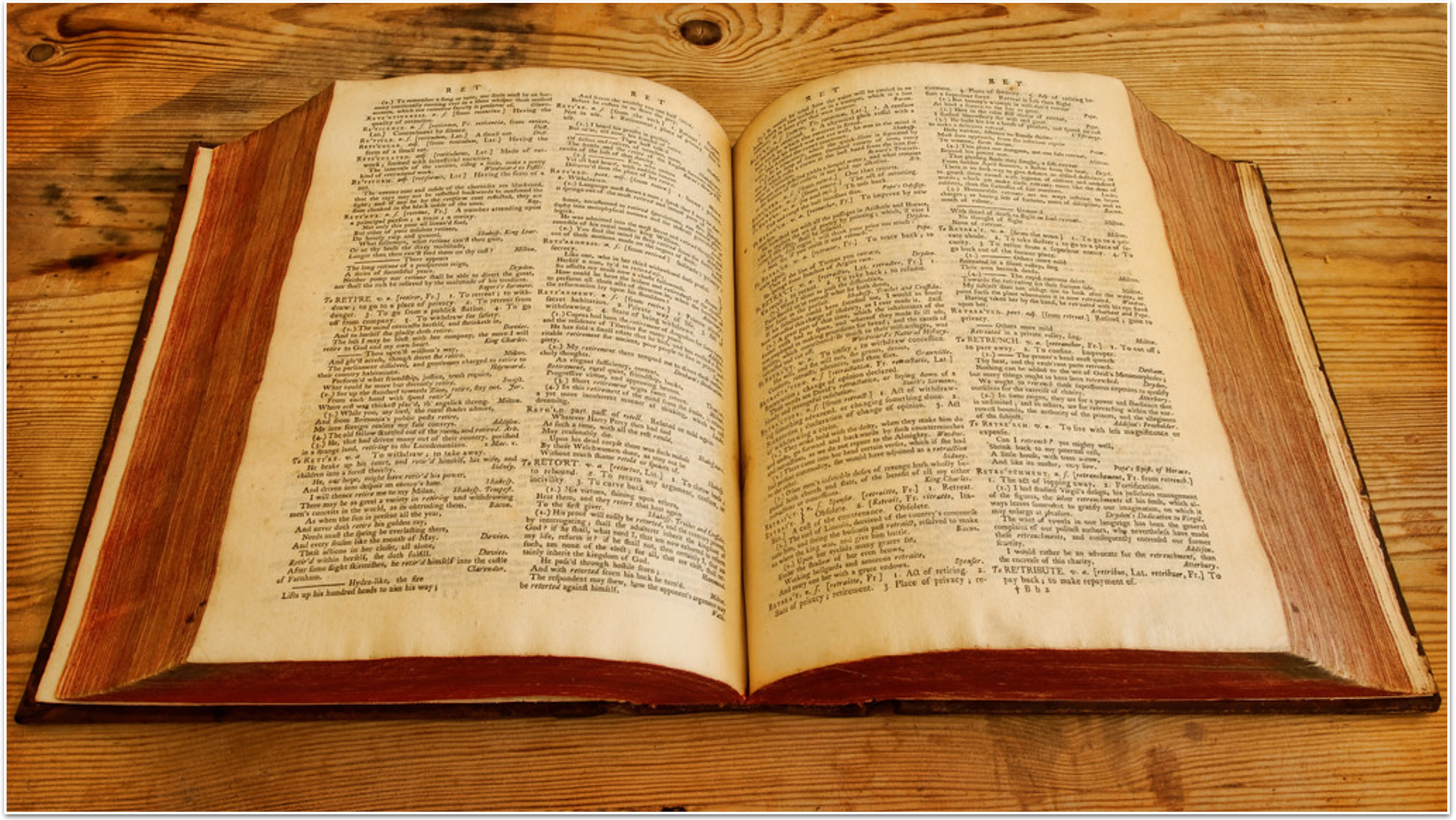
- **Search Problem.** Given a list L of length n and an item e, is item e present in L?
- In the worst case need to look through the entire list
- $O(n)$ algorithm

```python
def linearSearch(L, e):
    n = len(L)
    for item in L:
        if item == e:
            return True
    return False
```

What if the list is sorted?

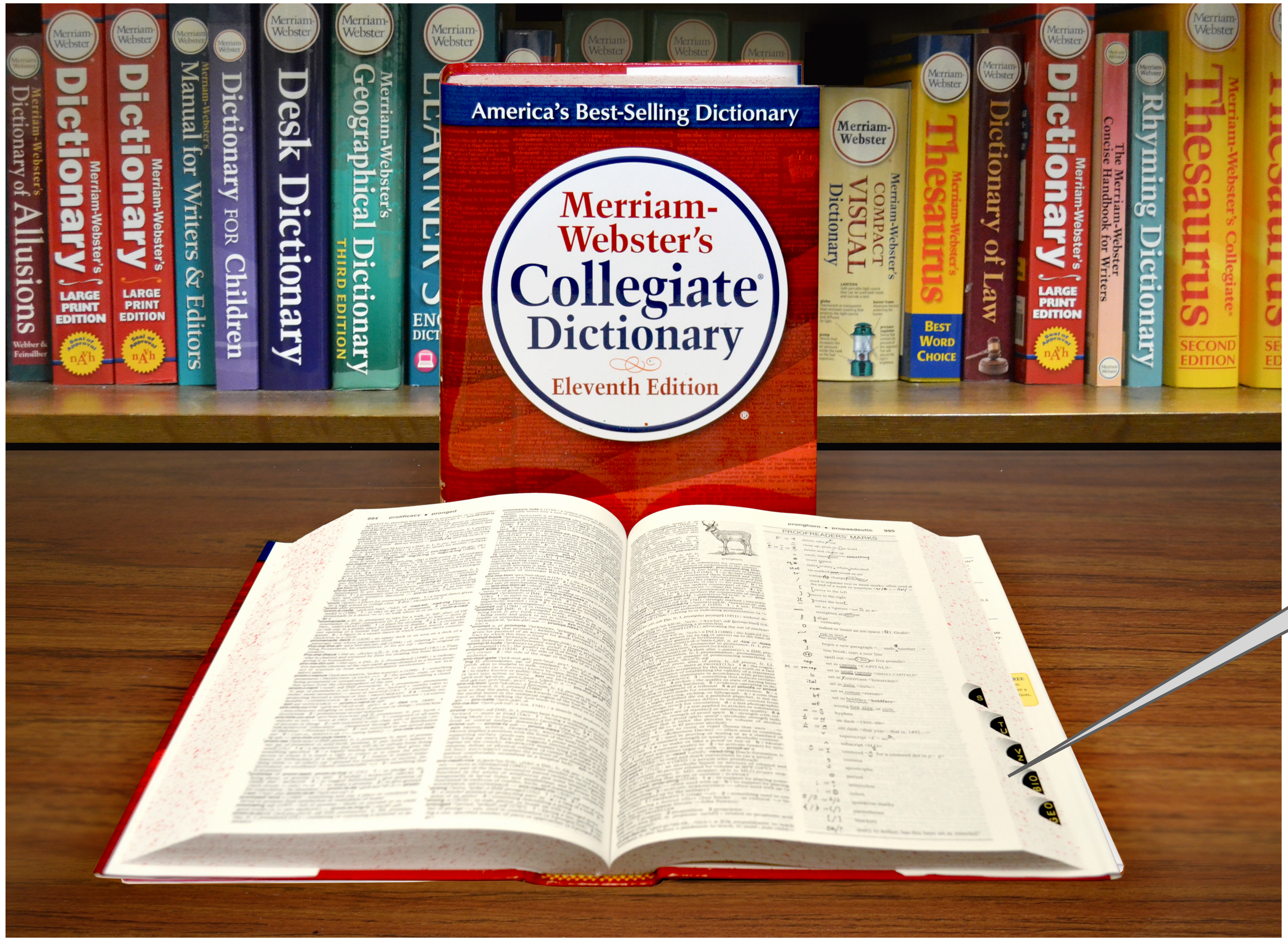
Example: Dictionary

- How do we look up a word in a dictionary?
- Words are listed in alphabetical order



Searching in a Dictionary

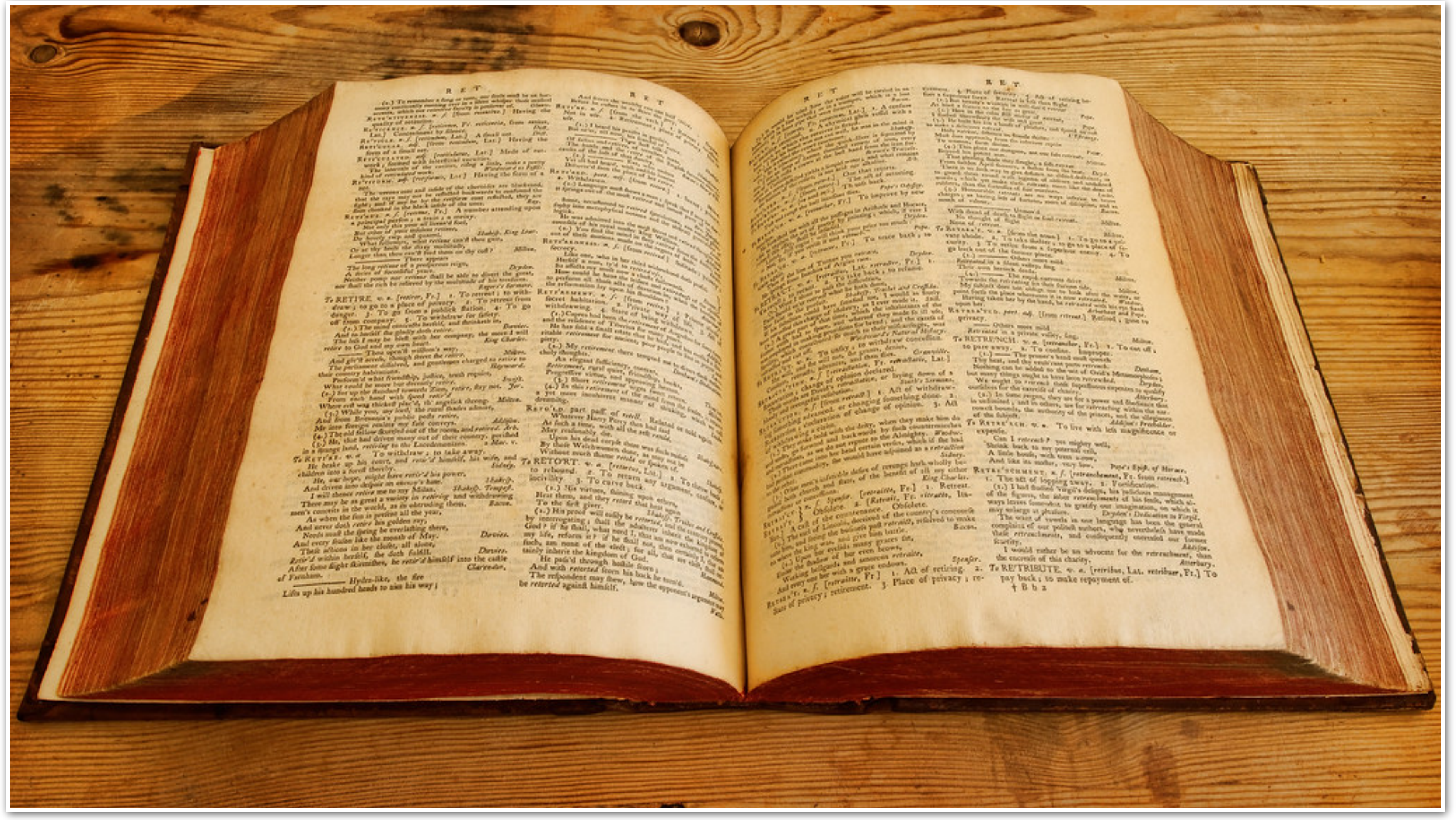
- How do we look up a word in a dictionary?
- Words are listed in alphabetical order



Let's assume we don't have these tabs to help us out

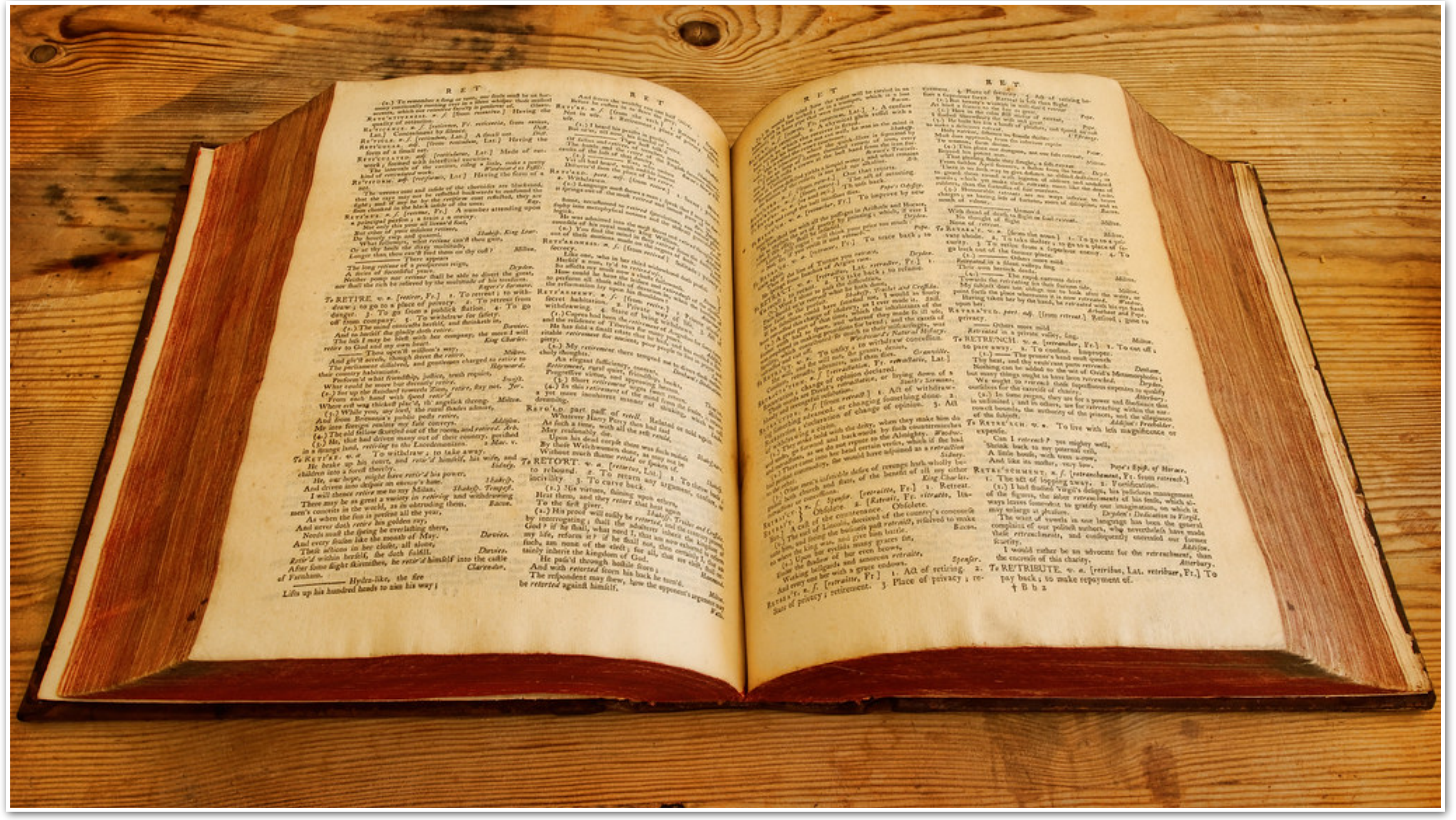
Search Algorithm

- Look at the middle page of the dictionary for our query word
- If we find our query, great!
- Otherwise:
  - If our query is later in alphabetical order to the words on the page, look for the query between the middle page and the last page
  - If our query is earlier in alphabetical order, look for the query between the middle page and the first page



How Good is This?

- **Goal:** Analyze how many pages we need to look at to look a word up in the dictionary
- Want the worst case: it's possible that I'm looking for a word that's right on the middle page
- Each time we look at the "middle" page remaining, the number of pages we need to look at is divided by 2 (Actually slightly better since we can rule out the middle page itself)
- A 1024-page dictionary requires at most 11 lookups: 1024 pages, < 512, <256, <128, <64, <32, <16, <8, <4, <2, <1 page.
- Just needed to look at 11 pages out of 1024 !
- Can we generalize this for an n page dictionary?



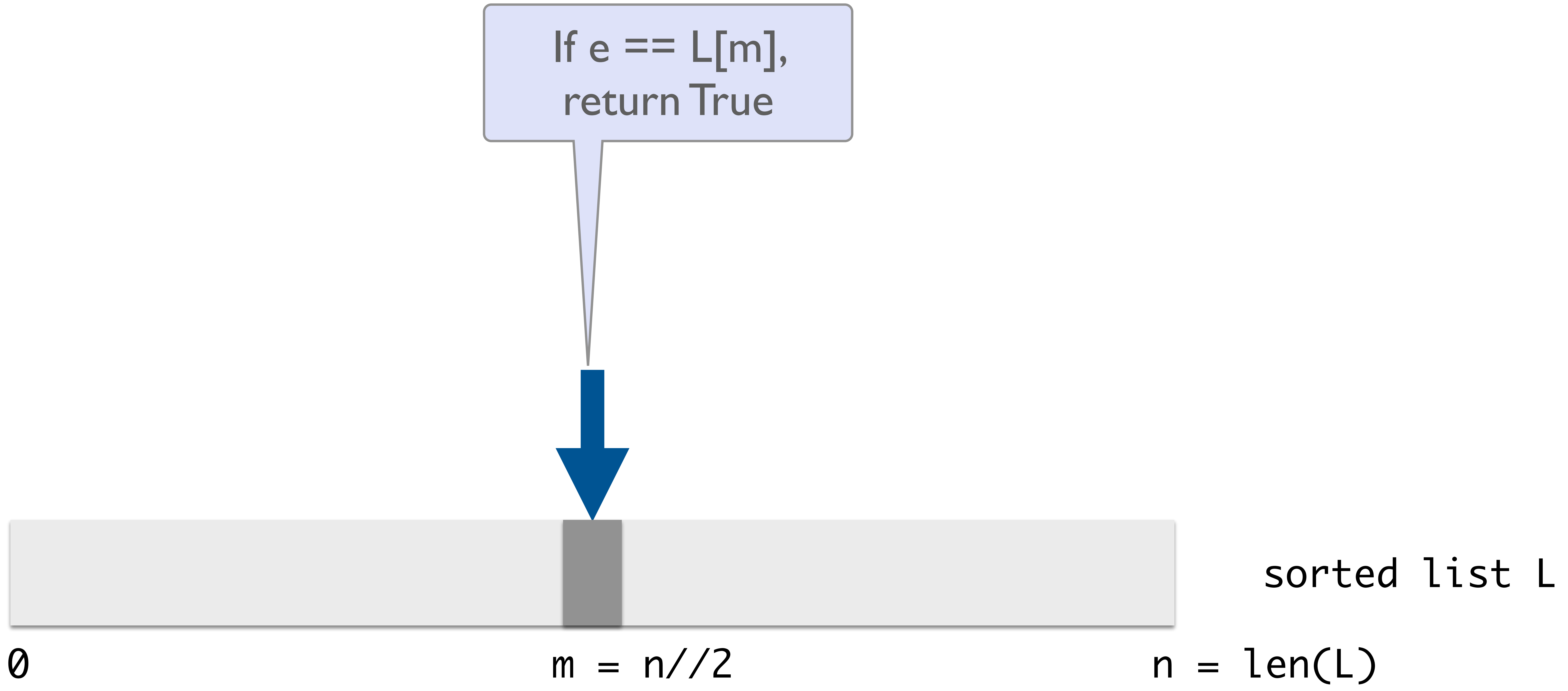
Review: Logarithm

- Logarithms are the inverse function to exponentiation
- Thus, $\log_b n$ describes the exponent to which base b must be raised to produce n
- That is, $b^{\log_b n} = n$
- Another way to look at logarithms:
    - $\log_b n$ is, essentially, the number of times n must be divided by b to reduce it to 1
- For us, important takeaway:
    - How many times can we divide n by 2 until we get down to 1
    - $\approx \log_2 n$

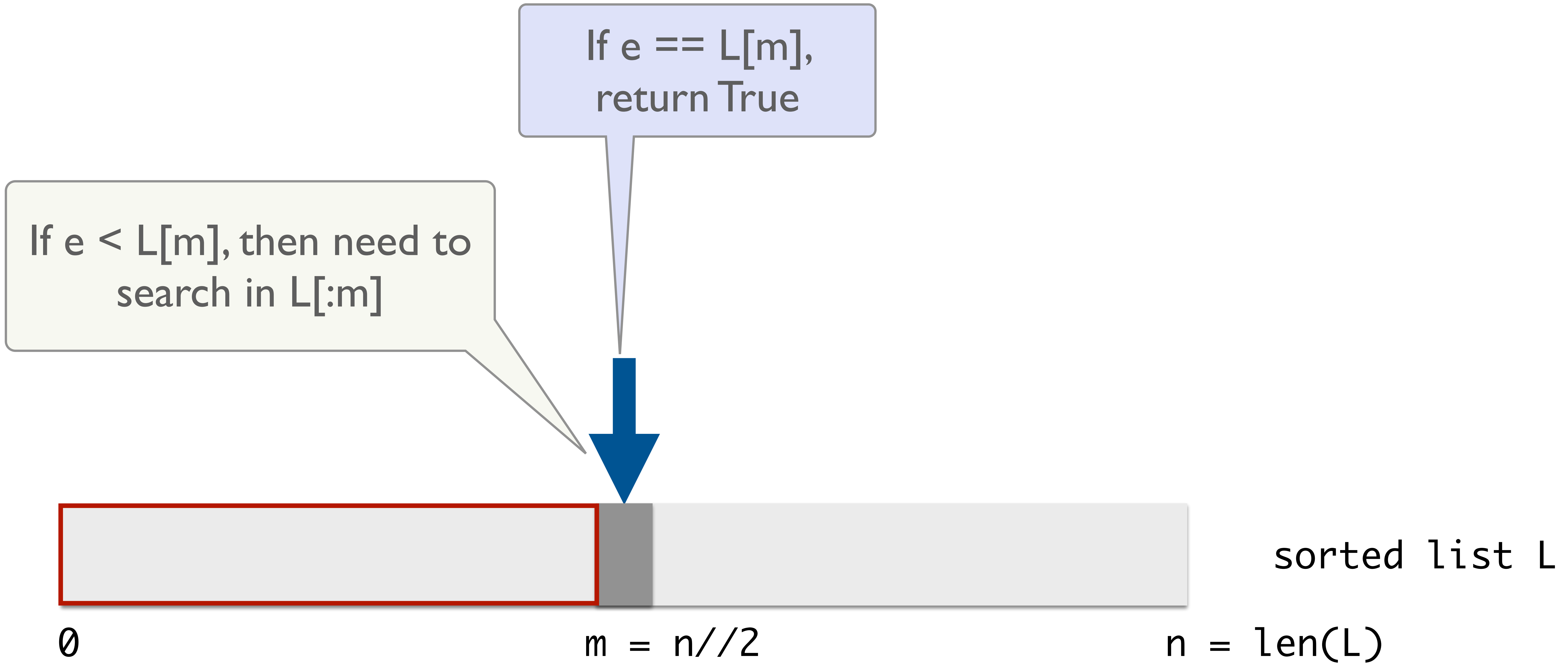
Log Summary

- Logarithms are defined by the relationship $n = b^{\log_b n}$
- The value $\log_b n$ is, essentially, the number of times n must be divided by b to reduce it to 1
- In CS, we often set $b = 2$ as we often design solutions where we divide the problem size in half
- We ignore base in Big Oh notation because they affect the value by a constant, that is,
$$\log_a n = \frac{\log_b n}{\log_b a} = \log_b n \cdot \log_a b = c \log_b n$$
- Thus we write $O(\log n)$ to describe logarithmic growth with respect to the input

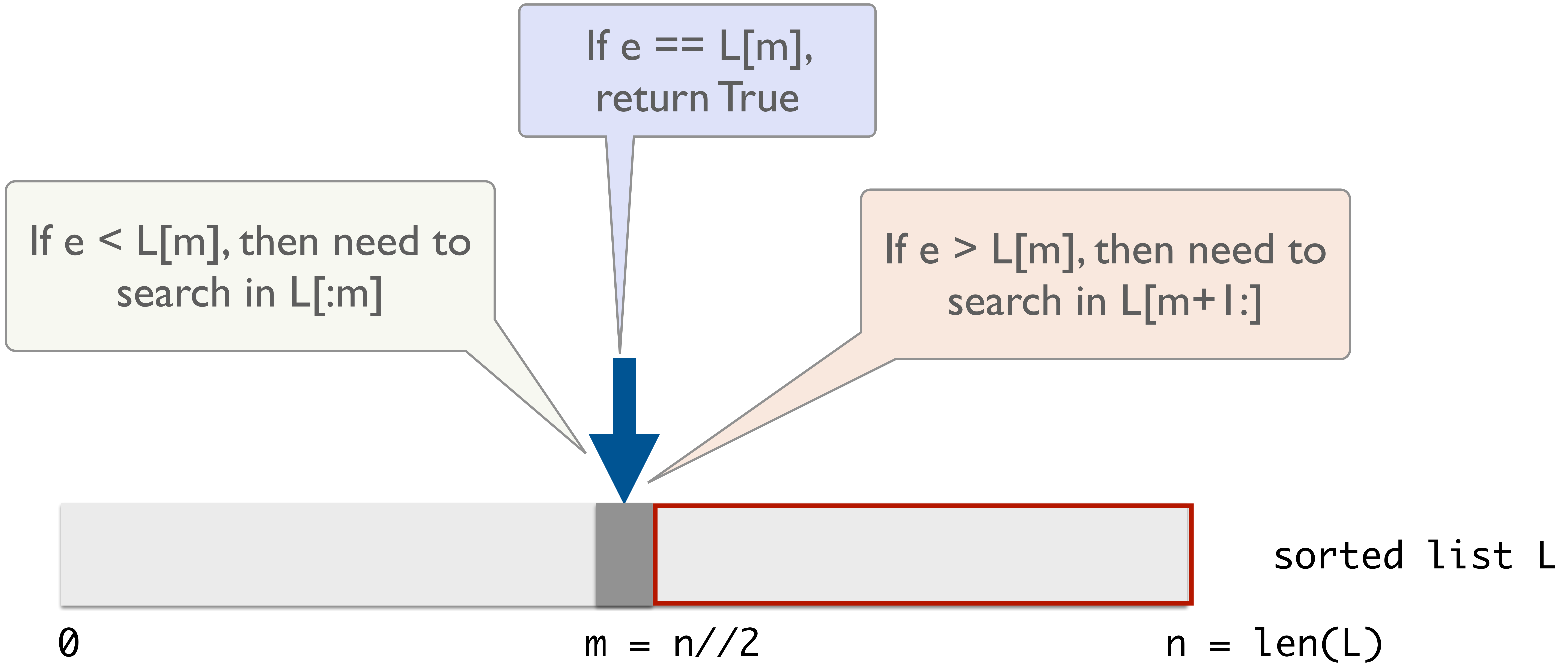
Binary Search Algorithm



Binary Search Algorithm

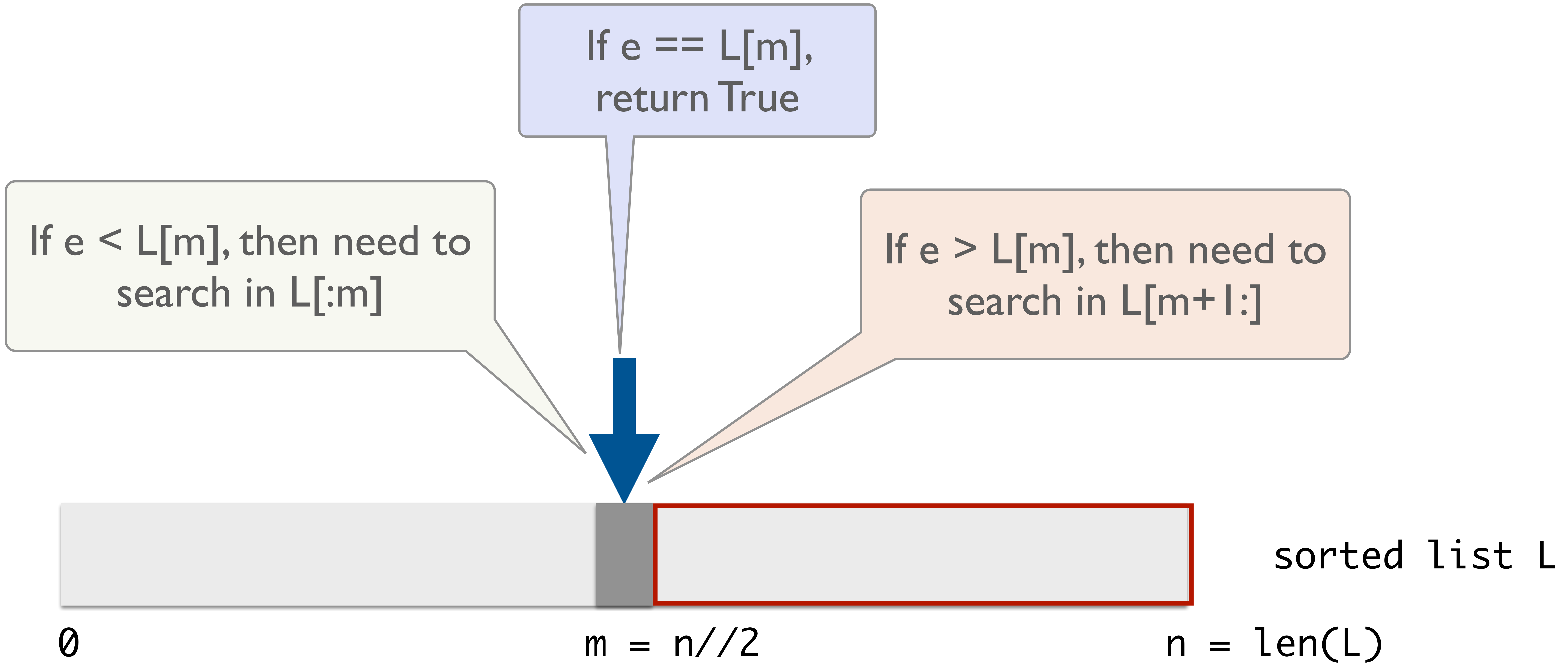


Binary Search Algorithm



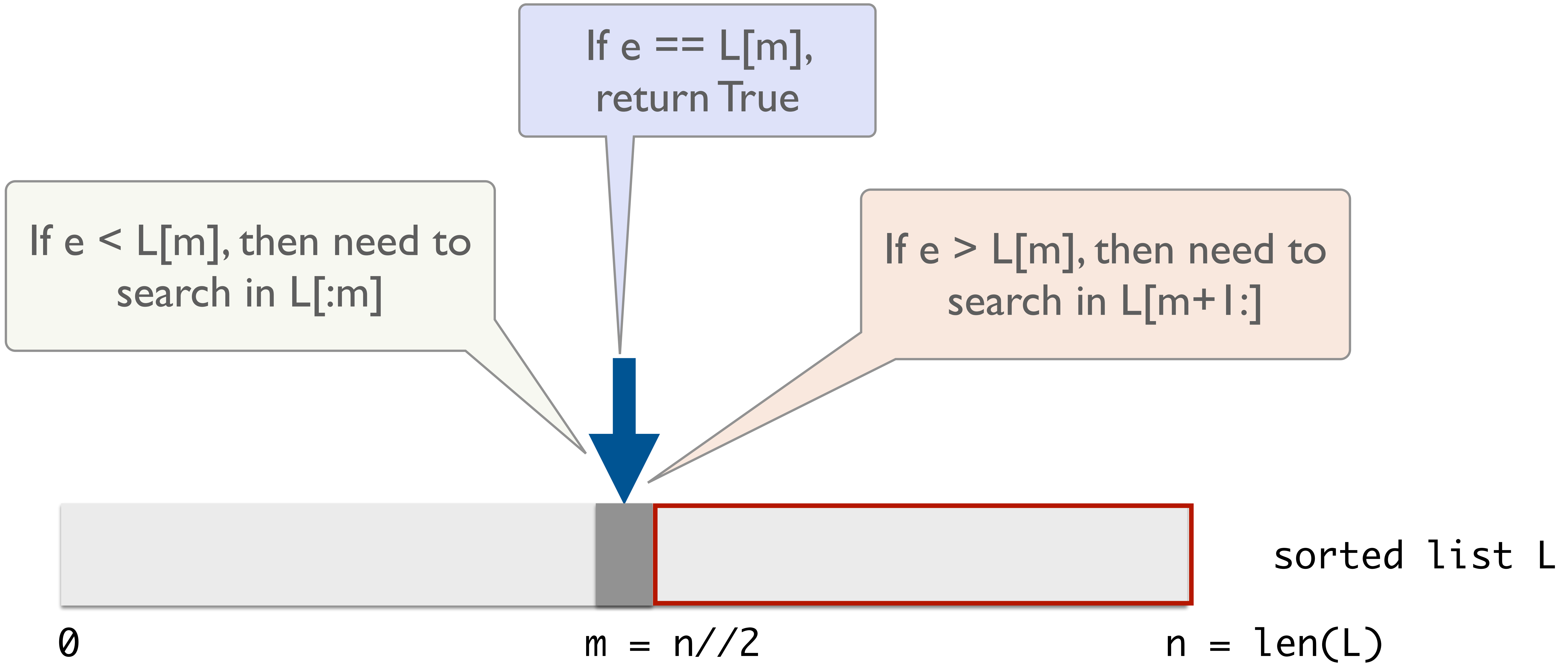
Binary Search Algorithm

- Base cases:
  - Positive: `if e == L[m]`, return `True`
  - Negative: `if len(L) == 0`, return `False`



Binary Search Code

- Let's **implement this algorithm**
- See Jupyter Notebook



Analysis of Binary Search

Binary Search Analysis

- Within a recursive call (function frame):
    - Constant number of steps (independent on n) and including at most one recursive call
- Total number of steps: O(# of recursive calls)

```python
def binarySearch(L, e):
    if len(L) == 0:
        return False
    else:  # recursive case
        if L[mid] == e:
            return True
        elif e <= L[mid]:
            return binarySearch(L[:mid], e)
        else:
            return binarySearch(L[mid+1:], e)
```

Binary Search Analysis

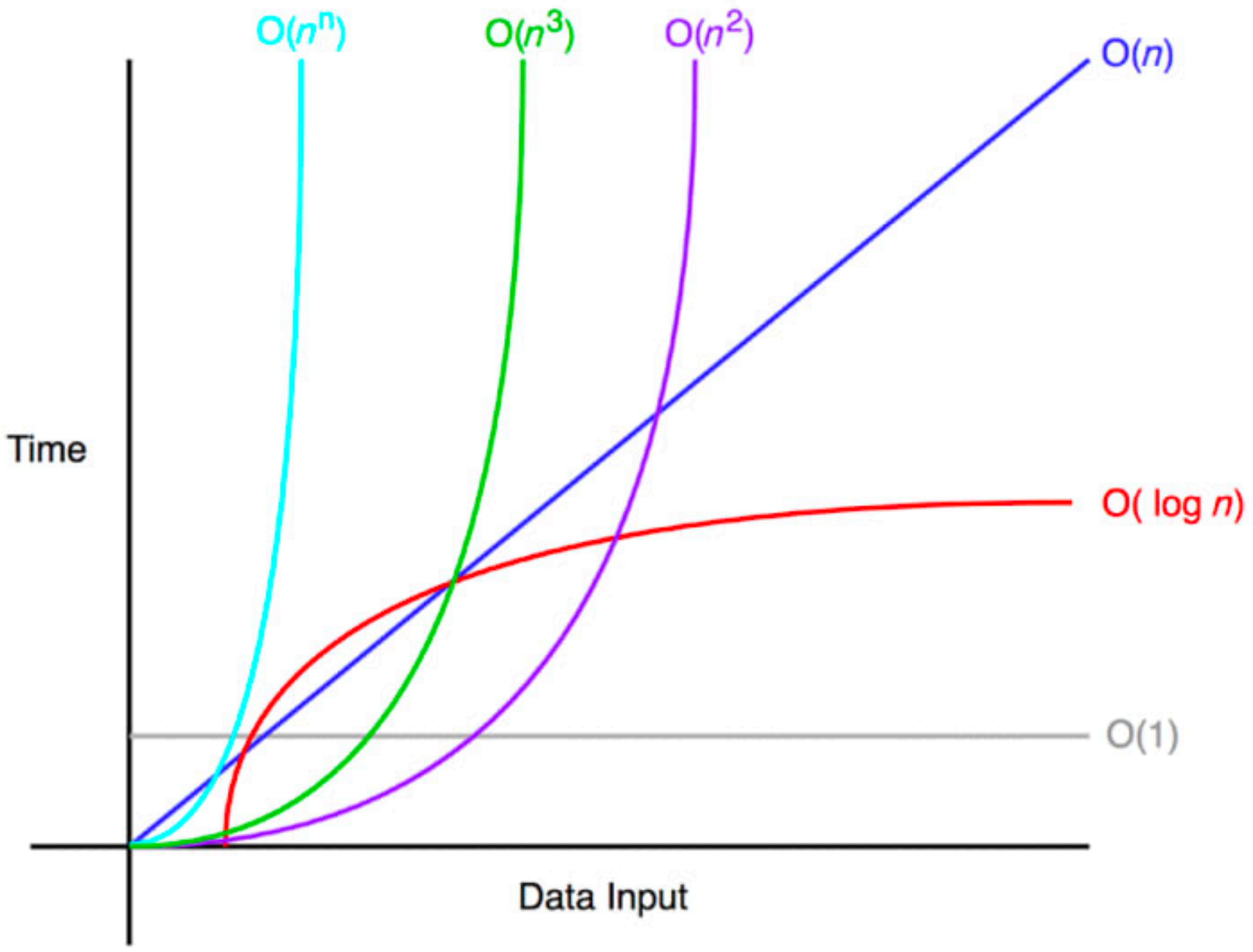
- How many recursive calls? How many timing can we halve L until either we find the element or L has size < 1
- Size goes down by half in each recursive call:

$$n \to n/2 \to n/4 \to n/8 \to \cdots \to n/2^i = 1$$

```python
def binarySearch(L, e):
    if len(L) == 0:
        return False
    else:   # recursive case
        if L[mid] == e:
            return True
        elif e <= L[mid]:
            return binarySearch(L[:mid], e)
        else:
            return binarySearch(L[mid+1:], e)
```

Binary Search Analysis

- Number of recursive calls at most $\log_2 n + 1$
- Overall binary search is a $O(\log n)$ algorithm
- Grows very slowly with respect to n !



$\log_2$ (1 billion) ~ 30

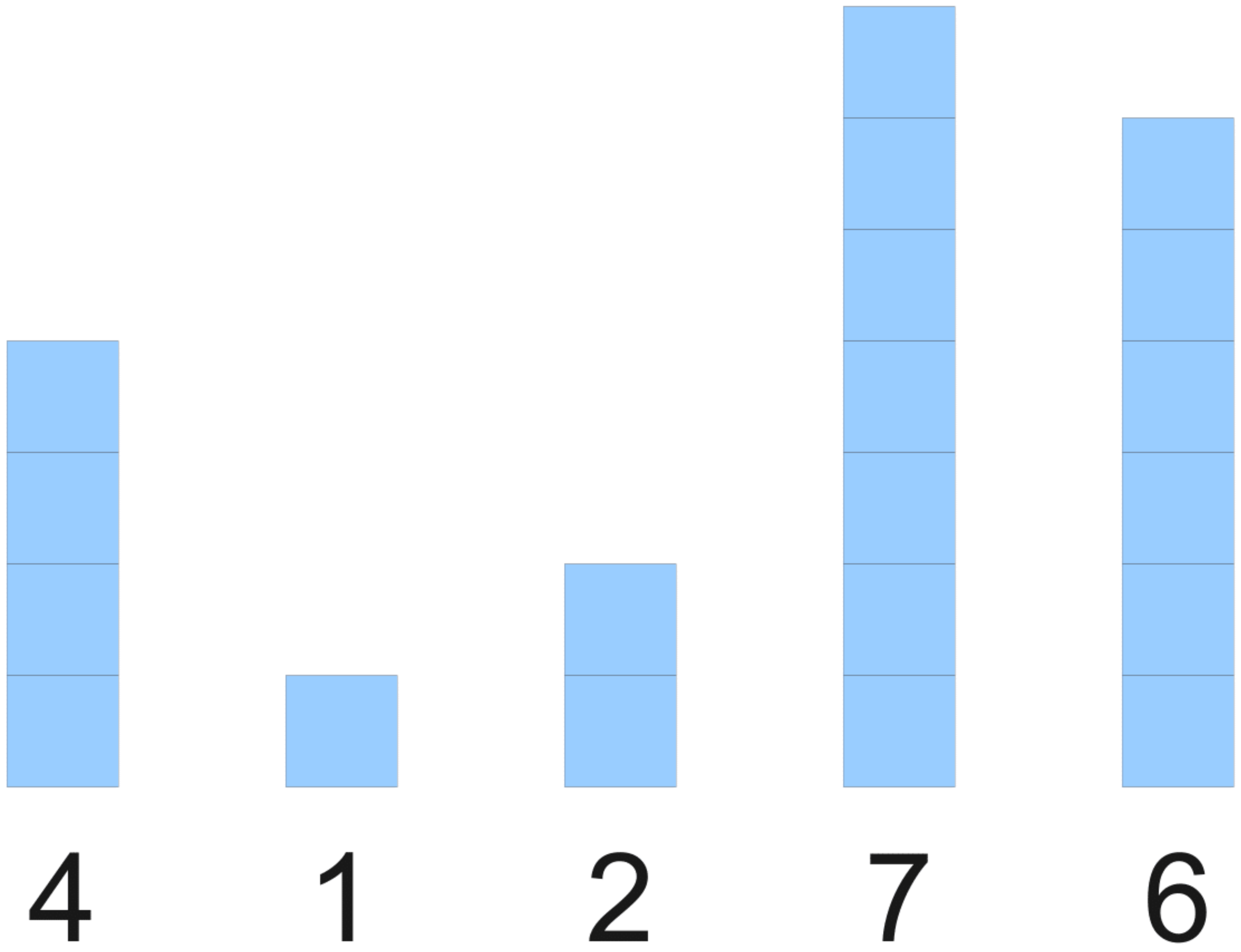
Sorting Algorithms

The Sorting Problem

- **Problem**. Given a list of elements, sort those elements in ascending order.
- There are many ways to solve this problem!
- Built-in sorting functions in Python
    - sorted: returns new sorted list
    - sort(): destructive sort that sorts the list its called on
- Today: how do we design our own sorting algorithm
- **Question.** What is the best way to sort n items?
- We will use Big Oh to find out!

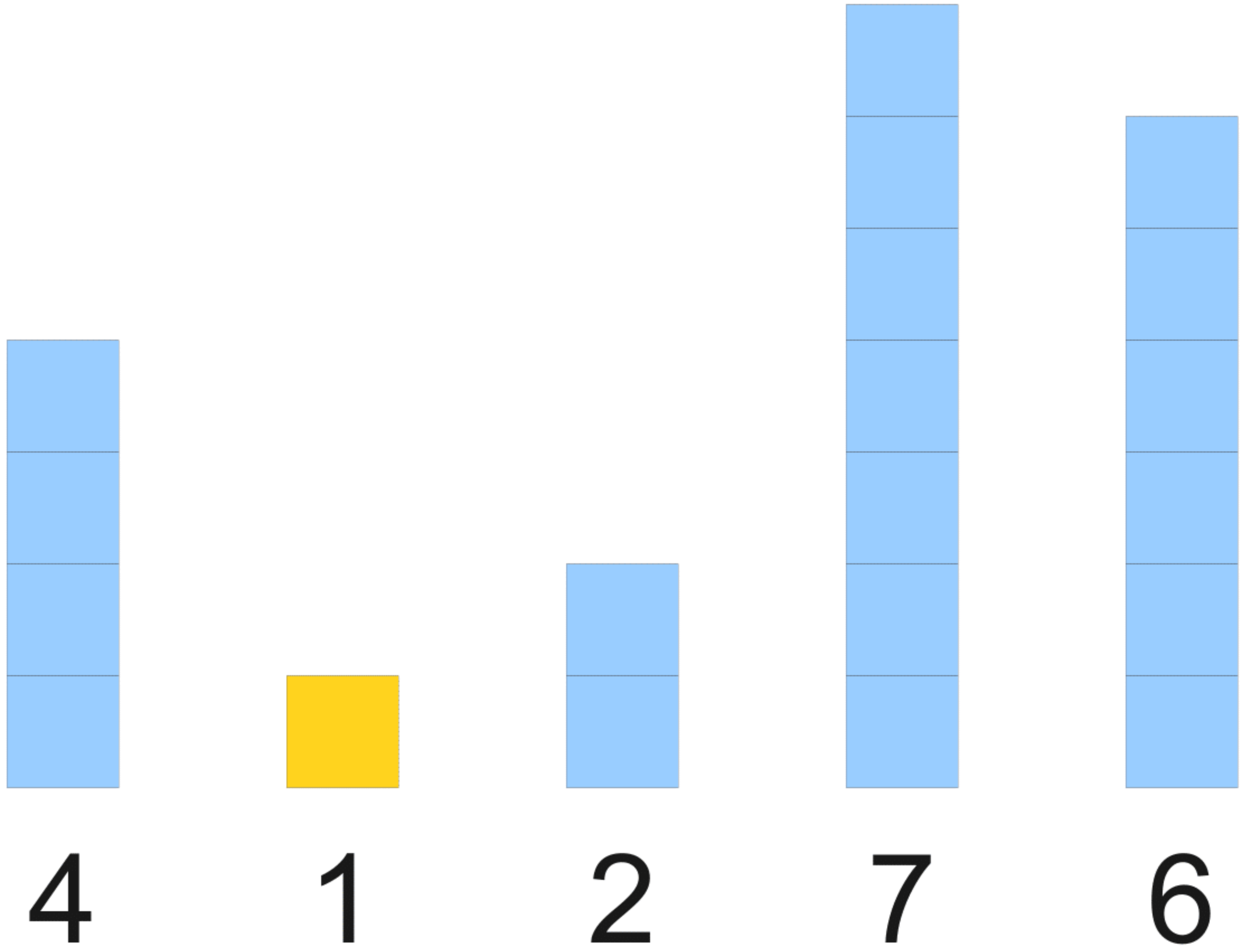
Selection Sort

- Find the smallest element and move it to the first position.
- Find the second-smallest element and move it to the second position, and so on



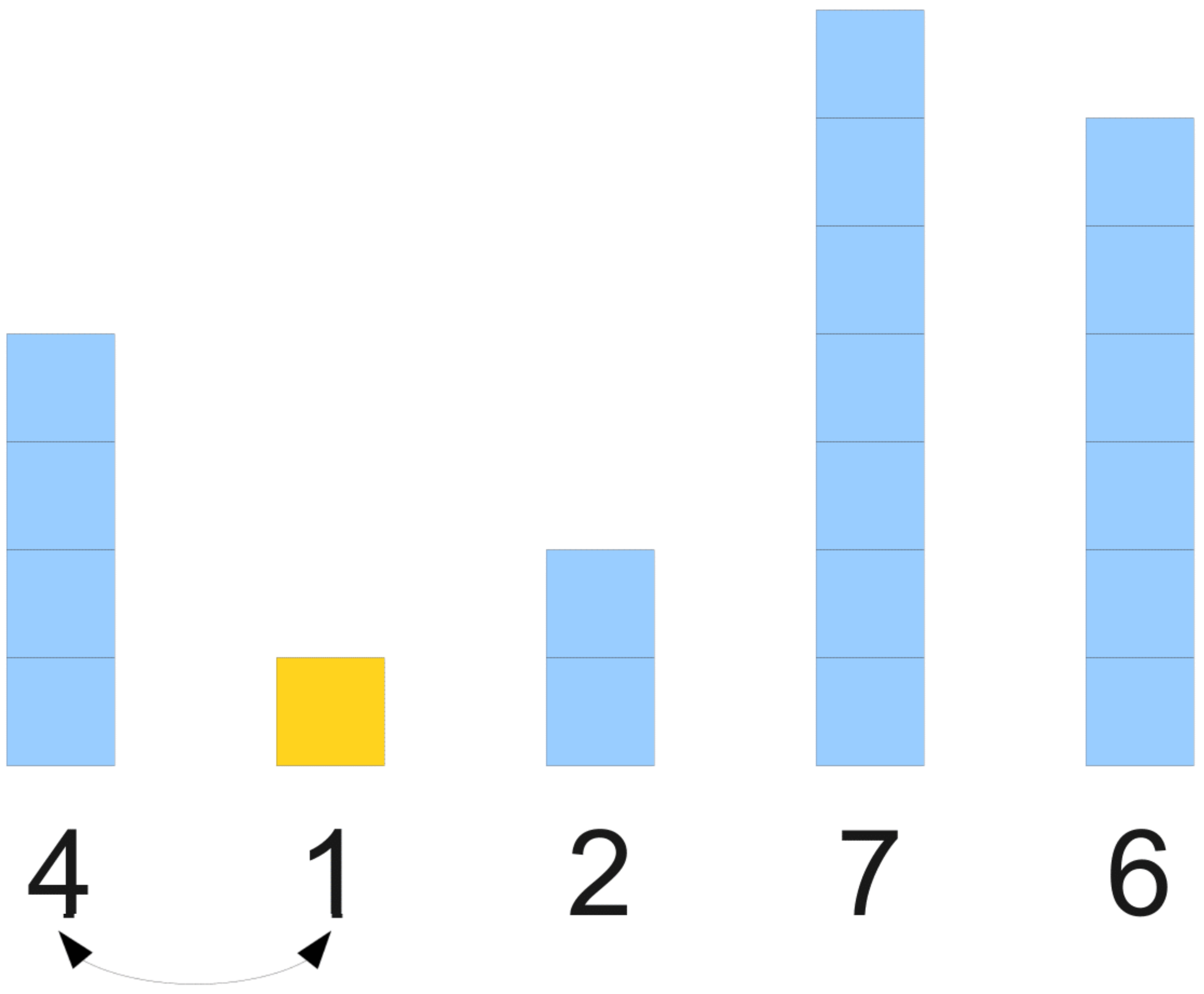
Selection Sort

- Find the smallest element and move it to the first position.
- Find the second-smallest element and move it to the second position, and so on



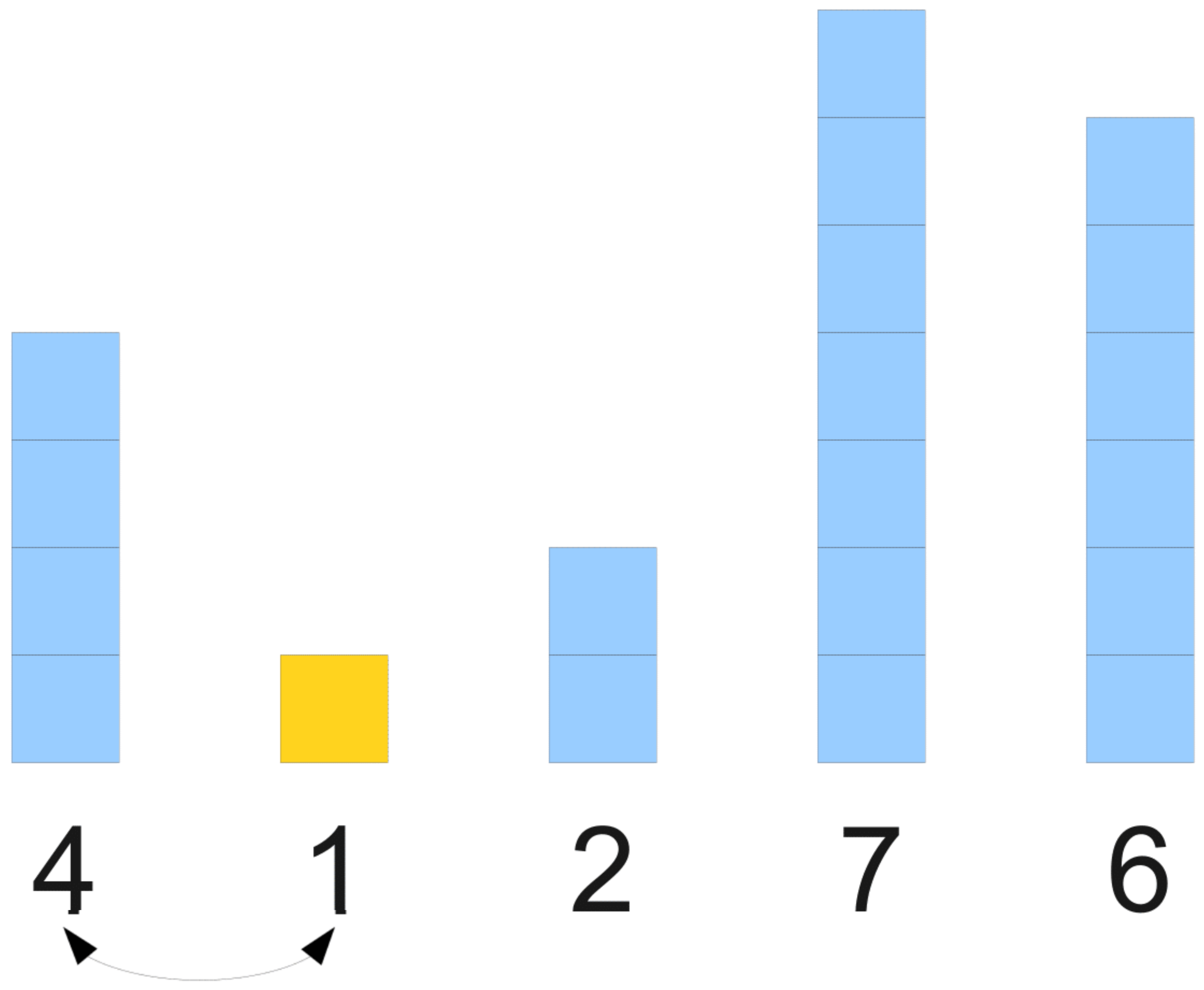
Selection Sort

- Find the smallest element and move it to the first position.
- Find the second-smallest element and move it to the second position, and so on



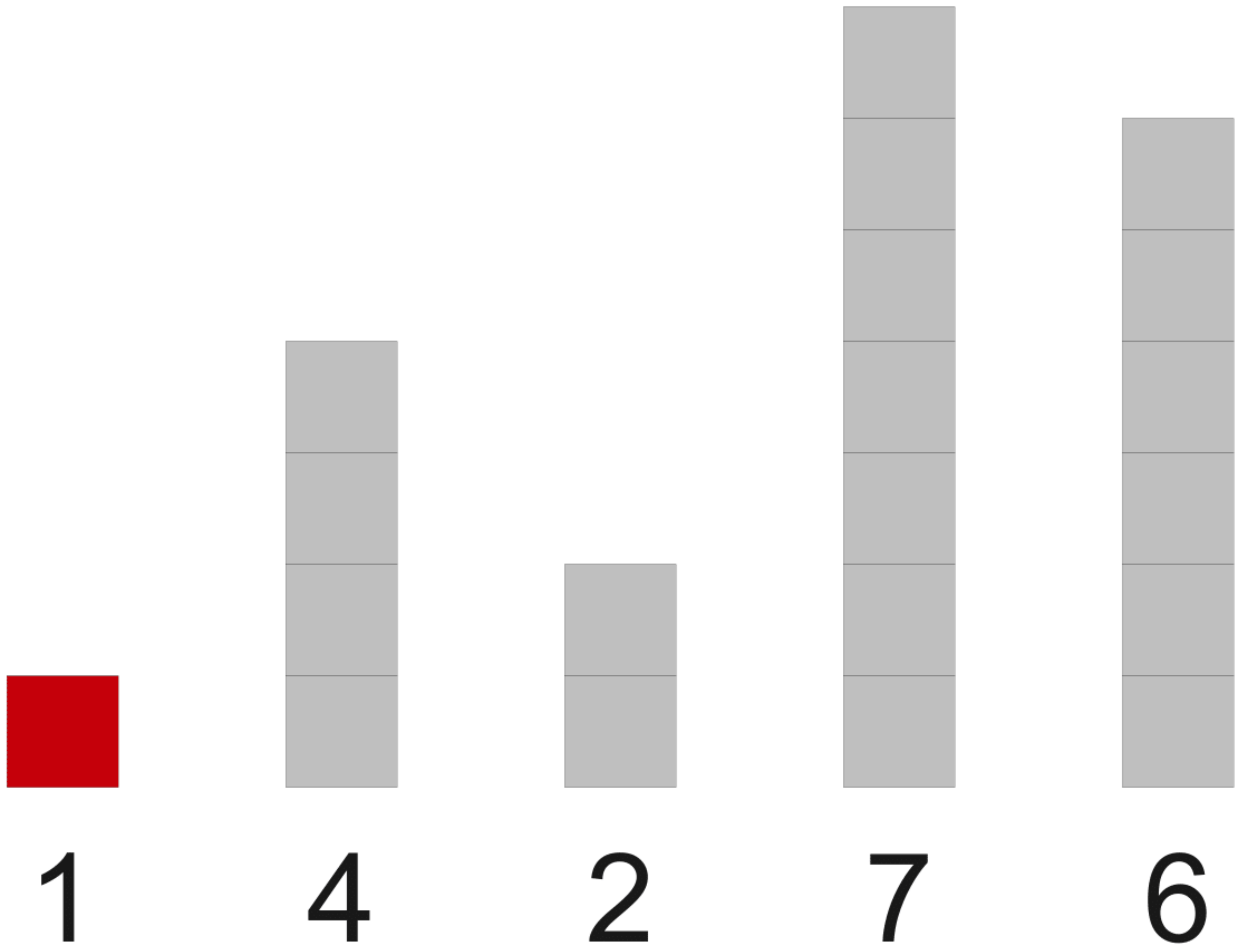
Selection Sort

- Find the smallest element and move it to the first position.
- Find the second-smallest element and move it to the second position, and so on



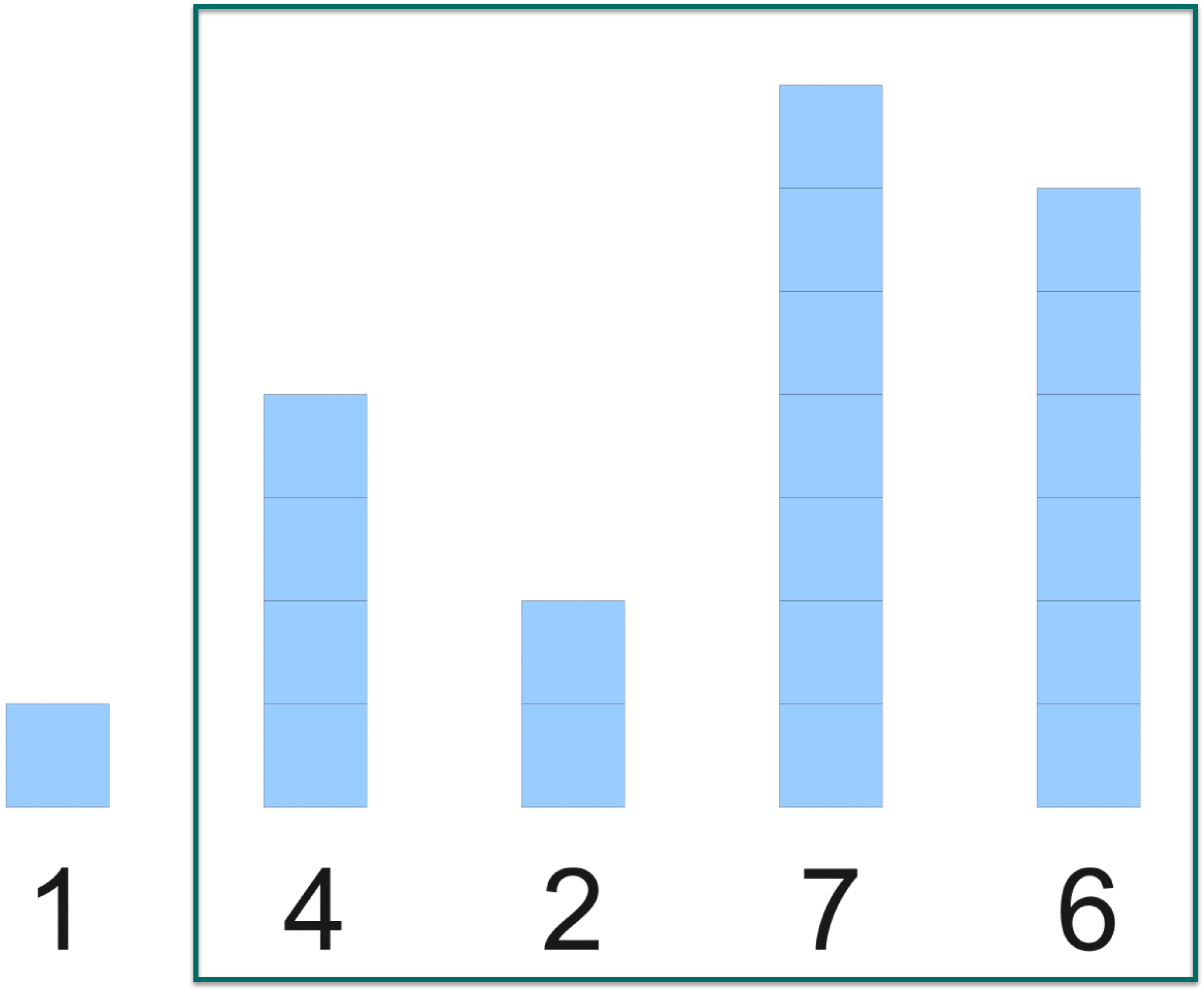
Selection Sort

- Find the smallest element and move it to the first position.
- Find the second-smallest element and move it to the second position, and so on



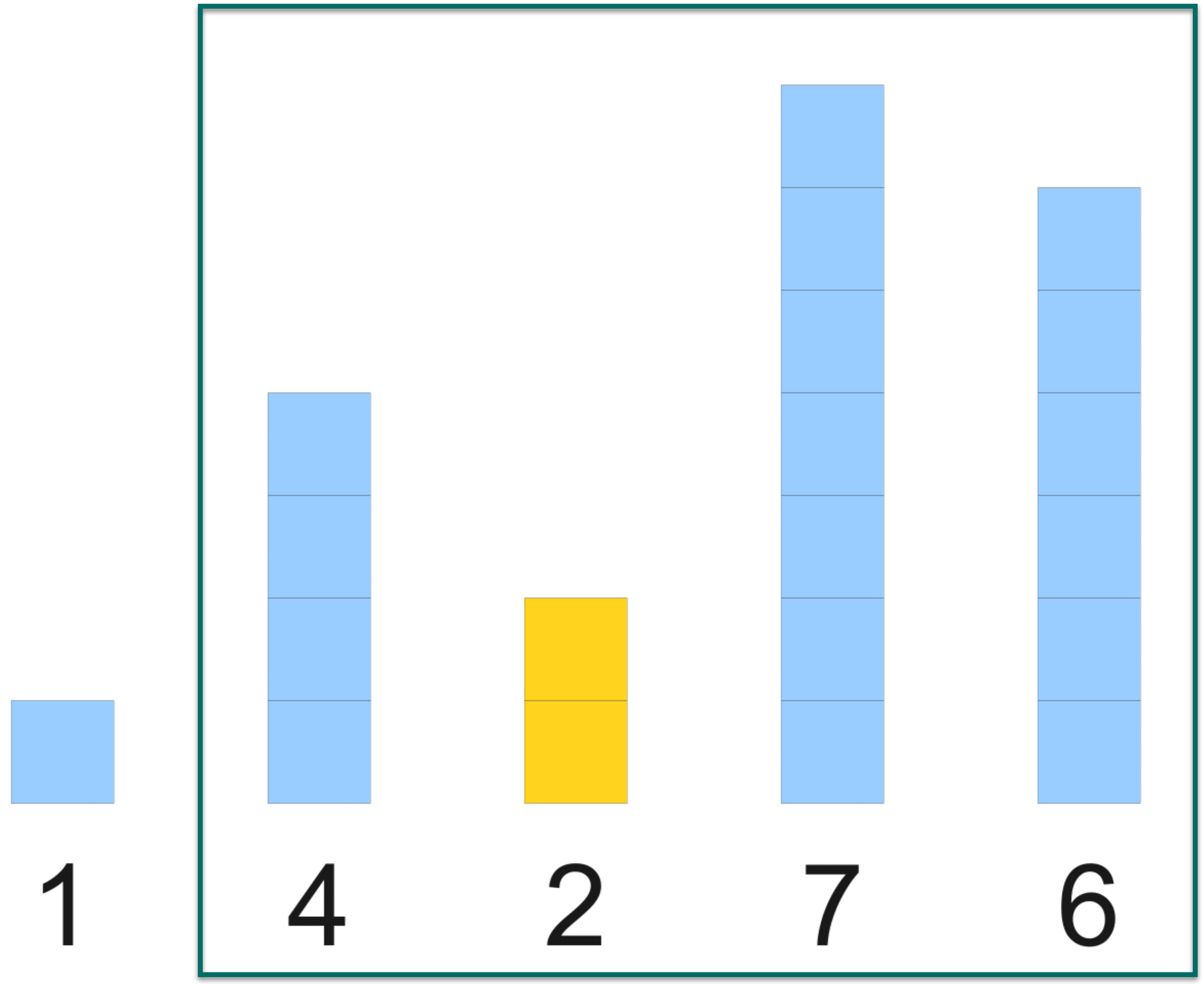
Selection Sort

- Find the smallest element and move it to the first position.
- Find the second-smallest element and move it to the second position, and so on



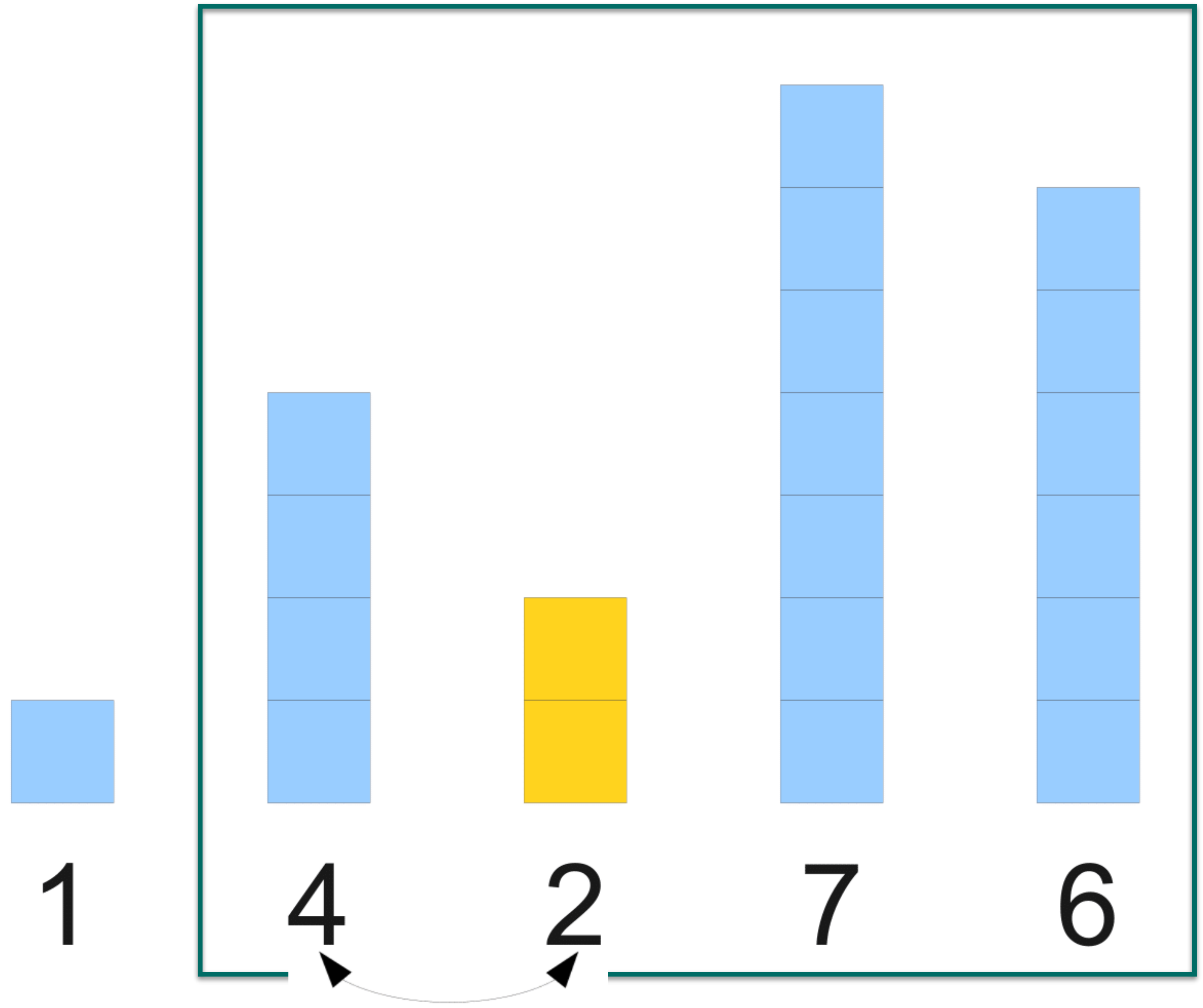
Selection Sort

- Find the smallest element and move it to the first position.
- Find the second-smallest element and move it to the second position, and so on



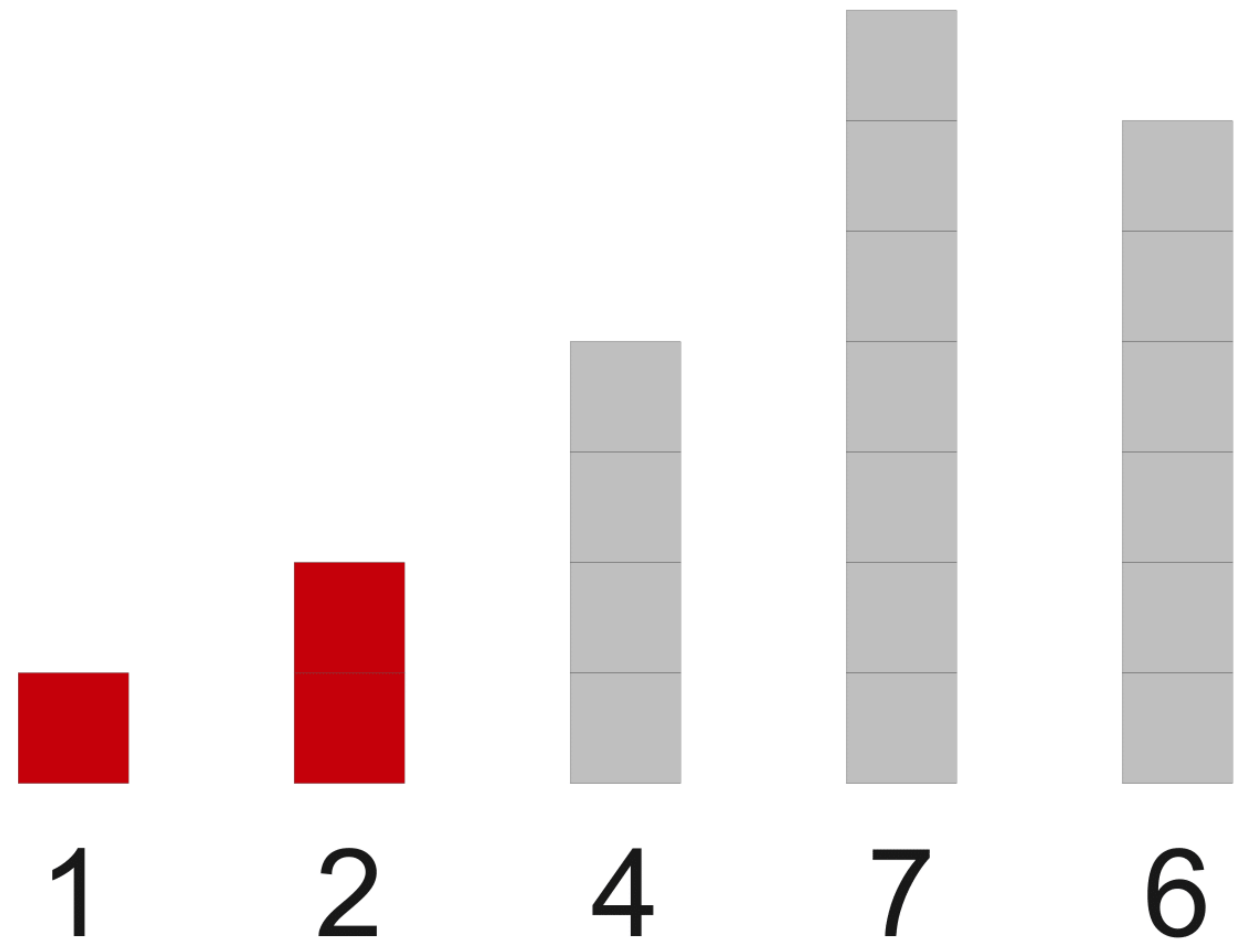
Selection Sort

- Find the smallest element and move it to the first position.
- Find the second-smallest element and move it to the second position, and so on



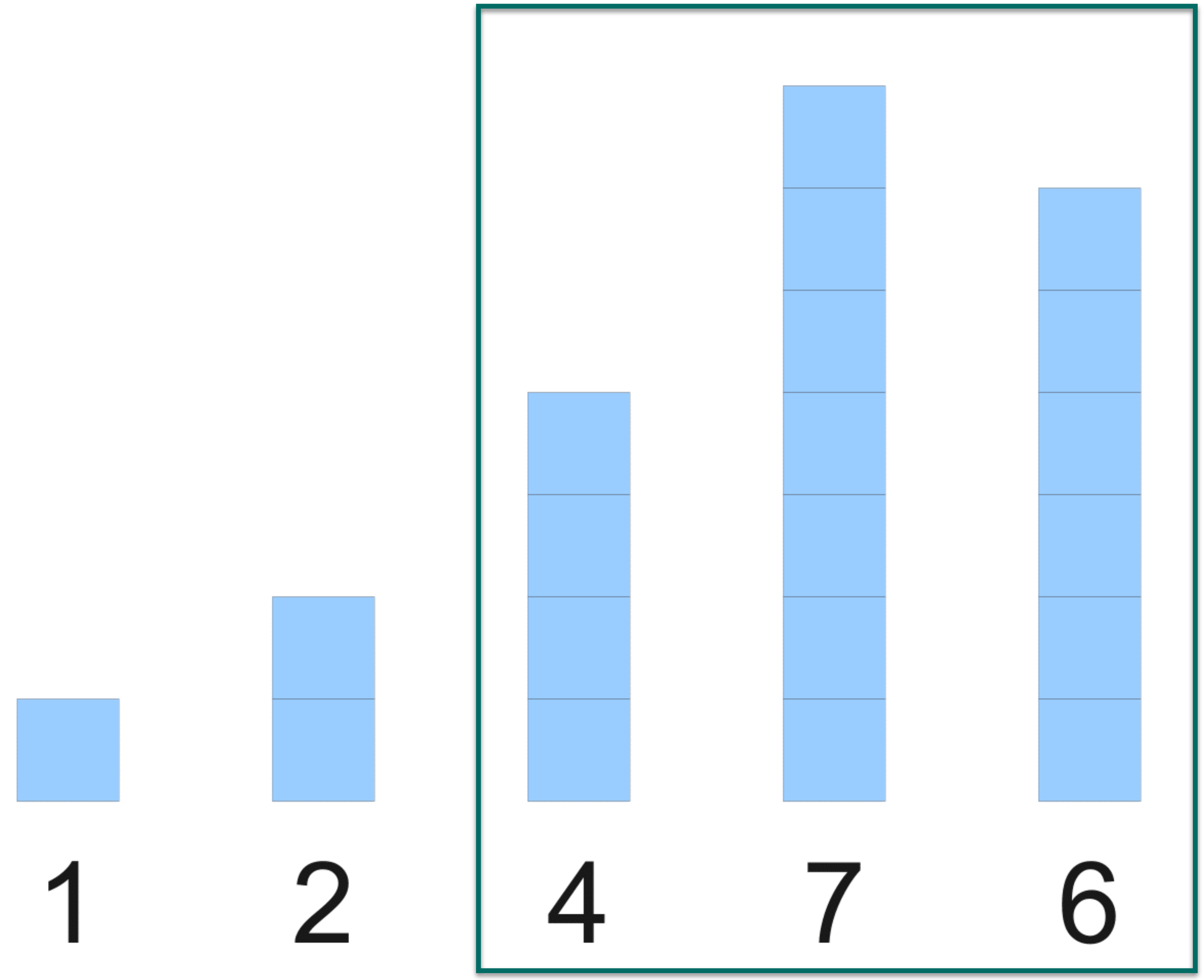
Selection Sort

- Find the smallest element and move it to the first position.
- Find the second-smallest element and move it to the second position, and so on



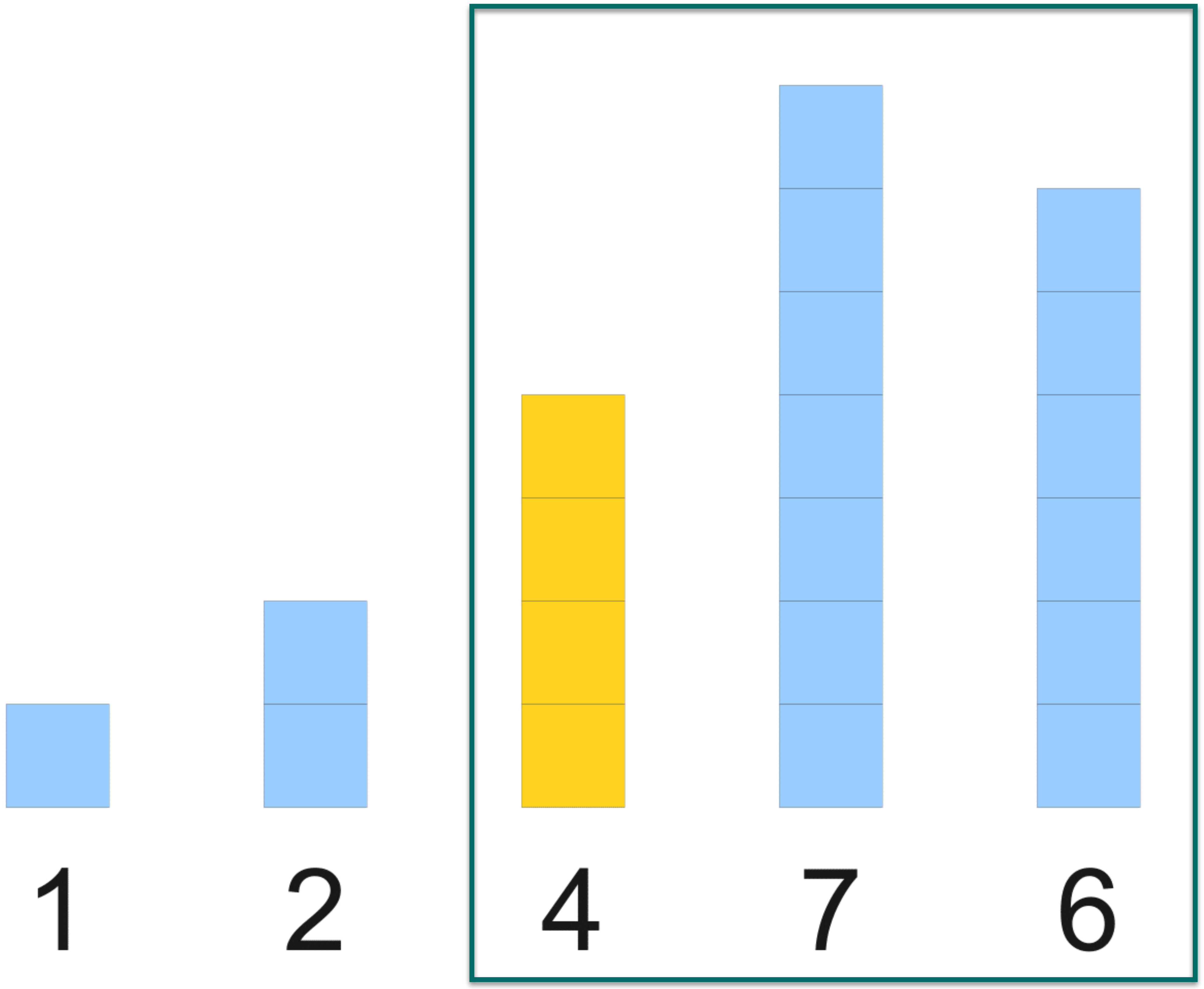
Selection Sort

- Find the smallest element and move it to the first position.
- Find the second-smallest element and move it to the second position, and so on



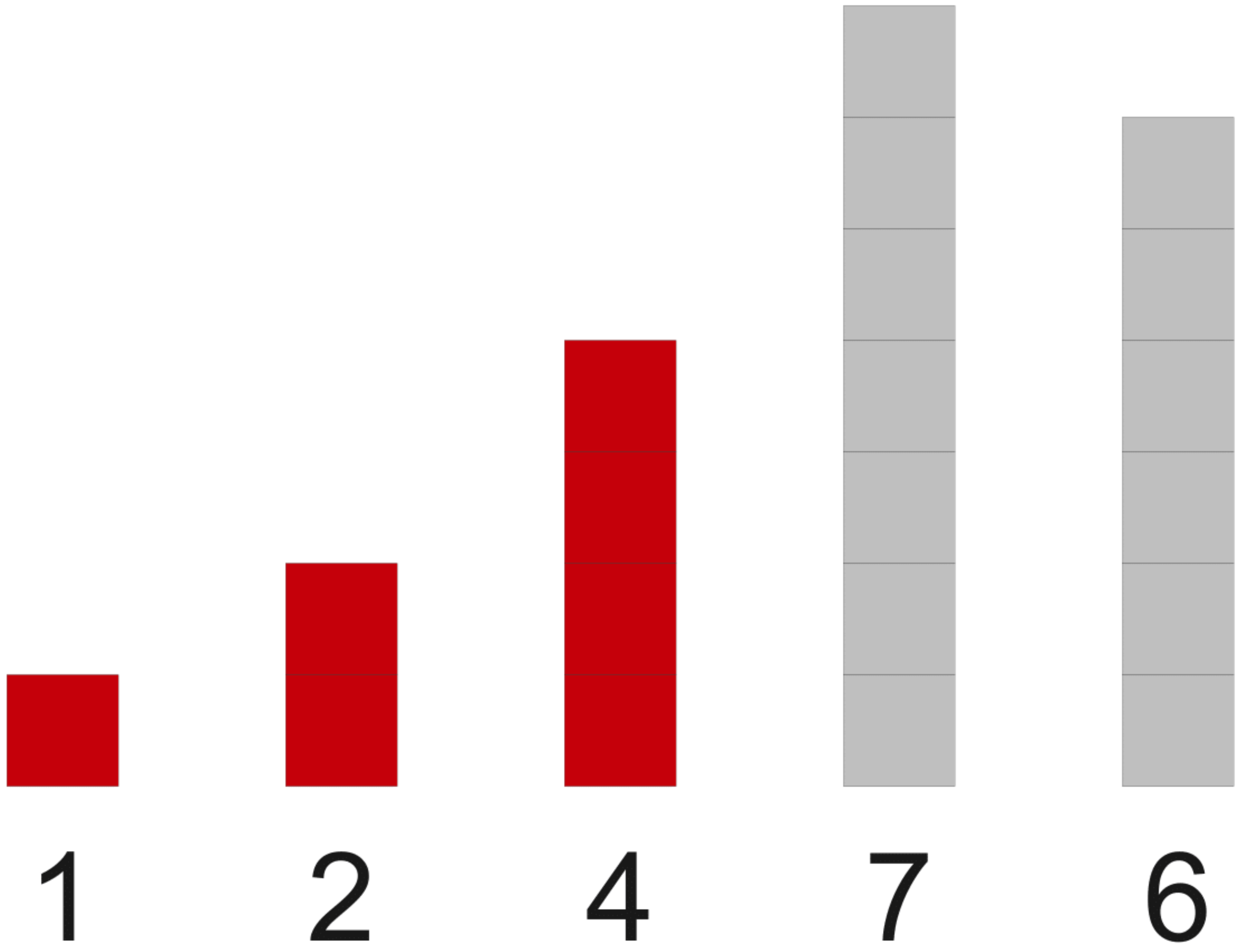
Selection Sort

- Find the smallest element and move it to the first position.
- Find the second-smallest element and move it to the second position, and so on



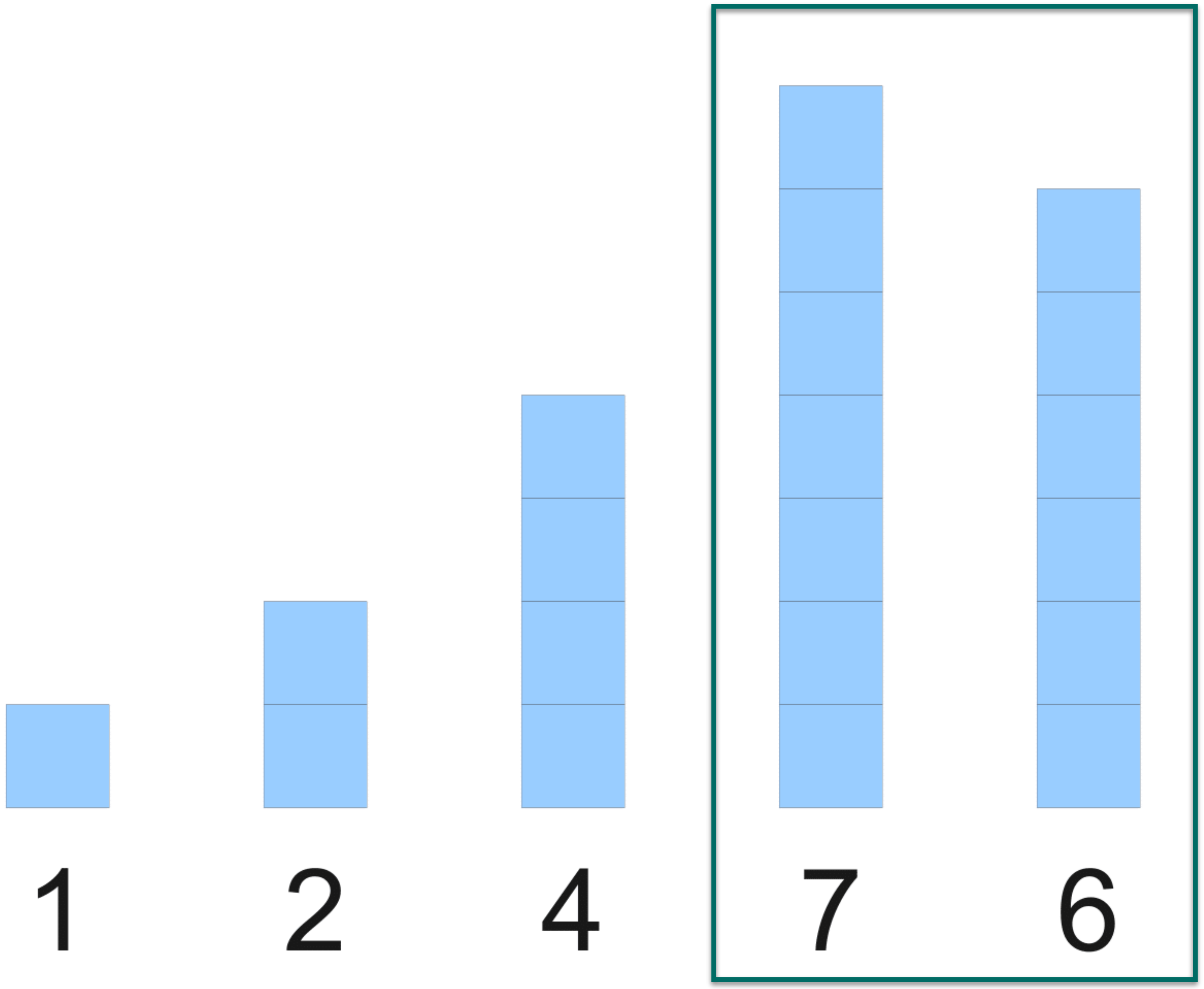
Selection Sort

- Find the smallest element and move it to the first position.
- Find the second-smallest element and move it to the second position, and so on



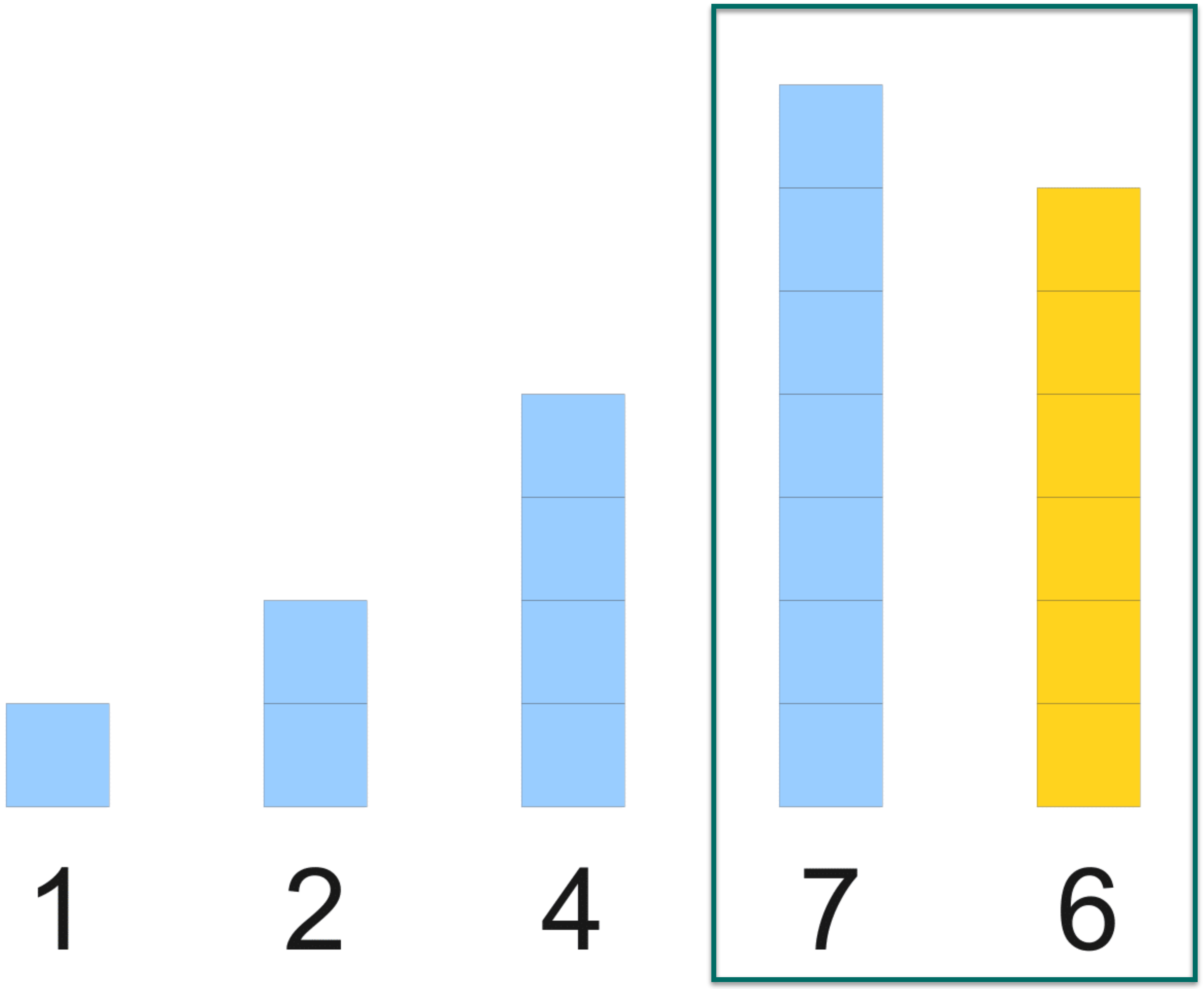
Selection Sort

- Find the smallest element and move it to the first position.
- Find the second-smallest element and move it to the second position, and so on



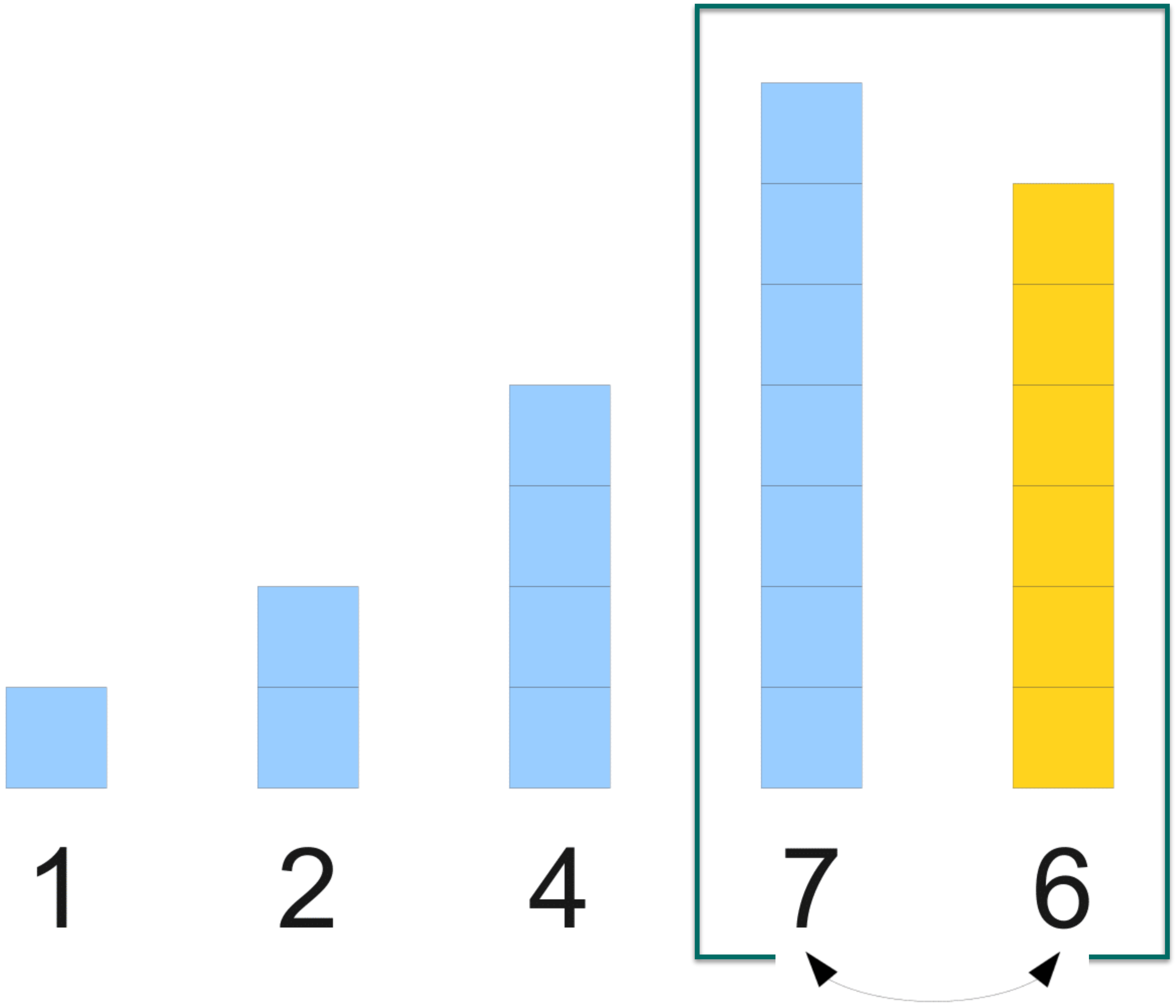
Selection Sort

- Find the smallest element and move it to the first position.
- Find the second-smallest element and move it to the second position, and so on



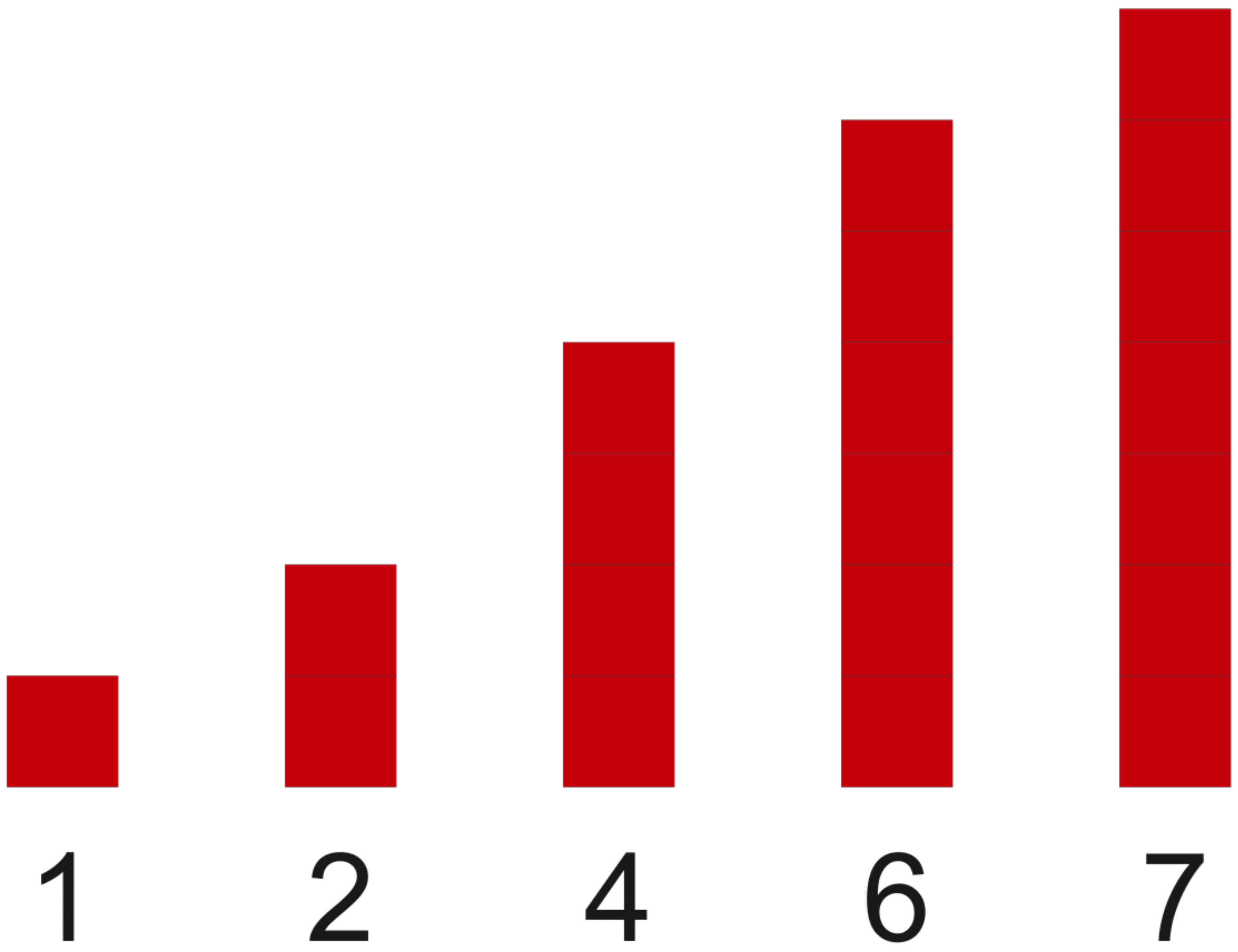
Selection Sort

- Find the smallest element and move it to the first position.
- Find the second-smallest element and move it to the second position, and so on



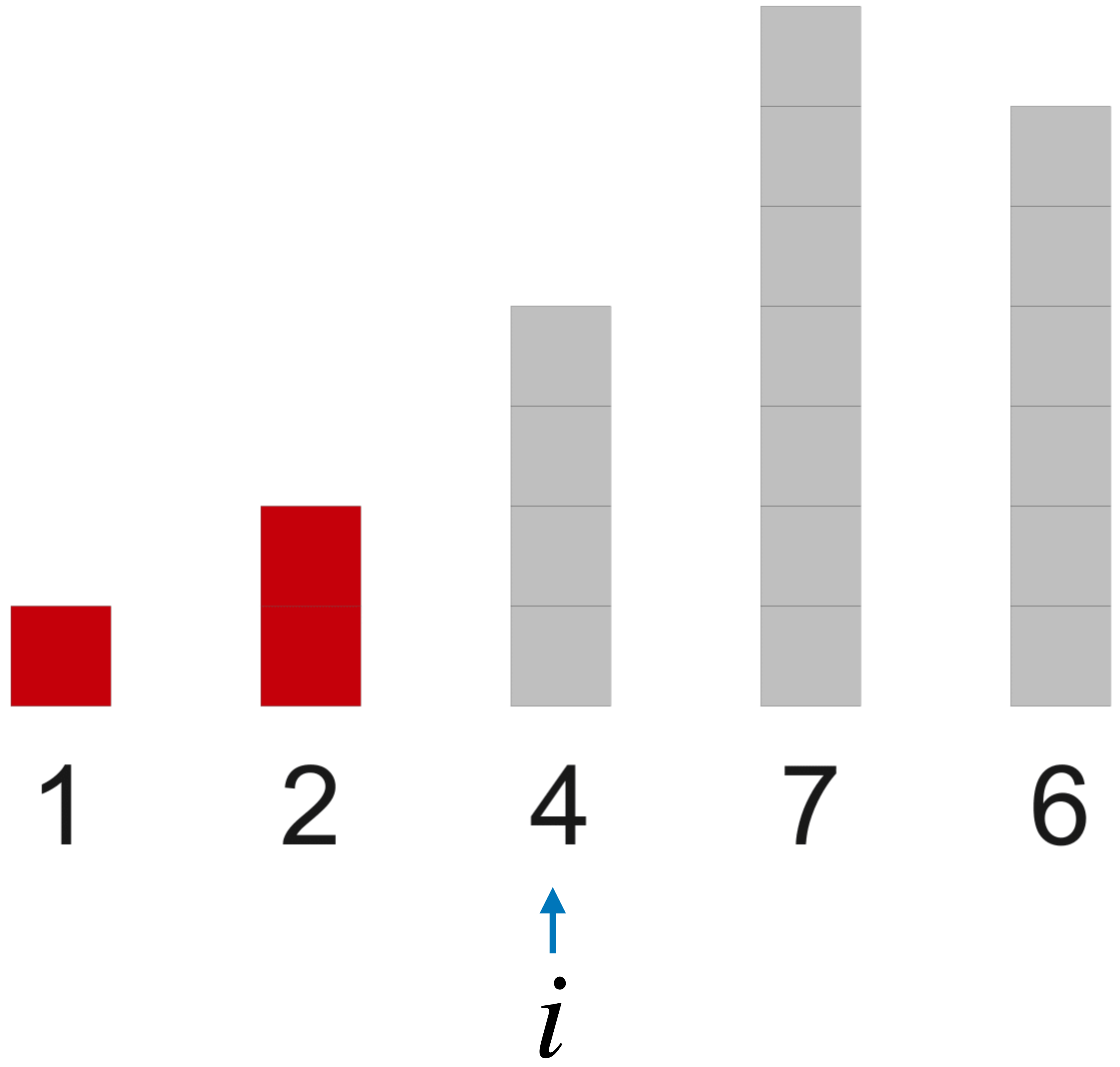
Selection Sort

- Find the smallest element and move it to the first position.
- Find the second-smallest element and move it to the second position, and so on



Selection Sort

- For each index i in the list `L`, we need to find the min item in `L[i+1:]`, if its smaller than `L[i]`, swap `L[i]` with that item



Selection Sort

- For each index i in the list `L`, we need to find the min item in `L[i+1:]` so we can replace `L[i]` with that item
- In fact we need to find the position `minPosition` of the item that is minimum in `L[i+1:]`
- Reminder: how to swap values of variables `a` and `b`?
    - Using tuple assignment in Python:
      `a, b = b, a`
- Let's **implement this algorithm**

Selection Sort: Analysis

Selection Sort Analysis

- For $i = 0$, inner loop runs $n - 1$ items
- For $i = 1$, inner loop runs $n - 2$ times
- ...
- For $i = n - 1$, inner loop runs 0 times

```python
def selectionSort(L):
    """Destructive sort of list L,
    returns sorted list."""
    n = len(L)
    for i in range(n):
        minPosition = i
        for j in range(i+1, n):
            if L[minPosition] > L[j]:
                minPosition = j
        L[i], L[minPosition] = L[minPosition], L[i]
    return L
```

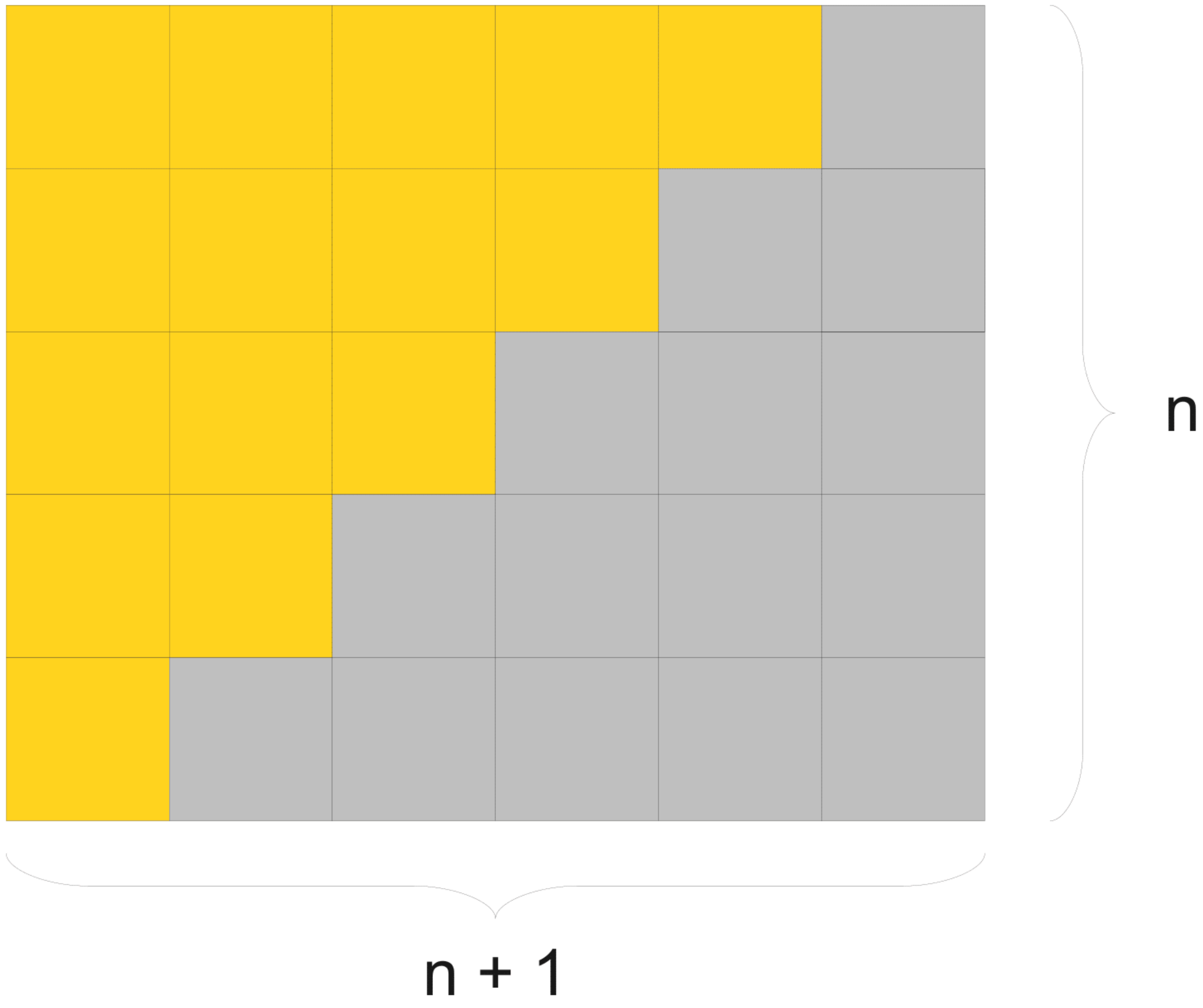
Selection Sort Analysis

- Within the inner loop we have $O(1)$ steps (constant)
- Overall number of steps $(n-1)+(n-2)+\cdots+0$
  $\leq n+(n-1)+(n-2)+\cdots+1$
- What is this sum?

```python
def selectionSort(L):
    """Destructive sort of list L,
    returns sorted list."""
    n = len(L)
    for i in range(n):
        minPosition = i
        for j in range(i+1, n):
            if L[minPosition] > L[j]:
                minPosition = j
        L[i], L[minPosition] = L[minPosition], L[i]
    return L
```

Selection Sort Analysis

$n + (n-1) + \ldots + 2 + 1 = n(n+1) / 2$



Guassian Summation

$$S = n + (n-1) + (n-2) + \cdots + 2 + 1$$

$$+ \quad S = 1 + 2 + \cdots + (n-2) + (n-1) + n$$

$$2S = (n+1) + (n+1) + \cdots + (n+1) + (n+1) + (n+1)$$

$$2S = (n+1) \cdot n$$

$$S = (n+1) \cdot n \cdot 1/2$$

Selection Sort Analysis

- Total number of steps

$$O(n(n+1)/2)$$

$$= O(n(n+1))$$

$$= O(n^2 + n)$$

$$= O(n^2)$$

- Is this the best we can do for sorting?
- No! Can sort in $O(n \log n)$ time !
- Next lecture: merge sort, a faster recursive sorting algorithm that is optimal

Acknowledgments

These slides have been adapted from:

- http://cs111.wellesley.edu/spring19 and
- https://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-0001-introduction-to-computer-science-and-programming-in-python-fall-2016/
- Selection sort images from: https://web.stanford.edu/class/archive/cs/cs106b/cs106b.1126/lectures/11/Slides11.pdf